ECE 201, Section 3
Lecture 35

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Real-World Significance of Power

- Energy used in homes, vehicles
- Battery life in mobile devices
- Safety in electrical wiring

Instantaneous and Average Power

- Instantaneous power given by:

$$P(t) = I(t)\,V(t)$$

- Average power over time interval [t_1,t_2] given by:

$$P_{ave} = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} dt\, P(t)$$

Average Power for SSS

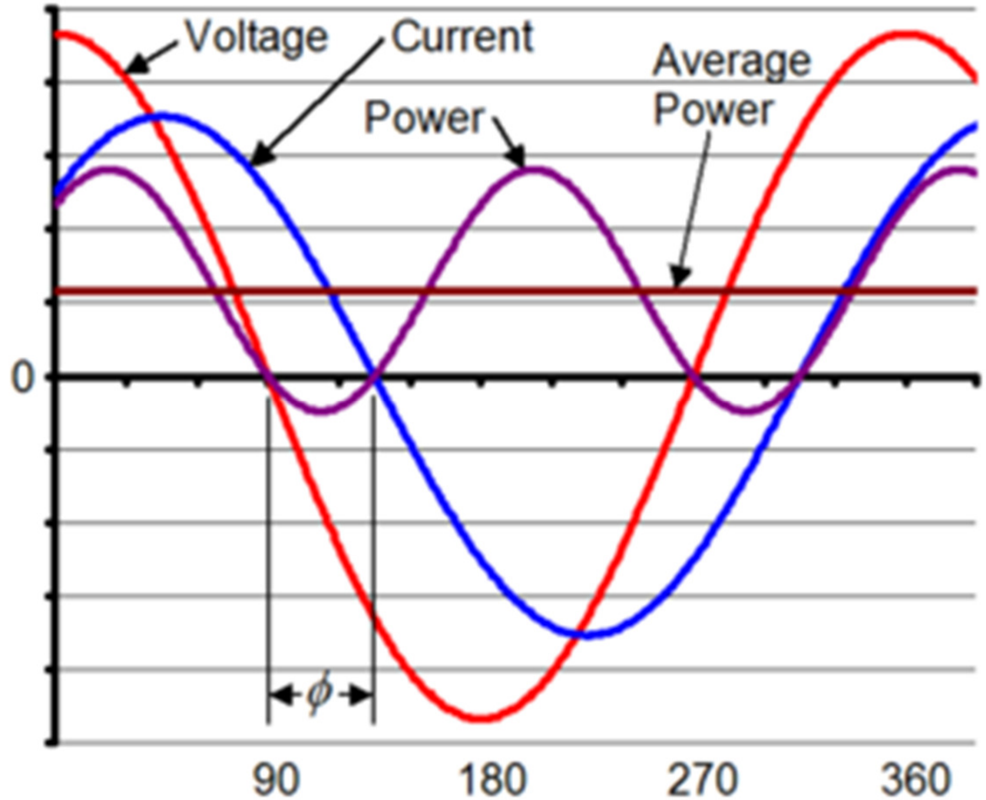
- Generally, currents and voltages driven at the same frequency ω – assume:

$$I(t) = I_o \cos \omega t$$
$$V(t) = V_o \cos(\omega t + \phi)$$

- Instantaneous power then given by:

$$P(t) = I_o V_o \cos \omega t \cos(\omega t + \phi)$$
$$P(t) = \frac{I_o V_o}{2} [\cos \phi + \cos(2\omega t + \phi)]$$

Average Power for SSS



Over half-integer periods, only the first term contributes, so that:

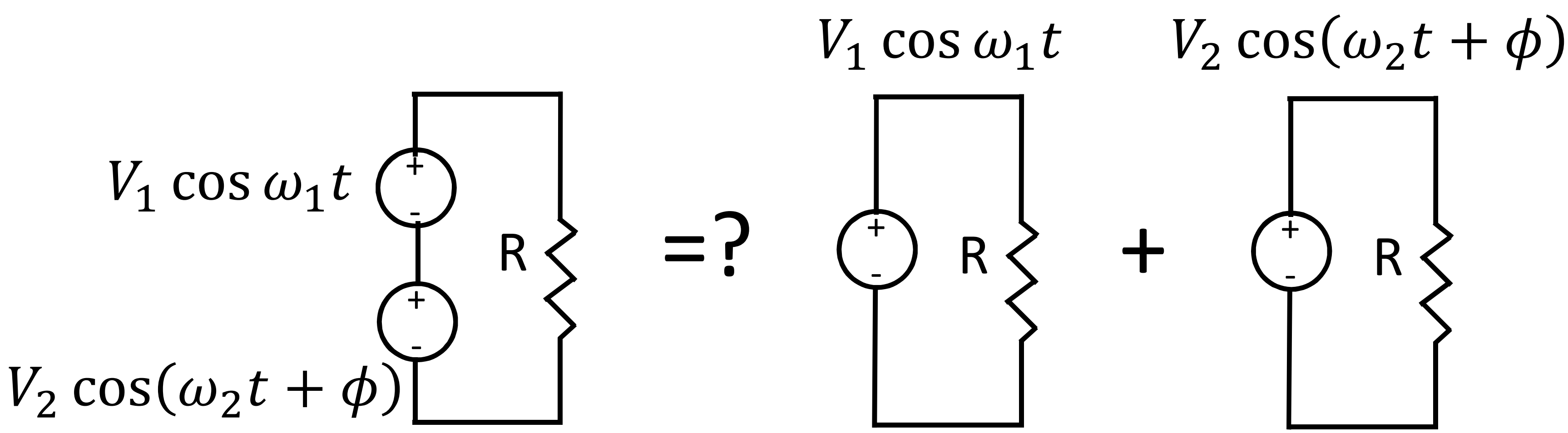
$$P_{ave} = \frac{1}{2} I_o V_o \cos\phi$$

Demonstration

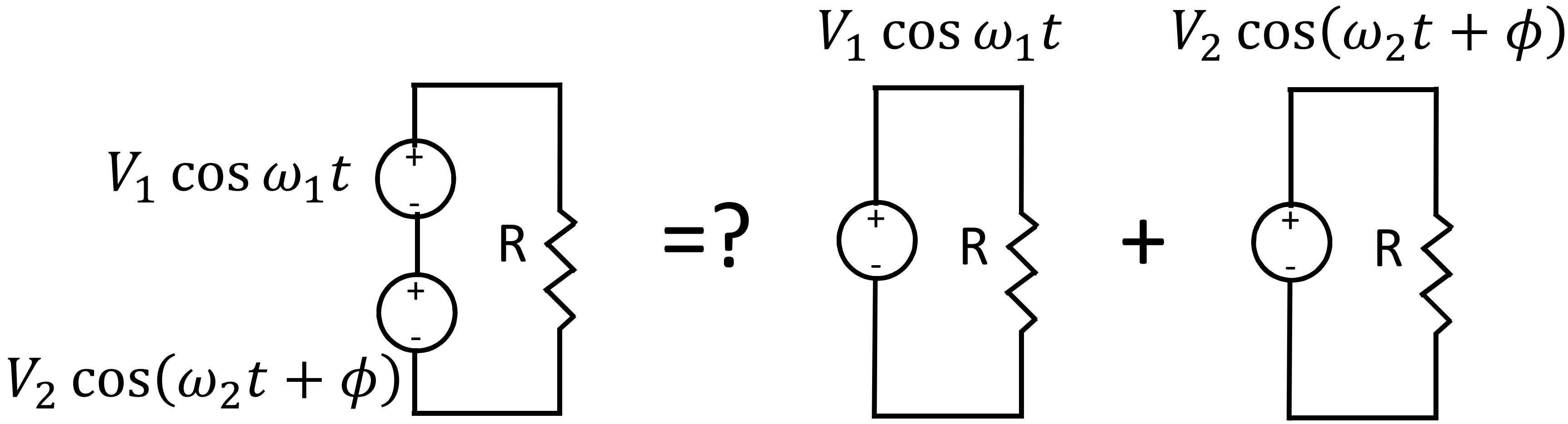
- RLC circuit with storage

Superposition of Average Power

- Example: how does the average power of 2 AC sources connected to a resistor compare to the average power of each one alone?



Superposition of Average Power



- By linearity:

$$P(t) = \frac{1}{R}\left[V_1 \cos \omega_1 t + V_2 \cos(\omega_2 t + \phi)\right]^2$$

$$P(t) = P_1 + P_2 + \frac{2V_1V_2}{R} \cos \omega_1 t \cdot \cos(\omega_2 t + \phi)$$

Superposition of Average Power

- 3 important cases:
  - $\omega_1 \neq \omega_2$, integer periods: superposition holds
  - $\omega_1 = \omega_2, \cos\phi = 0$, integer periods: superposition holds
  - Otherwise: superposition fails

Effective Values

- We can generally define effective values for periodic signals, $f(t+T) = f(t)$, such that:

$$F_{eff} = \sqrt{\frac{1}{T}\int_{t_o}^{t_o+T} dt\,[f(t)]^2}$$

- Also known as root-mean-square (rms) value

Effective Values

- For an AC signal given by:

$$f(t) = f_o \cos(\omega_o t + \phi)$$

- Plugging into our formula yields:

$$F_{eff} = \sqrt{\frac{1}{T}\int_{t_o}^{t_o+T} dt\, [f_o \cos(\omega_o t + \phi)]^2}$$

$$F_{eff}^{\,2} = \frac{f_o^{\,2}}{T}\int_{t_o}^{t_o+T} dt\, \frac{1}{2}[1 + \cos(\omega_o t + \phi)]$$

$$F_{eff}^{\,2} = \frac{f_o^{\,2}}{T}\frac{T}{2} = \frac{f_o^{\,2}}{2}$$

$$F_{eff} = \frac{f_o}{\sqrt{2}}$$

Power from Effective Values

- Revisiting our calculation from last time:

$$P_{ave} = \frac{1}{2} I_o V_o \cos\phi$$

- Rewriting in our new notation:

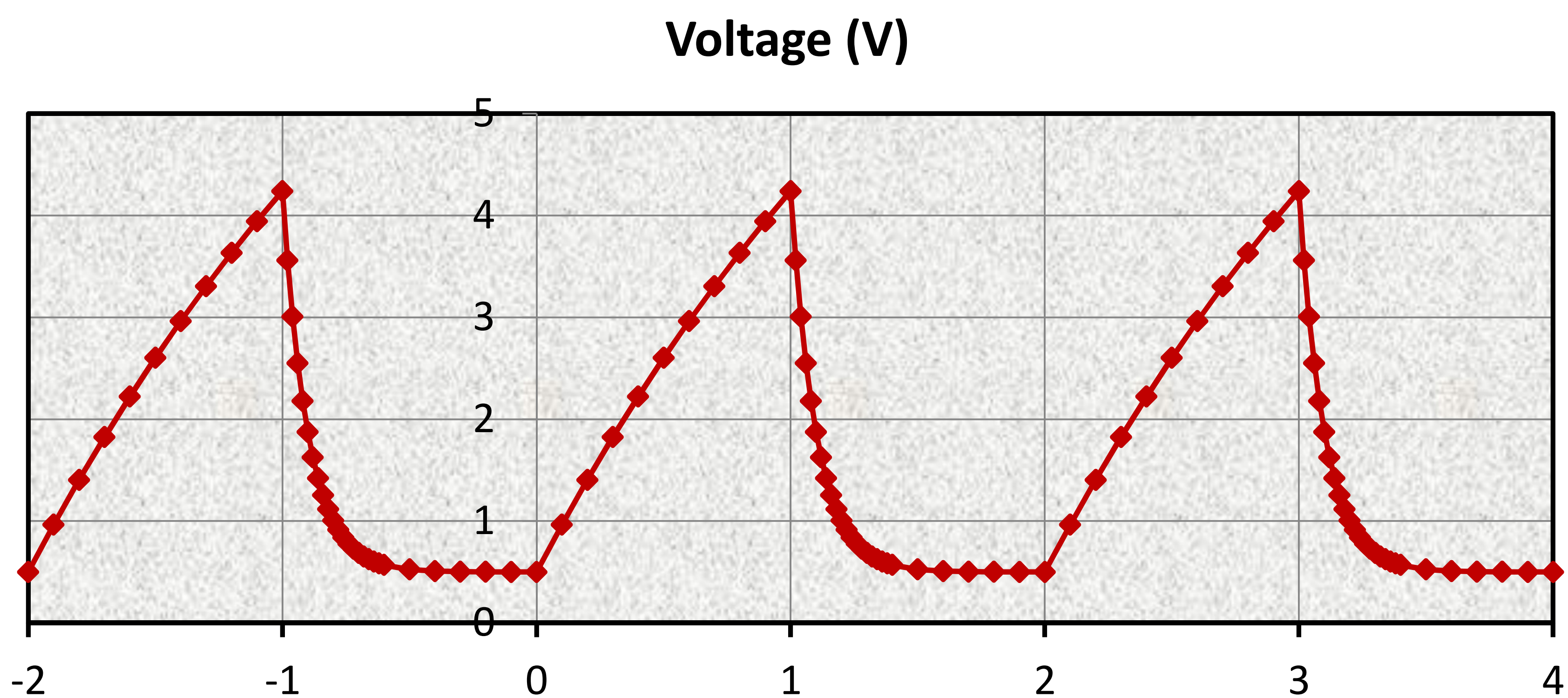
$$P_{ave} = I_{eff} V_{eff} \cos\phi$$

- In electrical engineering, drop the effective subscripts and write:

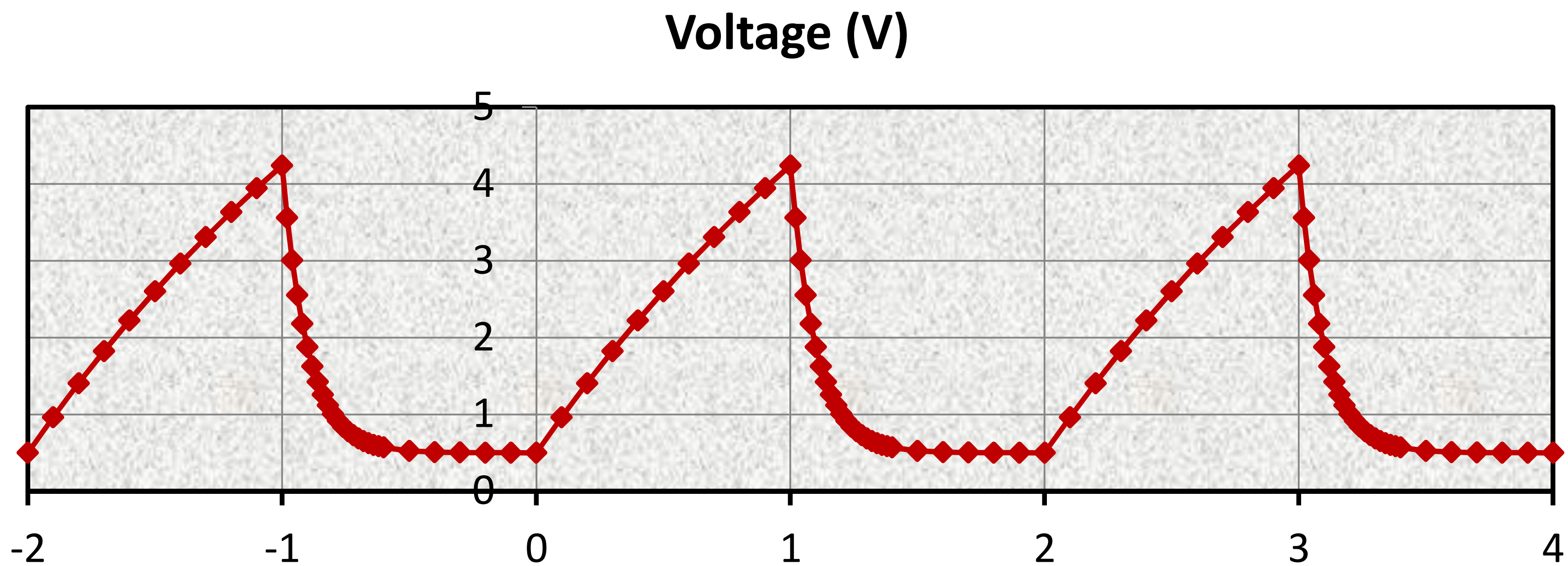
$$P_{ave} = IV \cos\phi$$

Effective Values Example

- What is the effective voltage of this waveform? How much power is dissipated by a 2 Ω resistor?



Effective Values Example



$$V_{eff}^{2} = \frac{1}{2}\int_0^2 dt\,[0.5^2 + (3.8t + 0.5)^2 u(1-t) + [0.5 + 3.8(6-5t)]^2 u(t-1)u(6-5t)]$$

$$V_{eff}^{2} \approx \frac{1}{2}\left[0.25 \cdot 0.8 + \frac{4.3^3 - 0.5^3}{3 \cdot 3.8} + \frac{4.3^3 - 0.5^3}{3 \cdot 19}\right] = 4.28\,V^2$$

$$P_{ave} = \frac{V_{eff}^{2}}{R} = \frac{4.28}{2} = 2.14\text{ W}$$

Complex Power

- Complex power is defined as:

$$\boldsymbol{S} = \boldsymbol{I}^{*}\boldsymbol{V}$$

- For AC signals:

$$\boldsymbol{S} = I_o e^{-j\omega t} V_o e^{j(\omega t+\phi)}$$

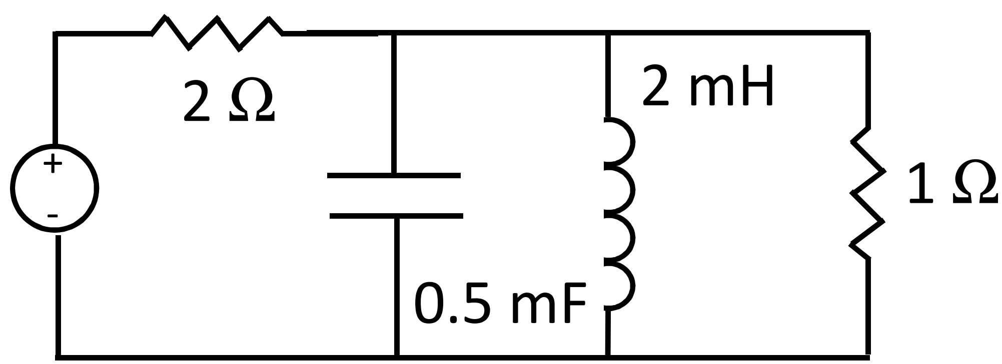
$$\boldsymbol{S} = I_o V_o e^{j\phi} = I_o V_o(\cos\phi + j\sin\phi)$$

- Key definitions:
  - Re S – average power (dissipated)
  - Im S – reactive power (stored)
  - $|\mathbf{S}|$ – apparent power

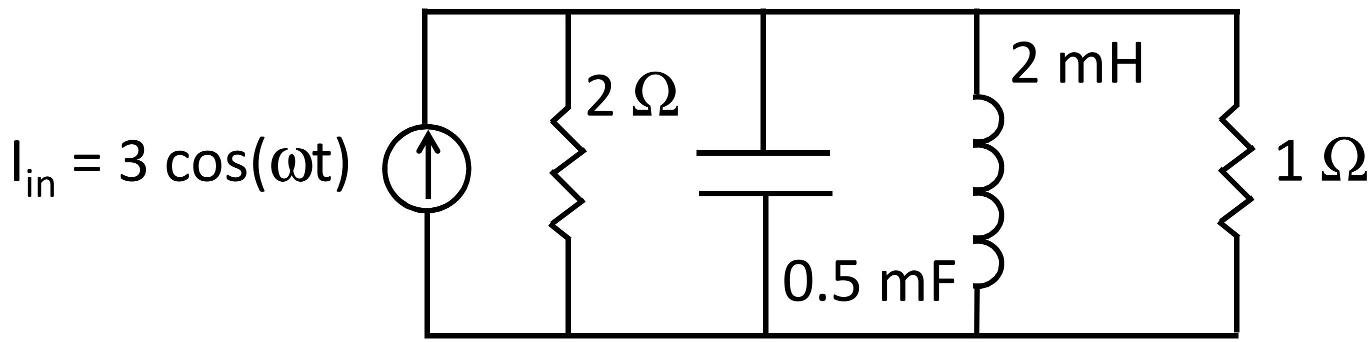
Complex Power Example

- What is the current and complex power dissipated by this circuit?

$V_{in} = 6\cos(\omega t)$



Complex Power: Solution



- The impedance is given by:

$$Z = \left[\frac{3}{2} + j\omega \cdot 5 \cdot 10^{-4} + \frac{1}{j\omega \cdot 2 \cdot 10^{-3}}\right]^{-1}$$

- The voltage drop is thus given by:

$$V = IZ = \frac{3e^{j\omega t}}{\frac{3}{2} + j \cdot (\omega/2000 - 500/\omega)}$$

$$V = \frac{2e^{j\left[\omega t - \tan^{-1}\left(\frac{\omega}{3000} - \frac{1000}{3\omega}\right)\right]}}{\left[1 + \left(\frac{\omega}{3000} - \frac{1000}{3\omega}\right)^2\right]^{1/2}}$$

Complex Power: Solution

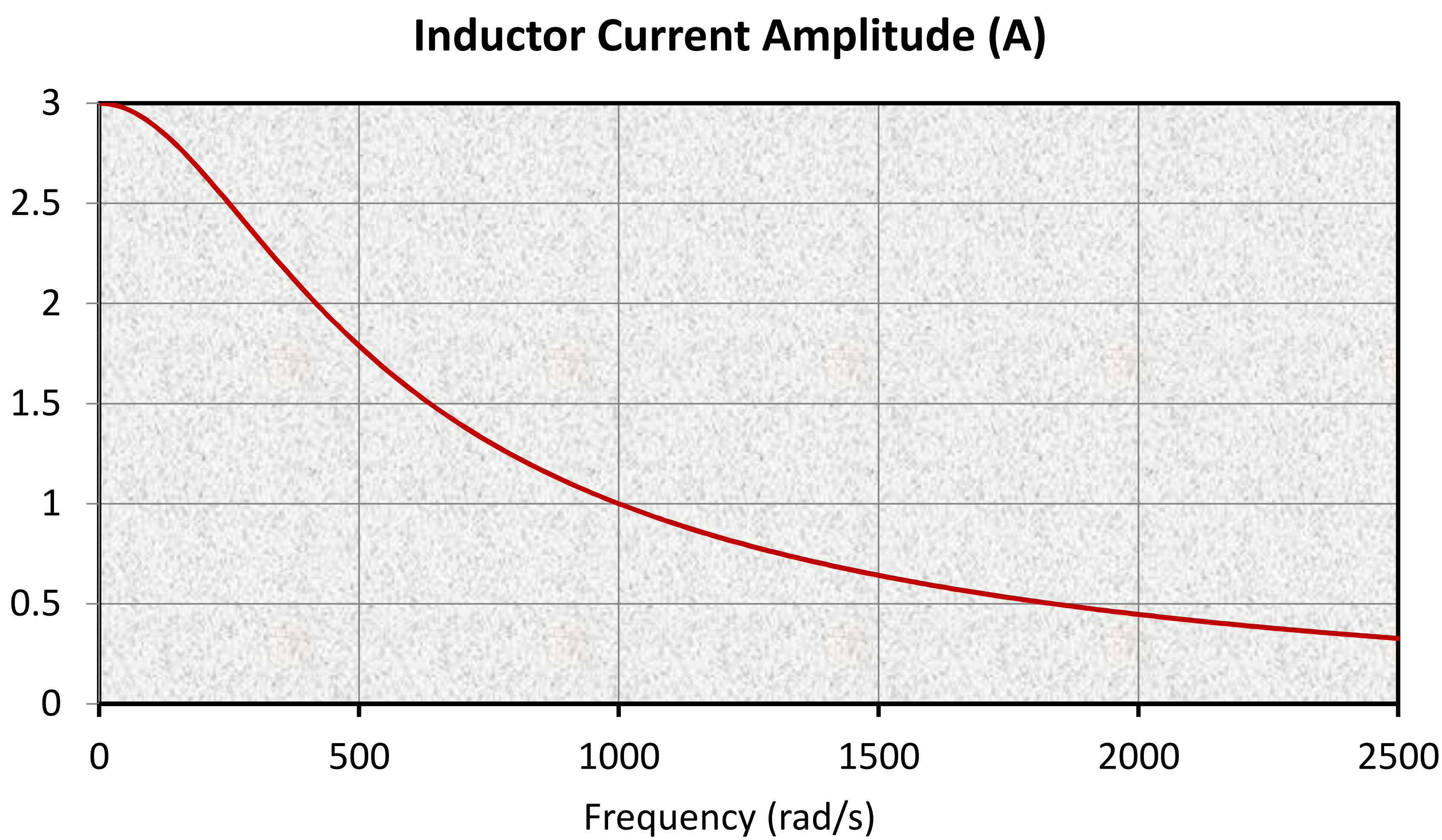
- Applying the current division rule:

$$I_L = \frac{Y_L}{Y} I_{tot}$$

$$I_L = \frac{(500/j\omega)3e^{j\omega t}}{\frac{3}{2} + j \cdot (\omega/2000 - 500/\omega)} = \frac{1000e^{j\omega t}}{j\omega + \frac{1000}{3} - \frac{\omega^2}{3000}}$$

$$I_L = \frac{1000e^{j[\omega t - \tan^{-1}(\omega/(333 - \omega^2/3000))]}}{\left[\omega^2 + \left(\frac{1000}{3} - \frac{\omega^2}{3000}\right)^2\right]^{1/2}}$$

Solution



Inductor Current Amplitude (A)
3
2.5
2
1.5
1
0.5
0
0
500
1000
1500
2000
2500
Frequency (rad/s)

Complex Power: Solution

- From our earlier definitions:

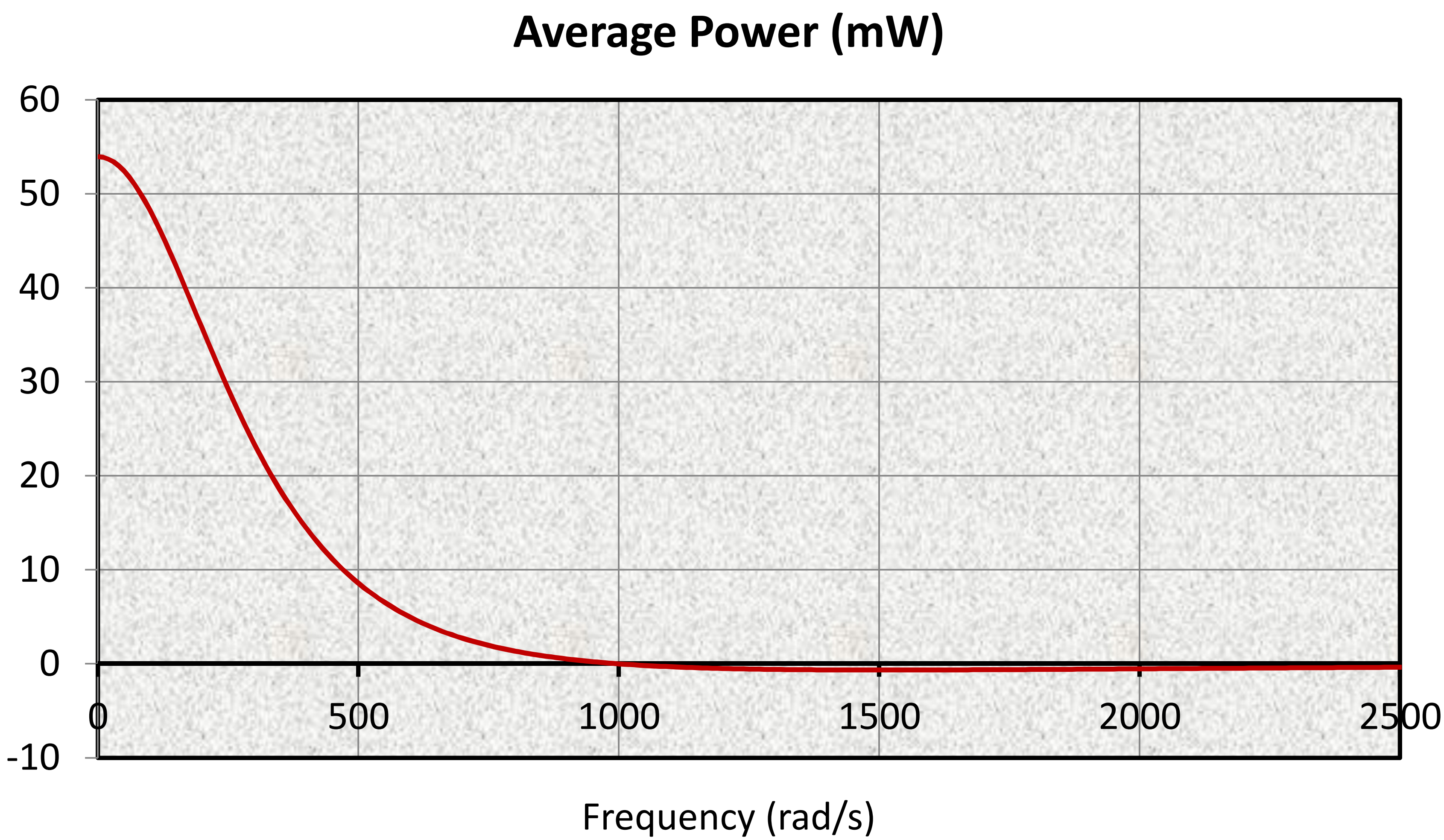
$$\boldsymbol{S} = \boldsymbol{I}^{*}\boldsymbol{V}$$

- Substitution yields:

$$\boldsymbol{S} = \frac{1000e^{-j\left[\omega t - \tan^{-1}\left(\omega/(333 - \omega^2/3000)\right)\right]}}{\left[\omega^2 + \left(\frac{1000}{3} - \frac{\omega^2}{3000}\right)^2\right]^{1/2}} \cdot 6e^{j\omega t}$$

$$\boldsymbol{S} = 6000 \cdot \frac{333 - \frac{\omega^2}{3000} + j\omega}{\left[\omega^2 + \left(\frac{1000}{3} - \frac{\omega^2}{3000}\right)^2\right]^{3/2}}$$

Solution


Average Power (mW)
60
50
40
30
20
10
0
-10
0
500
1000
1500
2000
2500
Frequency (rad/s)

Homework

- HW #34 due today by 4:30 pm in EE 325B
- HW #35 due Mon. Nov. 26: DeCarlo & Lin, Chapter 10:
  - Problem 64 [Correction: Delete "and determine the frequency ... of its maximum value." Also, either a plot from MATLAB or a sketch from an analytical formula is fine.]
  - Problem 66