ECE 201, Section 3
Lecture 34

Prof. Peter Bermel

November 14, 2012

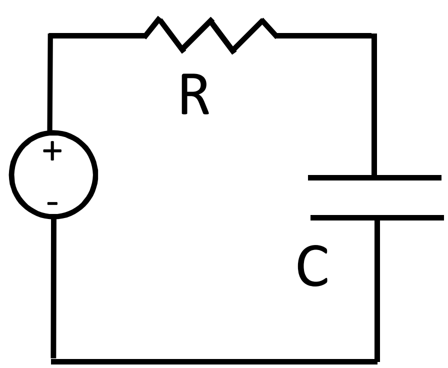
Exam #3

- Thurs., Nov. 15 at 6:30 pm in LILY 1105
- No class or HW next week (Nov. 19-23)!
- Covers Lectures 22-32, with 3 major topics:
  - Second-order circuits (primarily RLC)
  - Op-amps
  - Sinusoidal steady state
- Posted 7 sample exams with answers for Spring 2009, 2010, and 2011, and full solutions for '09 and '11

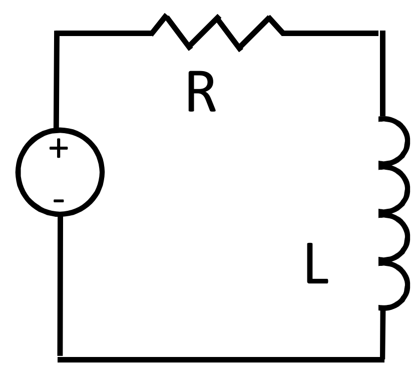
General Solutions: RC & RL Circuits

$$X(t) = X_\infty + [X(t_o{}^+) - X_\infty]e^{-(t-t_o)/\tau}$$

- Where X(t) is current or voltage at time t
- Voltages are continuous across capacitors; currents are continuous through inductors
- Capacitor currents and inductor voltages can jump discontinuously



RC Circuit



RL Circuit

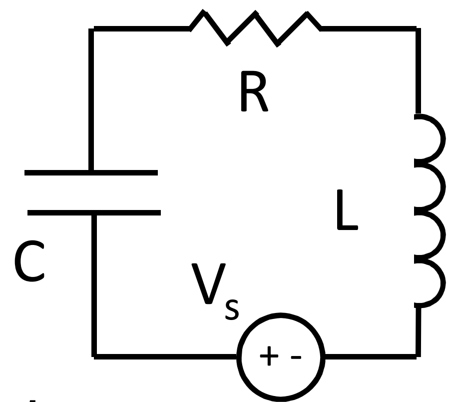
Driven Series RLC Circuits

From KVL:

$$\frac{d^2Q}{dt^2}+\frac{R}{L}\frac{dQ}{dt}+\frac{1}{LC}Q=\frac{V_s}{L}$$

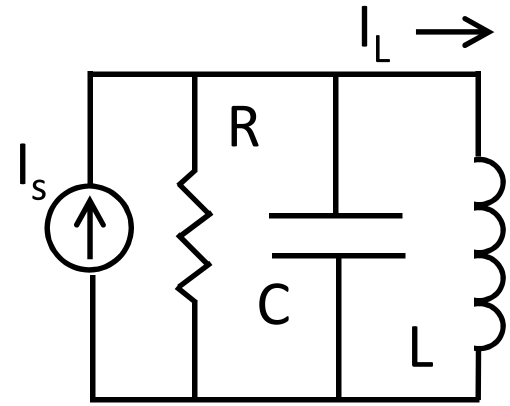
$$\frac{d^2V_C}{dt^2}+2\Gamma\frac{dV_C}{dt}+\omega_o^2 V_C=\omega_o^2 V_s$$

$$\Gamma=R/2L;\ \omega_o=1/\sqrt{LC};\ \omega'=\sqrt{\omega_o^2-\Gamma^2}=-i\Gamma'$$



Regime	Range	Solution	Behavior
Under-damped	$\Gamma<\omega_o$	$V_C(t)=V_o e^{-\Gamma t}\cos(\omega' t+\phi)+V_s$	Oscillate & decay
Critically damped	$\Gamma=\omega_o$	$V_C(t)=e^{-\Gamma t}(A_1+A_2 t)+V_s$	Decay
Over-damped	$\Gamma>\omega_o$	$V_C(t)=e^{-\Gamma t}\left(A_+e^{\Gamma' t}+A_-e^{-\Gamma' t}\right)+V_s$	Decay

Driven Parallel RLC Circuits



From KCL:

$$\frac{d^2 I_L}{dt^2} + \frac{1}{RC}\frac{dI_L}{dt} + \frac{1}{LC} I_L = \frac{I_S}{LC}$$

$$\frac{d^2 I_L}{dt^2} + 2\Gamma\frac{dI_L}{dt} + \omega_o^2 I_L = \omega_o^2 I_s$$

$$\Gamma = 1/(2RC);\ \omega_o = 1/\sqrt{LC};\ \omega' = \sqrt{\omega_o^2 - \Gamma^2} = -i\Gamma'$$

Regime	Range	Solution	Behavior
Under-damped	$\Gamma < \omega_o$	$I_L(t) = I_o e^{-\Gamma t}\cos(\omega' t + \phi) + I_s$	Oscillate & decay
Critically damped	$\Gamma = \omega_o$	$I_L(t) = e^{-\Gamma t}(A_1 + A_2 t) + I_s$	Decay
Over-damped	$\Gamma > \omega_o$	$I_L(t) = e^{-\Gamma t}\left(A_+ e^{\Gamma' t} + A_- e^{-\Gamma' t}\right) + I_s$	Decay

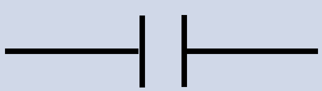
Phasor Review

- Shorthand for writing complex numbers:

$$\boldsymbol{V} = V_m \angle \phi = V_m e^{j(\omega t + \phi)}$$

- Ohm's law with phasors:

$$\boldsymbol{V} = Z(j\omega)\boldsymbol{I}$$
$$\boldsymbol{I} = Y(j\omega)\boldsymbol{V}$$

Circuit Element	**Impedance**	**Admittance**
	$Z(j\omega) = R$	$Y(j\omega) = \frac{1}{R}$
	$Z(j\omega) = j\omega L$	$Y(j\omega) = \frac{1}{j\omega L}$
	$Z(j\omega) = \frac{1}{j\omega C}$	$Y(j\omega) = j\omega C$

Impedance Properties

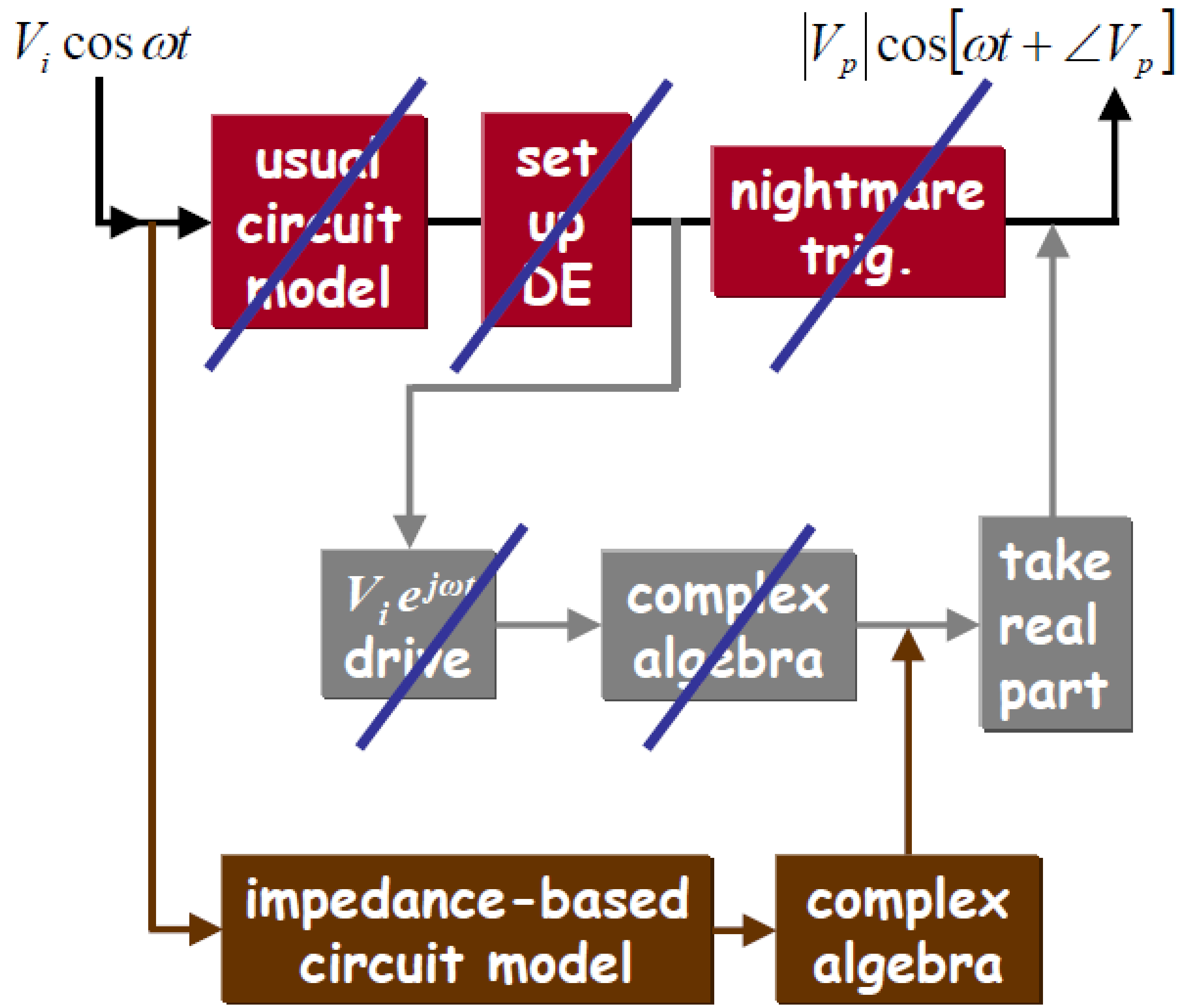
- For circuit elements in series, voltage division rule becomes:

$$V_k = \frac{Z_k}{Z_{eq}} V_{tot}$$

- For circuit elements in parallel, current division rule becomes:

$$I_k = \frac{Y_k}{Y_{eq}} I_{tot}$$

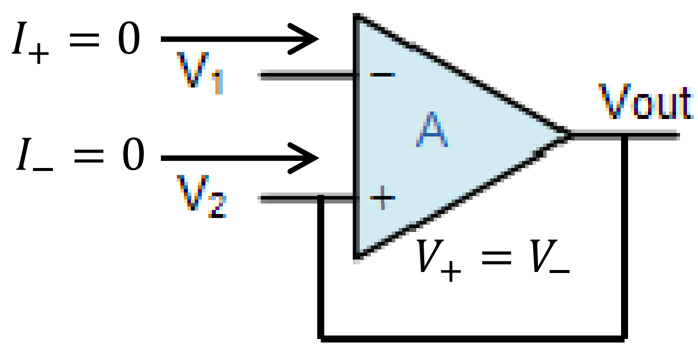
Second Order Circuit Overview



Adapted from A. Agarwal & J. Lang, Course Materials for MIT 6.002, Spring 2007

Ideal Op-Amps

- Golden rules:
  - Both input currents are zero
  - For closed loops: both input voltages are equal



Exam 3 Spring 2010

1. At $t = 0$ sec, the inductor current is $i_L(0^+) = 5A$ and the capacitor voltage is $v_c(0^+) = 0V$. Find $v_c(t)$ for $t \geq 0$ s (in V).

(1) $v_c(t) = 25\cos(10^6 t) + 25\sin(10^6 t)$

(2) $v_c(t) = 25\cos(10^6 t)$

(3) $v_c(t) = 25\sin(10^6 t)$

(4) $v_c(t) = 12.5\cos(10^3 t) + 12.5\sin(10^3 t)$

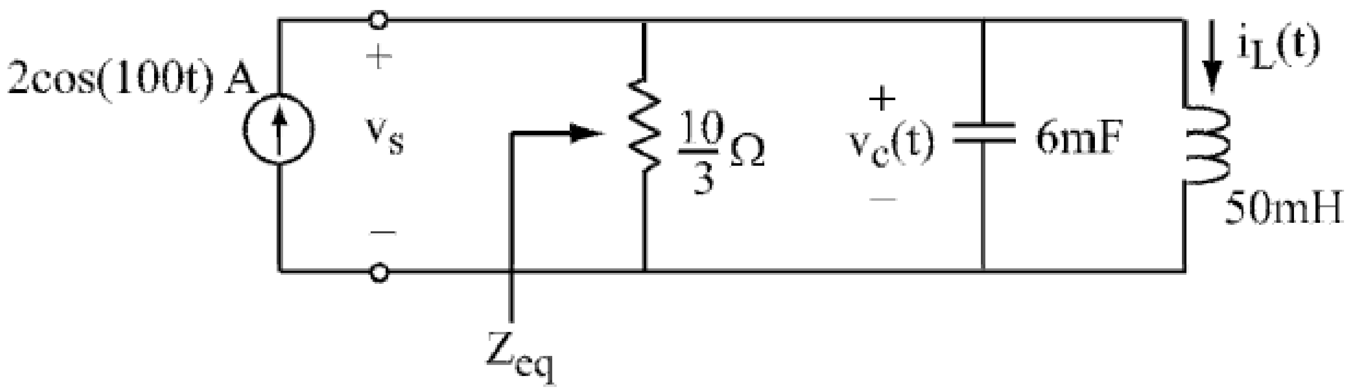
(5) $v_c(t) = 12.5\cos(10^3 t)$

(6) $v_c(t) = -12.5\sin(10^3 t)$

(7) $v_c(t) = 0$

Exam 3 Spring 2010

11. Calculate the equivalent impedance, Z_{eq} (in Ω), for the circuit below.

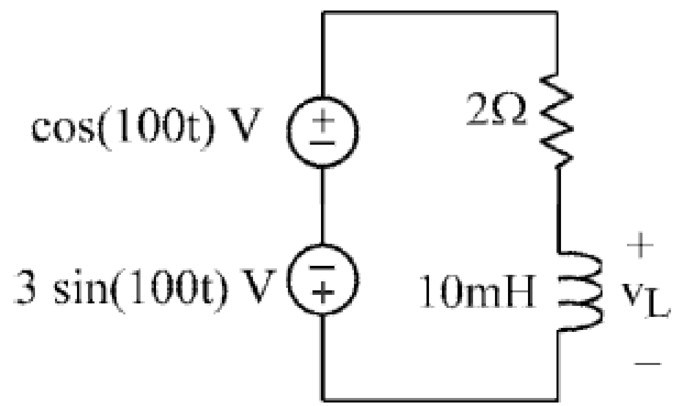


(1) $2\angle 53.13°$ (2) $2\angle -53.13°$ (3) $2\angle 36.7°$ (4) $2\angle -36.7°$

(5) $1\angle 40°$ (6) $1\angle -40°$ (7) $1\angle 45°$

Exam 3 Spring 2010

13. The circuit shown below is in steady state. Find $v_L(t)$.



(1) $v_L(t)=\sqrt{2}\cos(100t+90°)$

(2) $v_L(t)=2\cos(100t+90°)$

(3) $v_L(t)=\sqrt{2}\cos(100t+135°)$

(4) $v_L(t)=2\cos(100t+135°)$

(5) $v_L(t)=\sqrt{2}\cos(100t+170°)$

(6) $v_L(t)=2\cos(100t+170°)$

(7) $v_L(t)=\sqrt{2}\cos(200t+170°)$

Equation Sheet for Exam

$I=dQ/dt$

$V=IR$ or $V=IZ$; $I=GV$ or $I=YV$

$Q=CV$ and $V=L\, dI/dt$

In series:

$Z_{eq}=\Sigma_k Z_k$

$V_k=VZ_k/Z_{eq}$

In parallel:

$Y_{eq}=\Sigma_k Y_k$

$I_k=VY_k=IY_k/Y_{eq}$

$P=IV$

For first-order circuits:

$$X = X_\infty + (X_o - X_\infty)e^{-(t-t_o)/\tau}$$

For RL circuits: $\tau=L/R$

For RC circuits: $\tau=RC$

For second-order (RLC) circuits:

$$X(t) = Ae^{-\Gamma t}\cos(\omega' t + \phi) + X_s$$

$$X(t) = (A_1 + A_2 t)e^{-\Gamma t} + X_s$$

$$X(t) = e^{-\Gamma t}\left(A_+ e^{\Gamma' t} + A_- e^{-\Gamma' t}\right) + X_s$$

where: $\Gamma=R/2L$ (series) or $\Gamma=1/(2RC)$ (parallel);

$\omega_o = 1/\sqrt{LC}$; $\omega' = \sqrt{\omega_o^2 - \Gamma^2}$; and $\Gamma' = \sqrt{\Gamma^2 - \omega_o^2}$

Homework

- HW #33 due today by 4:30 pm in EE 325B
- HW #34 due Fri. – DeCarlo & Lin, Chapter 10:
  - Problem 58
  - Problem 62