ECE 201, Section 3
Lecture 39

Prof. Peter Bermel

pbermel@purdue.edu

December 3, 2012

Final Exam

- Friday, December 14 from 3:30-5:30 pm in EE 129
- Review sessions:
  - Fri., Dec. 7 from 6:30-7:30 pm in MATH 175
  - Wed., Dec. 12 from 4:30-5:30 pm in ARMS 1010
- Office hours will be held MWF 12:30-1:30 pm (including Dec. 14) in EE 331A & by appointment
- TA office hours will be M-F, 8:30 am-4:30 pm, including December 14 (until 2:30 pm)

Final Exam

- Key Concepts (from course outcomes):
  - Basic concepts: current, voltage, charge, Ohm's law, KCL, KVL, current & voltage division
  - Resistor networks: node, supernode, loop analysis
  - Combining resistors, inductors, and capacitors
  - Source transformation; Thévenin & Norton equivalents
  - Op-amps
  - First and second order circuits: DC with stepped sources, AC in sinusoidal steady state
  - Phasor analysis, including complex power
  - Maximum power transfer for 'realistic' sources

Basic Concepts

- Current *I* represents the flow of charge:

$$I = \frac{dq}{dt}$$

- Voltage *V* creates potential energy *U* for charges:

$$U = qV$$

- Power dissipated (passive sign convention):

$$P = IV$$

- Ohm's Law for resistors:

$$V = IR$$

Basic Concepts: Kirchoff's Laws

Kirchoff's Current Law (KCL)

– Sum of all currents entering a node or Gaussian surface is zero at all times:

$$\sum_{k=1}^{N} I_k(t) = 0 \text{, for all t}$$

Kirchoff's Voltage Law (KVL)

– Voltage drop between any two nodes is direction-dependent and path-independent (i.e., $V_{AB} = V_A - V_B$)

– Sum of voltage drops over any closed loop is zero

Voltage and Current Division in Resistors; Resistor Networks

- Series resistors:

$$R_{eq} = \sum R_l$$

$$V_k = VR_k/R_{eq}\text{; currents equal}$$

- Parallel resistors

$$G_{eq} = \sum G_l$$

$$I_k = IR_{eq}/R_k\text{; voltages equal}$$

- Series-parallel circuits
  - Analyzed iteratively

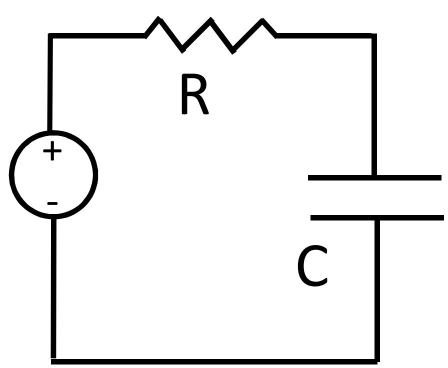
Resistor Network Analysis Approaches

- **Nodal analysis**
- Modified nodal analysis
- Nodal analysis with floating voltage sources
- **Loop analysis**
- All approaches should generally yield the same physical results
- Best choice generally involves least number of unknowns, and will depend on details of problem

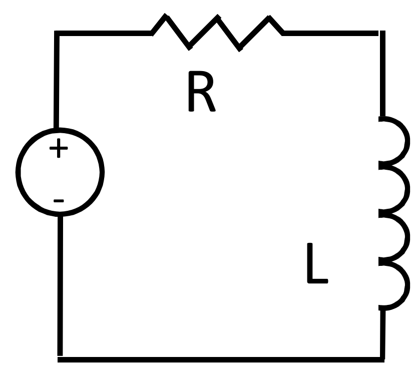
General Solutions: RC & RL Circuits

$$X = X_\infty + (X_o - X_\infty)e^{-(t-t_o)/\tau}$$

- Solution steps:
  - Choose X: RC circuits: X=Q or X=V; LR circuits: X=I
  - Find X_o (simplifying diagram)
  - Find X_∞ (simplifying diagram)
  - Find R_{th} (for inductor/capacitor)
  - Find τ: RC circuits: $\tau = R_{th}C$; LR circuits: $\tau = L/R_{th}$
  - If circuit changes at t_1, use $X(t_1)$ from prior solution for initial values



RC Circuit



RL Circuit

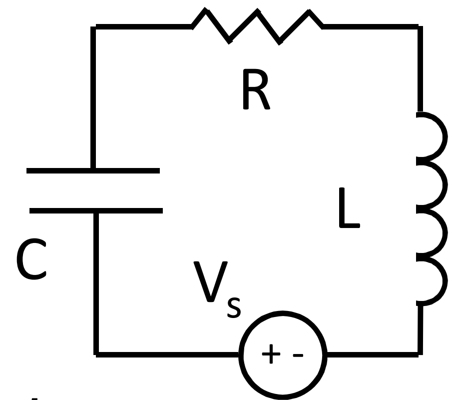
Driven Series RLC Circuits

From KVL:

$$\frac{d^2Q}{dt^2} + \frac{R}{L}\frac{dQ}{dt} + \frac{1}{LC}Q = \frac{V_s}{L}$$

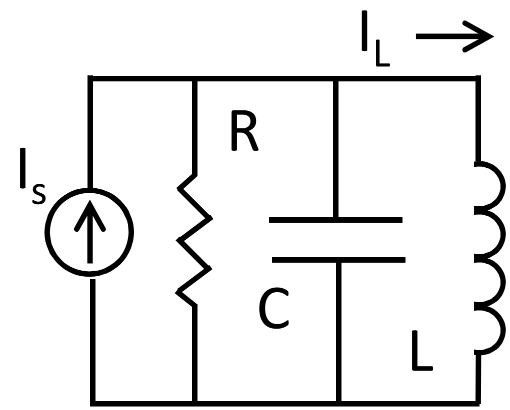
$$\frac{d^2V_C}{dt^2} + 2\Gamma\frac{dV_C}{dt} + \omega_o^2 V_C = \omega_o^2 V_s$$

$$\Gamma = R/2L;\ \omega_o = 1/\sqrt{LC};\ \omega' = \sqrt{\omega_o^2 - \Gamma^2} = -i\Gamma'$$



Regime	Range	Solution	Behavior
Under-damped	$\Gamma < \omega_o$	$V_C(t) = V_o e^{-\Gamma t}\cos(\omega' t + \phi) + V_s$	Oscillate & decay
Critically damped	$\Gamma = \omega_o$	$V_C(t) = e^{-\Gamma t}(A_1 + A_2 t) + V_s$	Decay
Over-damped	$\Gamma > \omega_o$	$V_C(t) = e^{-\Gamma t}\left(A_+ e^{\Gamma' t} + A_- e^{-\Gamma' t}\right) + V_s$	Decay

Driven Parallel RLC Circuits



From KCL:

$$\frac{d^2 I_L}{dt^2} + \frac{1}{RC}\frac{dI_L}{dt} + \frac{1}{LC} I_L = \frac{I_S}{LC}$$

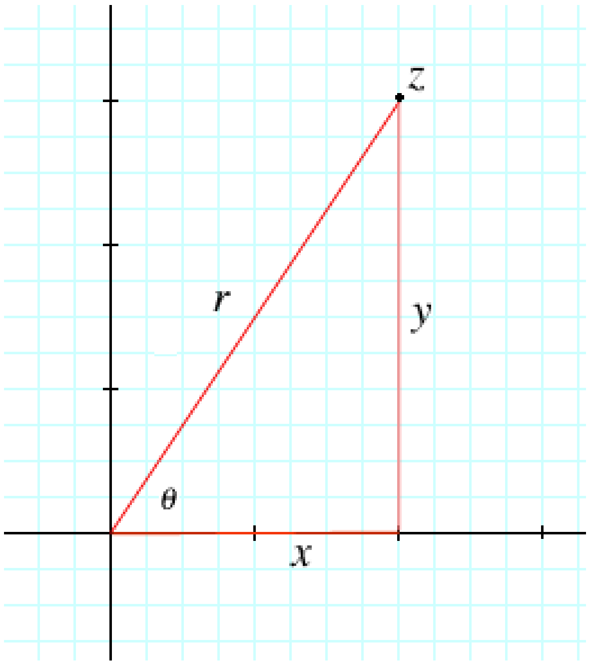
$$\frac{d^2 I_L}{dt^2} + 2\Gamma\frac{dI_L}{dt} + \omega_o^2 I_L = \omega_o^2 I_s$$

$\Gamma = 1/(2RC);\ \omega_o = 1/\sqrt{LC};\ \omega' = \sqrt{\omega_o^2 - \Gamma^2} = -i\Gamma'$

Regime	**Range**	**Solution**	**Behavior**
Under-damped	$\Gamma < \omega_o$	$I_L(t) = I_o e^{-\Gamma t}\cos(\omega' t + \phi) + I_s$	Oscillate & decay
Critically damped	$\Gamma = \omega_o$	$I_L(t) = e^{-\Gamma t}(A_1 + A_2 t) + I_s$	Decay
Over-damped	$\Gamma > \omega_o$	$I_L(t) = e^{-\Gamma t}\left(A_+ e^{\Gamma' t} + A_- e^{-\Gamma' t}\right) + I_s$	Decay

Complex Numbers (for Phasors)

- Representations
  - Cartesian: $z = x + jy = r(\cos\theta + j\sin\theta)$
  - Polar: $z = re^{j\theta} = \sqrt{x^2+y^2}e^{j\tan^{-1}\frac{y}{x}}$
- Complex math
  - Absolute value: $|z| = r = \sqrt{x^2+y^2}$
  - Complex conjugation: $z^* = x - jy = re^{-j\theta}$
  - Natural log: $\ln z = \ln r + j\theta$
  - Multiplication: $z_1 z_2 = r_1 r_2 e^{j(\theta_1+\theta_2)}$
  - Division: $\frac{z_1}{z_2} = \frac{r_1}{r_2}e^{j(\theta_1-\theta_2)} = \frac{(x_1+jy_1)(x_2-jy_2)}{x_2^2+y_2^2}$
- Complex exponential properties:
  - Euler's equation: $e^{j\omega t} = \cos\omega t + j\sin\omega t$
  - Converting to phasors:
    - $\cos\omega t \rightarrow e^{j\omega t}$
    - $\sin\omega t \rightarrow e^{j(\omega t-90°)}$



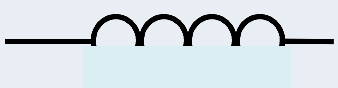
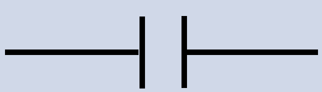
Phasor Review

- Shorthand for writing complex numbers:

$$\boldsymbol{V} = V_m \angle \phi = V_m e^{j(\omega t + \phi)}$$

- Ohm's law with phasors:

$$\boldsymbol{V} = Z(j\omega)\boldsymbol{I}$$
$$\boldsymbol{I} = Y(j\omega)\boldsymbol{V}$$

Circuit Element	Impedance	Admittance
	$Z(j\omega) = R$	$Y(j\omega) = \frac{1}{R}$
	$Z(j\omega) = j\omega L$	$Y(j\omega) = \frac{1}{j\omega L}$
	$Z(j\omega) = \frac{1}{j\omega C}$	$Y(j\omega) = j\omega C$

Impedance Properties

- For circuit elements in series, voltage division rule becomes:

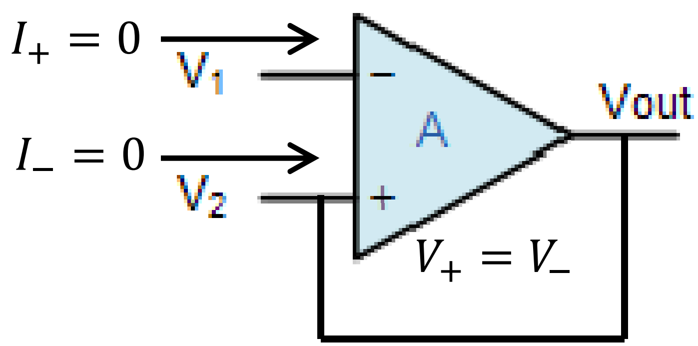
$$V_k = \frac{Z_k}{Z_{eq}} V_{tot}$$

- For circuit elements in parallel, current division rule becomes:

$$I_k = \frac{Y_k}{Y_{eq}} I_{tot}$$

Ideal Op-Amps

- Golden rules:
  - Both input currents are zero
  - For closed loops: both input voltages are equal



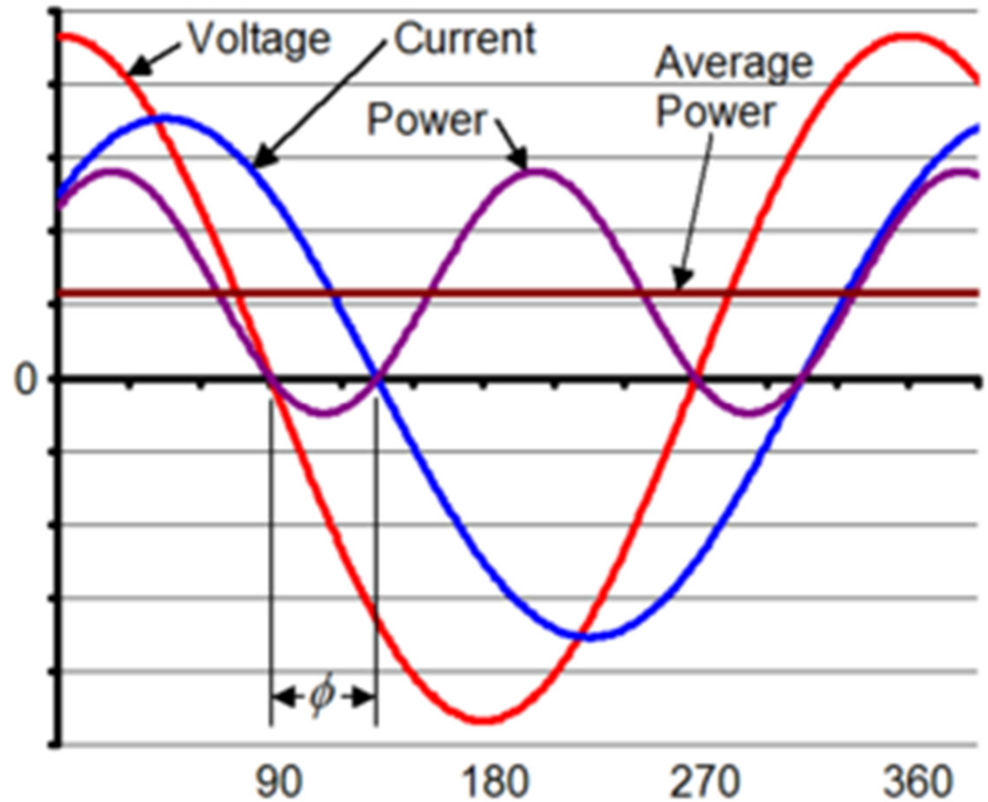
Effective Values

- We can generally define effective values for periodic signals, $f(t+T) = f(t)$, such that:

$$F_{eff} = \sqrt{\frac{1}{T}\int_{t_o}^{t_o+T} dt\,[f(t)]^2}$$

- Also known as root-mean-square (rms) value
- For AC signals, $F_{eff} = \frac{f_o}{\sqrt{2}}$

Average Power for SSS



Over half-integer periods, only the first term contributes, so that:

$$P_{ave} = IV\cos\phi$$

Complex Power

- Complex power is defined as:

$$\boldsymbol{S} = \boldsymbol{I}^{*}\boldsymbol{V}$$

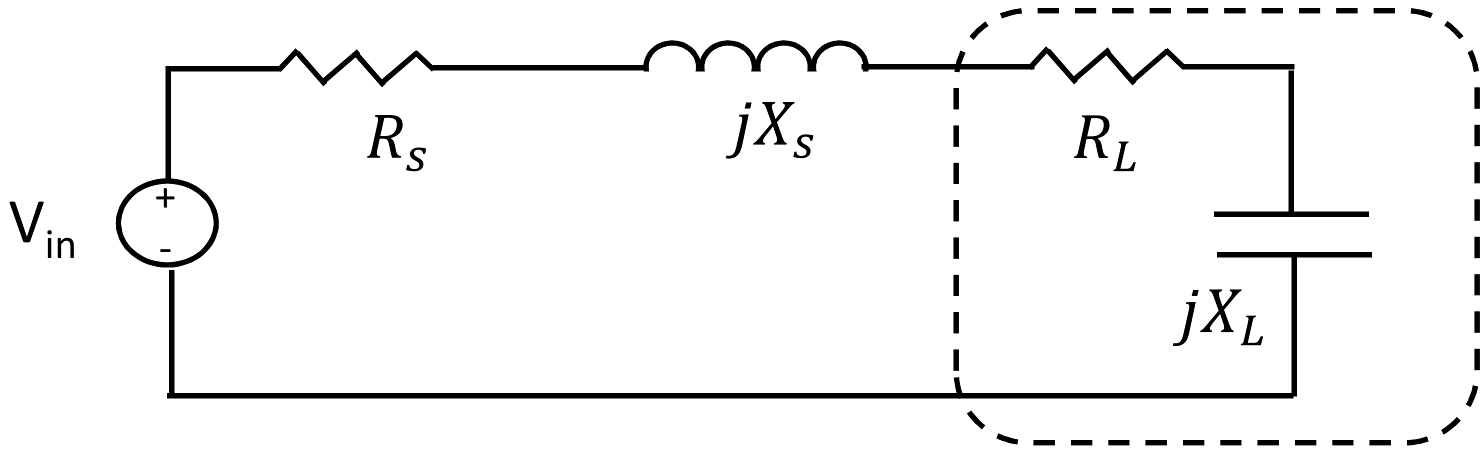
- For AC signals:

$$\boldsymbol{S} = Ie^{-j\omega t}Ve^{j(\omega t+\phi)}$$

$$\boldsymbol{S} = IVe^{j\phi} = IV(\cos\phi + j\sin\phi)$$

- Key definitions:
  - Re S – average power
  - Im S – reactive power
  - |S| – apparent power
  - $\cos\phi$ – power factor

Maximum Power Transfer



- For a source with impedance $Z_s = R_s + jX_s$, maximum power is transferred when load impedance $Z_L = Z_s^*$
  - Resistances are equal: $R_L = R_s$
  - Reactances are opposite: $X_L = -X_s$
- Magnitude of maximum power transfer:

$$P_{max} = \frac{V_{s,eff}^2}{4R_s}$$

Homework

- HW #38 due today at 4:30 pm in EE 325B
- HW #39 due Wed.: DeCarlo & Lin, Chapter 11:
  - Problem 24
  - Problem 26 [Correction: the power factor is 0.866.]