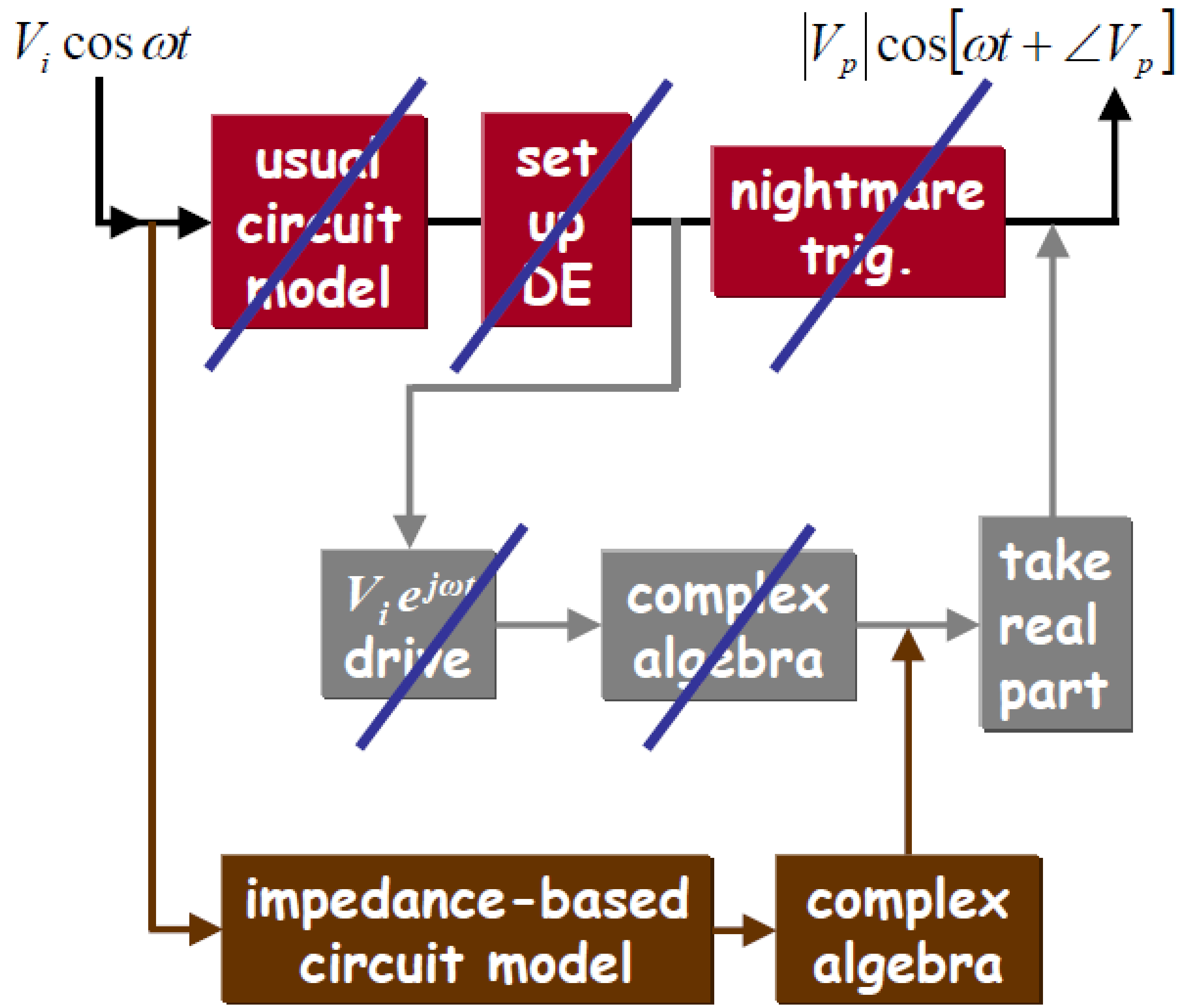
ECE 201, Section 3
Lecture 33

Prof. Peter Bermel

November 12, 2012

Second Order Circuit Overview



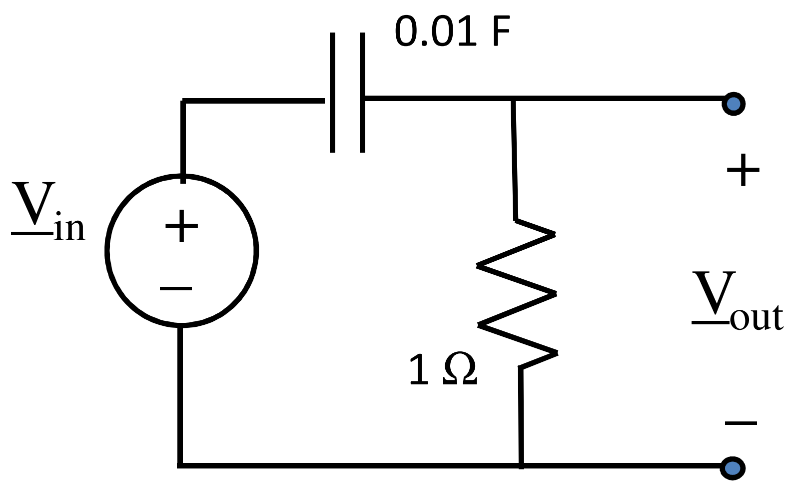
Adapted from A. Agarwal & J. Lang, Course Materials for MIT 6.002, Spring 2007

Frequency Response

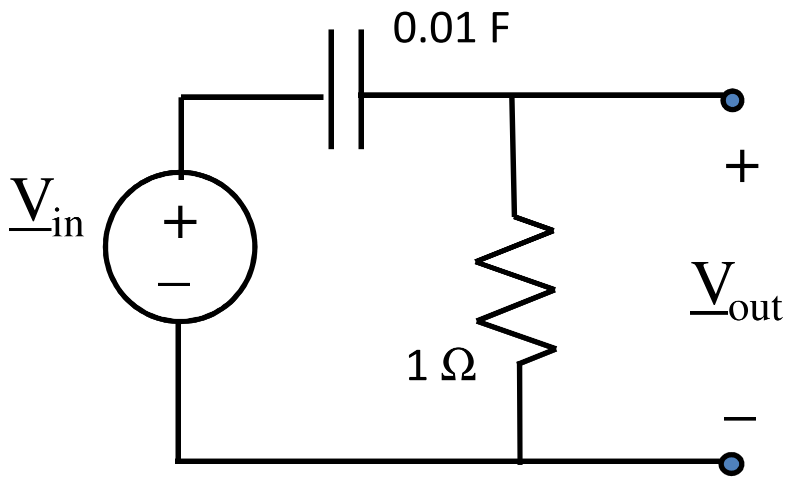
- Frequency response: the ratio of phasor output to phasor input as a function of frequency
- Consists of two plots:
  - Magnitude of phasor ratio: $\left|\frac{V_p}{V_i}\right|$
  - Phase of the phasor ratio: $\angle V_p - \angle V_i$

Example #1

- Plot the frequency response of the circuit below:



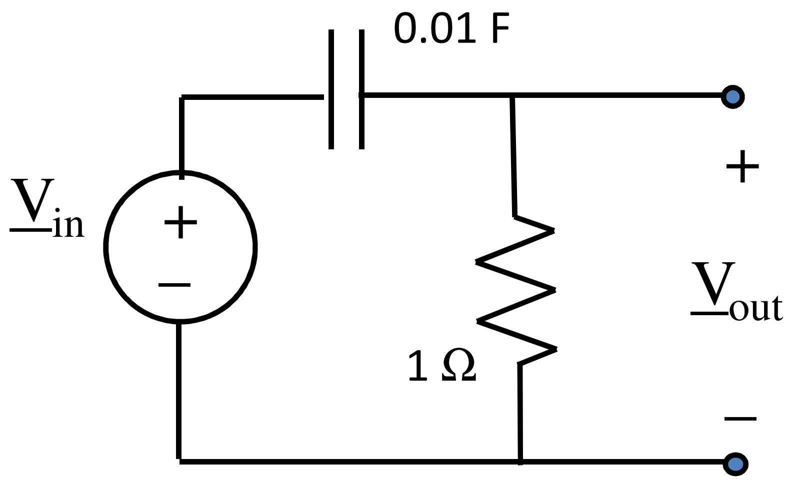
Solution



Step 1: Find impedance of the capacitor and resistor pair.

$$Z_{in} = R + (1/j\omega C) = 1 + (1/j0.01\omega)$$

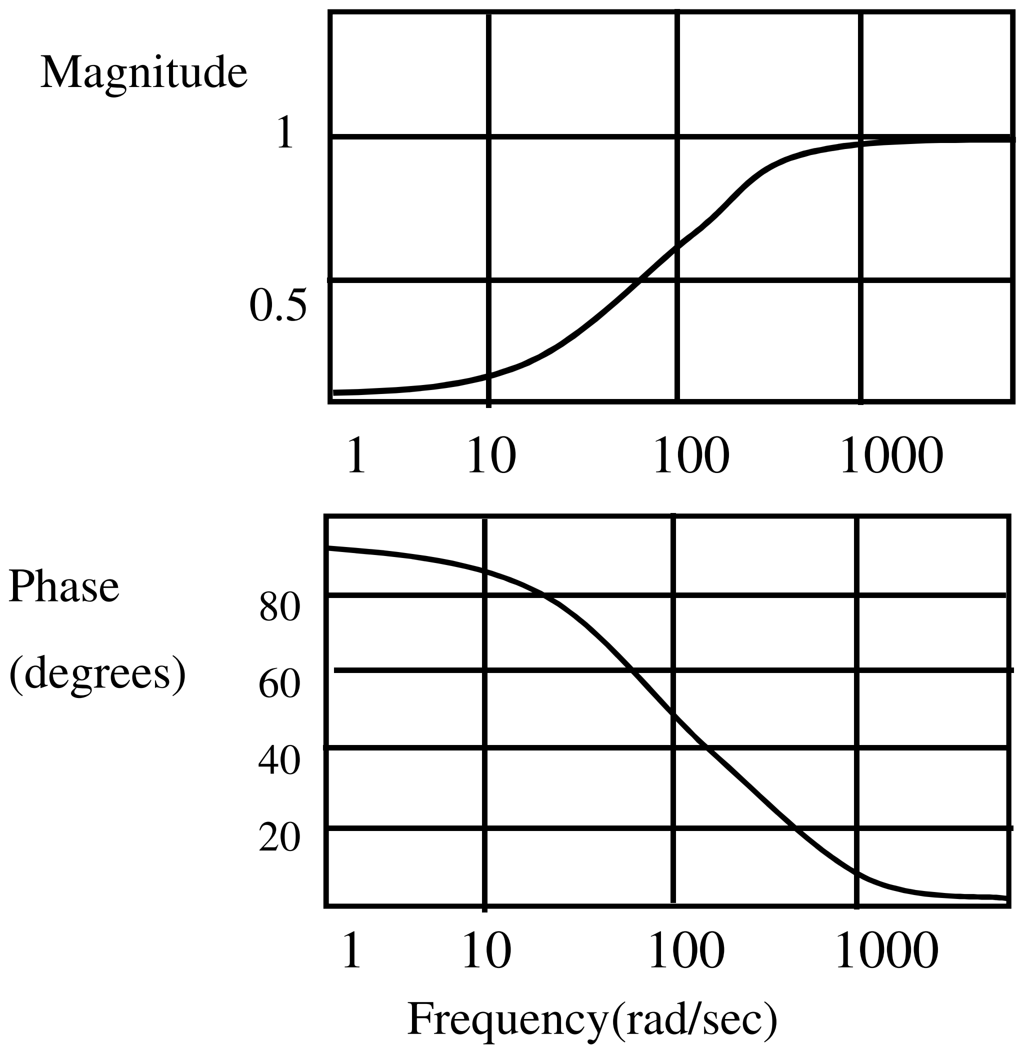
Solution

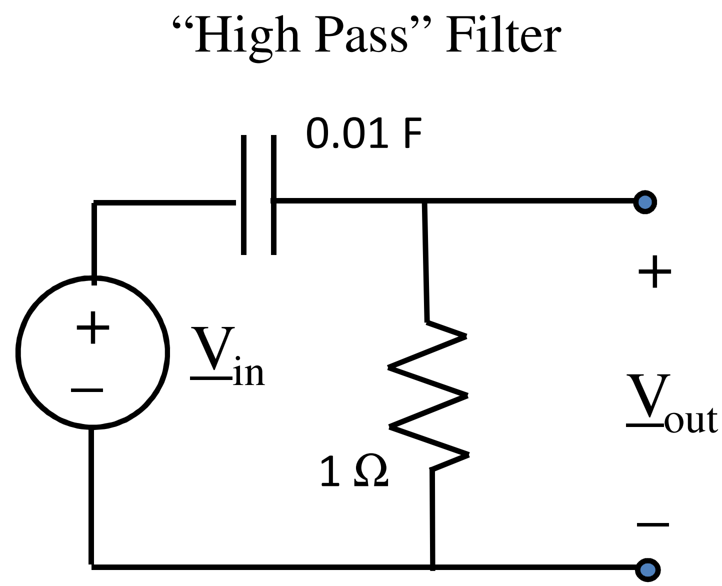


Step 2: Use voltage division to establish output to input ratio.

$$\frac{\underline{V}_{out}}{\underline{V}_{in}} = \frac{1}{1+(1/j0.01\omega)} = \frac{j0.01\omega}{1+j0.01\omega} = H(j\omega)$$

Frequency Response





Two plots, one for magnitude and one for phase angle, are needed because H(jω) is a complex quantity.

Limiting Frequency Responses

Important frequencies for RC circuits.

$\omega = 0$ and $\omega = \infty$

$H(j0) = 0 \angle 90^o$

$H(j\infty) = 1 \angle 0^o$

For magnitude:

$|H(j\omega)| \longrightarrow 0$ as $\omega \longrightarrow 0$

$|H(j\omega)| \longrightarrow 1$ as $\omega \longrightarrow \infty$

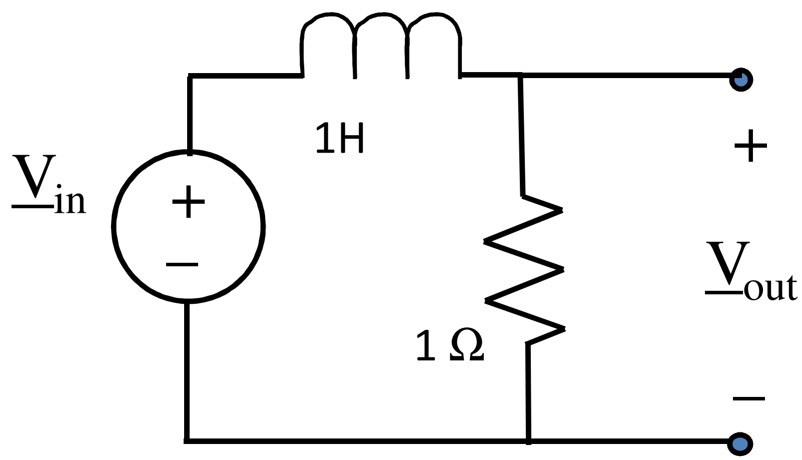
For phase:

$\angle H(j\omega) \longrightarrow 0^o$ as $\omega \longrightarrow \infty$

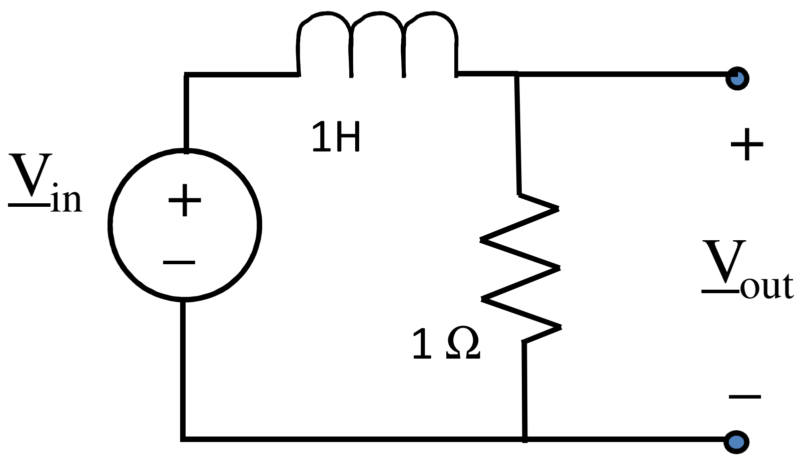
$\angle H(j\omega) \longrightarrow 90^o$ as $\omega \longrightarrow 0$

Example #2

Plot the frequency response for the circuit below.



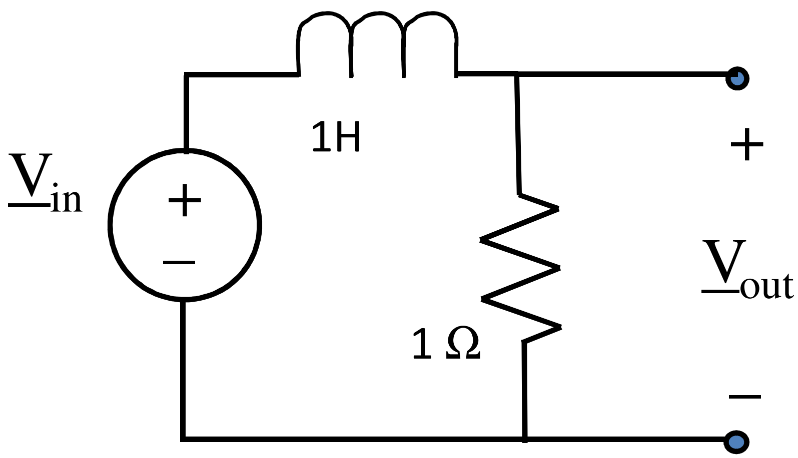
Solution



Step 1: Find the impedance of inductor and resistor pair.

$$Z_{in} = R + (j\omega L) = 1 + (j\omega)$$

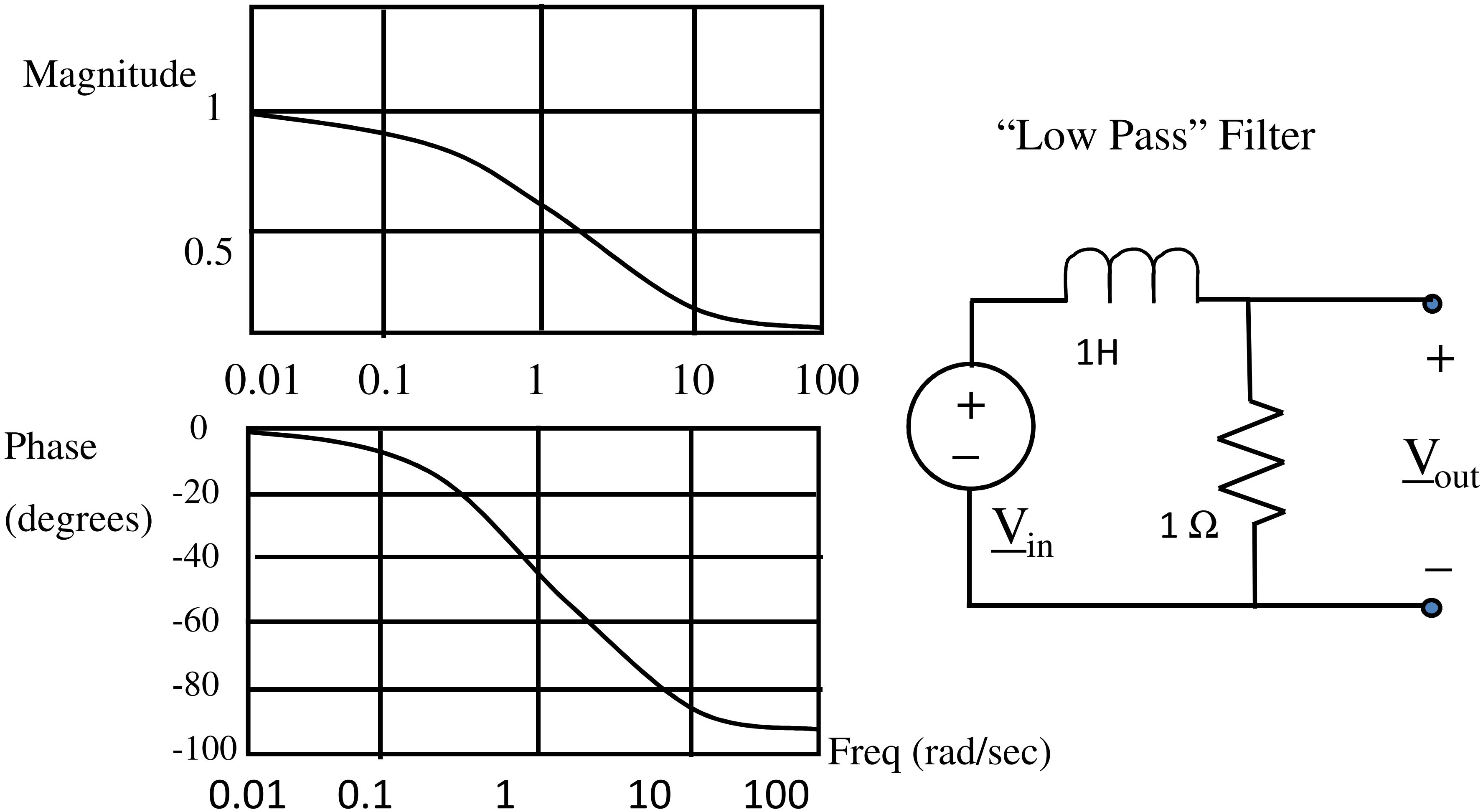
Solution



Step 2: Use voltage division to establish output to input ratio.

$$\frac{\underline{V}_{out}}{\underline{V}_{in}} = \frac{R}{R+j\omega L} = \frac{1}{1+j\omega} = H(j\omega)$$

Frequency Response



Limiting Frequency Responses

Important frequencies for RL circuits:

$\omega = 0$ and $\omega = \infty$

$H(j0) = 1 \angle 0^o$

$H(j\infty) = 0 \angle -90^o$

For magnitude:

$|H(j\omega)| \longrightarrow 1$ as $\omega \longrightarrow 0$

$|H(j\omega)| \longrightarrow 0$ as $\omega \longrightarrow \infty$

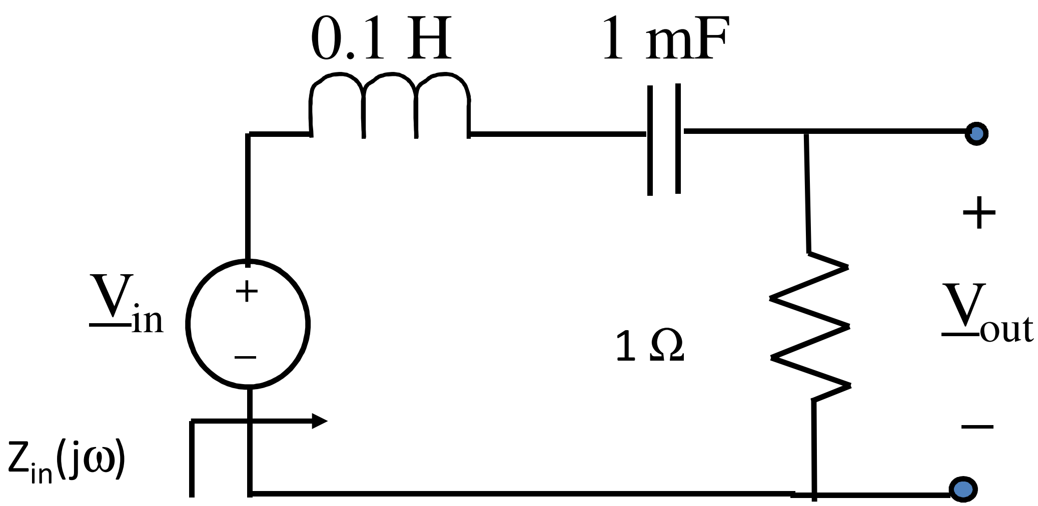
For phase:

$\angle H(j\omega) \longrightarrow 0^o$ as $\omega \longrightarrow 0$

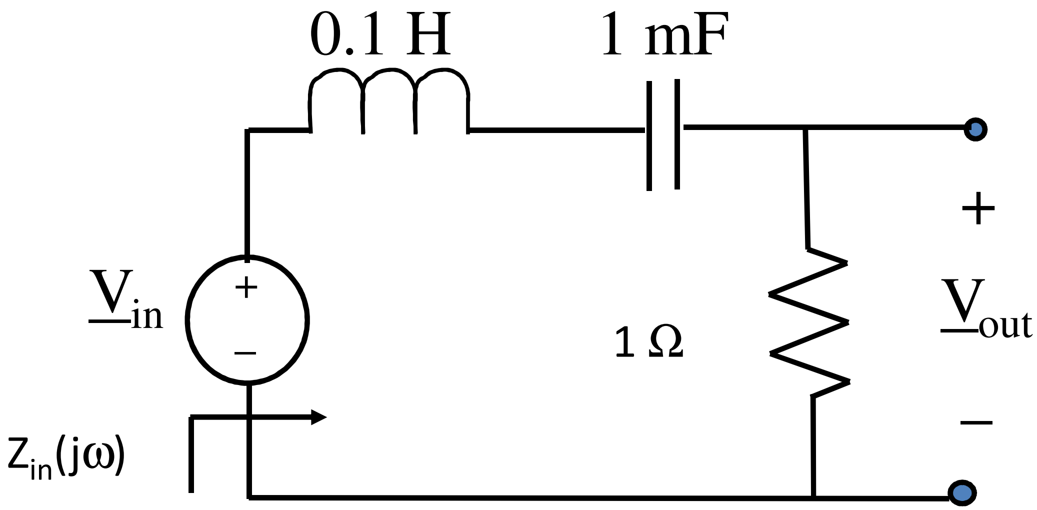
$\angle H(j\omega) \longrightarrow -90^o$ as $\omega \longrightarrow \infty$

Example #3

- Find the frequency response across the resistor of this series RLC circuit



Solution



Find ratio of phasor output to input

$Z_{in} = R+j\omega L+(1/j\omega C) = 1 + j0.1\omega + (1/j0.01\omega)$

By voltage division

$$\frac{\underline{V}_{out}}{\underline{V}_{in}} = \frac{R}{R+j\omega L+(1/j\omega C)} = \frac{1}{1+j0.01\omega+(1/j0.001\omega)}$$

Limiting Frequency Responses

$|H(j\omega)| \longrightarrow 0$ as $\omega \longrightarrow \infty$

$|H(j\omega)| \longrightarrow 0$ as $\omega \longrightarrow 0$

$H(j\omega) \longrightarrow -90^{\circ}$ as $\omega \longrightarrow \infty$

$H(j\omega) \longrightarrow 90^{\circ}$ as $\omega \longrightarrow 0$

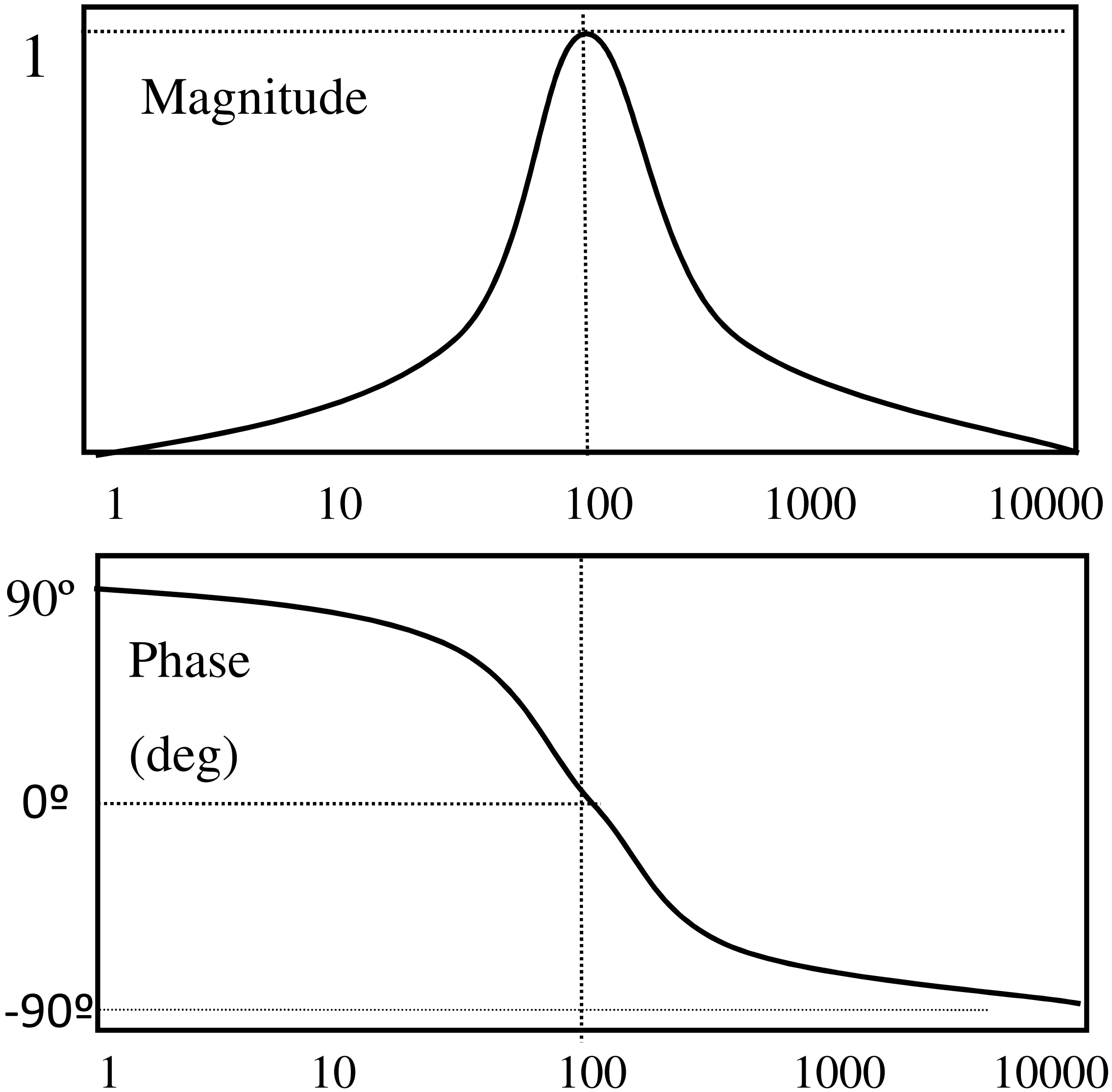
Look at other frequencies

$|H(j\omega)| \longrightarrow 0.01$ as $\omega \longrightarrow 10$ rad/sec

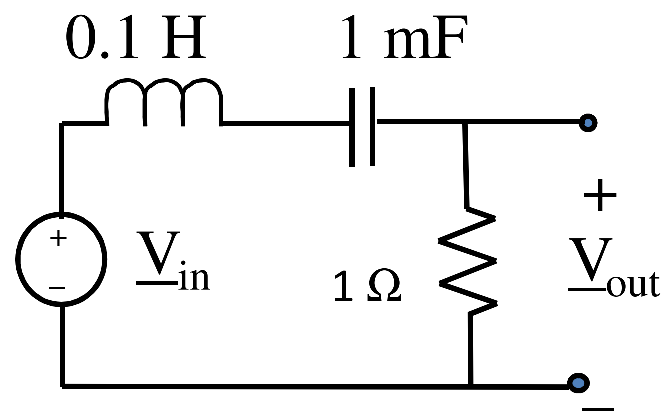
$|H(j\omega)| \longrightarrow 1$ as $\omega \longrightarrow 100$ rad/sec

$|H(j\omega)| \longrightarrow 0.01$ as $\omega \longrightarrow 1000$ rad/sec

Frequency Response



"Band Pass" Response



Homework

- HW #32 due today by 4 pm in EE 325B
- HW #33 due Wed.: DeCarlo & Lin, Chapter 10:
  - Problem 52
  - Problem 55