Recap from Monday

**Kirchoff’s Current Law (KCL)**
- Sum of all currents entering a node or Gaussian surface is zero at all times:
  \[
  \sum_{k=1}^{N} I_k(t) = 0, \text{ for all } t
  \]

**Kirchoff’s Voltage Law (KVL)**
- Voltage drop between any two nodes is direction-dependent and path-independent (i.e., \( V_{AB} = V_A - V_B \))
- Sum of voltage drops over any closed loop is zero
Gaussian Surfaces

• Hypothetical surface through which sum of currents must always be zero
• Clever choice of Gaussian surface can vastly simplify problem
• Example:
Series Resistive Circuits

- Calculate the effective resistance of $N$ resistors in series:

  ![Diagram of resistors in series](image)

- According to KVL:

  $$V_{N+1,1} = \sum_{l=1}^{N} V_{l+1,l}$$

  $$IR_{eq} = \sum_{l=1}^{N} IR_l$$

  $$R_{eq} = \sum_{l=1}^{N} R_l$$
Series Resistive Circuits

• Calculate the power dissipated by $N$ resistors in series
  — For each element:
    \[ P_k = I^2 R_k \]
  — For the circuit as a whole:
    \[ P = \sum_{l=1}^{N} I^2 R_l = I^2 \sum_{l=1}^{N} R_l \]
    \[ P = I^2 R_{eq} \]
Parallel Resistive Circuits

• Given a current source $I_o$ connected to $N$ parallel resistors, what is the equivalent resistance?

• Equal voltage drop across every resistor implies:
  \[ V = I_k R_k = I_o R_{eq} \]

• Kirchoff’s current law tells us that:
  \[
  I_o = \sum_{l=1}^{N} I_l \\
  \frac{V}{R_{eq}} = \sum_{l=1}^{N} \frac{V}{R_l} \\
  G_{eq} = \sum_{l=1}^{N} G_l
  \]
Parallel Resistive Circuits

• Calculate the power dissipated by \( N \) resistors in parallel
  
  – For each element:
  
  \[ P_k = I_k^2 R_k \]
  
  – For the circuit as a whole:
  
  \[ P = \sum_{l=1}^{N} I_l^2 R_l = V \sum_{l=1}^{N} I_l \]
  
  \[ P = I_o^2 R_{eq} \]
Series-Parallel Circuits

• Combines features of series and parallel circuits
• Depend on two reference points
• Solve iteratively with previous equations
Series-Parallel Circuits

• Example:

\[
\begin{align*}
\frac{1}{R_{12}} &= \frac{1}{R_3} + \frac{1}{R_2 + R_5} \\
\frac{1}{R_{eq}} &= \frac{1}{R_4} + \frac{1}{R_1 + R_{12}}
\end{align*}
\]
Summary

• Series resistors:

\[ R_{eq} = \sum R_l \]

\[ V_k = VR_k/R_{eq}; \text{ currents equal} \]

• Parallel resistors

\[ G_{eq} = \sum G_l \]

\[ I_k = IR_{eq}/R_k; \text{ voltages equal} \]

• Series-parallel circuits
  – Analyzed iteratively
Loading Effect

• Measuring voltage requires using a large but finite resistance

• Example:
  – 1.25 V nominal
  – 1.21 V measured

![Circuit Diagram]

200 Ω

1.5 V

1 kΩ

5 kΩ
Internal Resistance

- Internal resistance distinguishes real battery from ideal battery
- Would like internal resistance to be as low as possible
- Example:
  - 1.5 V nominal
  - 1.48 V measured
  - 80 mW heat loss
Internal Resistance

• Another example:
  – Current source
  – 1.5 V load
  – 1 A nominal
  – 0.97 A measured
  – 45 mW heat loss

\[ 1 \, \text{A} \quad \frac{1}{50 \, \Omega} \]
Voltage Sources

• Ideal voltage source:

\[ V = V_s \]

• Non-ideal voltage source:

\[ V = V_s - IR \]

Note: R is generally small
Ideal Current Sources

• Ideal current source

- Note: $R$ is generally large

• Non-ideal current source:
Homework

• HW #3 solution now posted
• HW #4 due today by 4:30 pm in EE 325B
• HW #5 due Friday: DeCarlo & Lin, Chapter 2:
  – Problem 25
  – Problem 32(b)
  – Problem 35
  – Problem 41