ECE 201, Section 4
Lecture 6

Prof. Peter Bermel

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Recap from Friday

- Series resistors:

$$R_{eq} = \sum R_l$$

$$V_k = VR_k/R_{eq}\text{; currents equal}$$

- Parallel resistors

$$G_{eq} = \sum G_l$$

$$I_k = IR_{eq}/R_k\text{; voltages equal}$$

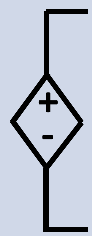
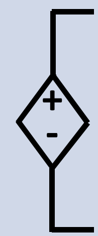
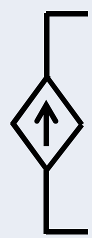
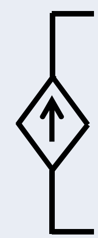
- Series-parallel circuits
  - Analyzed iteratively

Dependent Sources

- Can use current or voltage to control output current or voltage

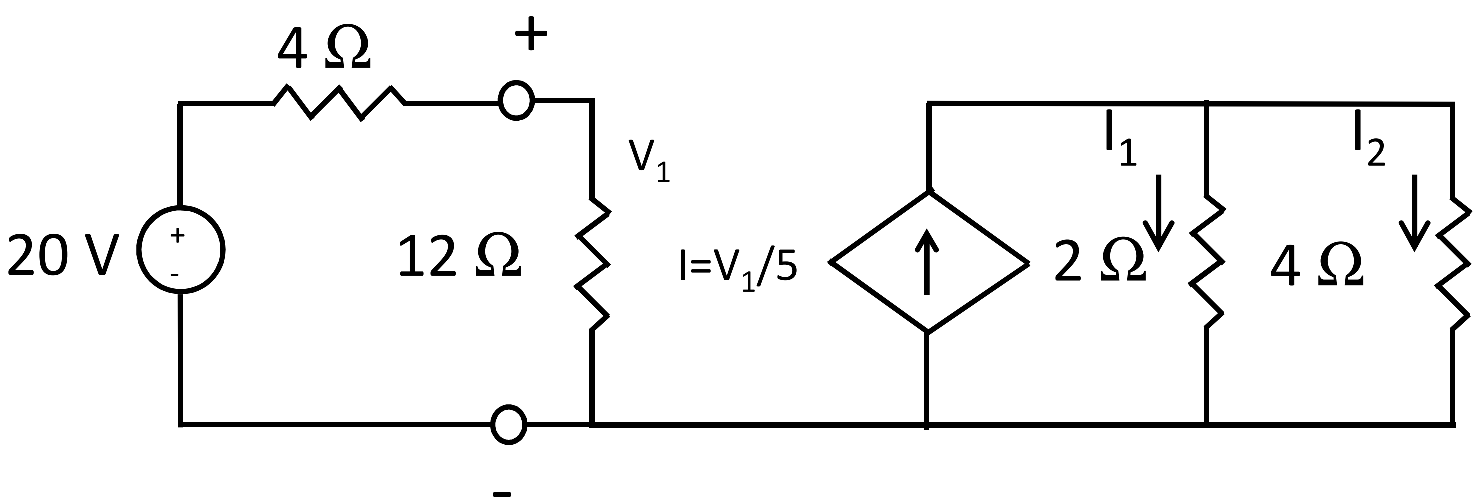
Control type

Output type

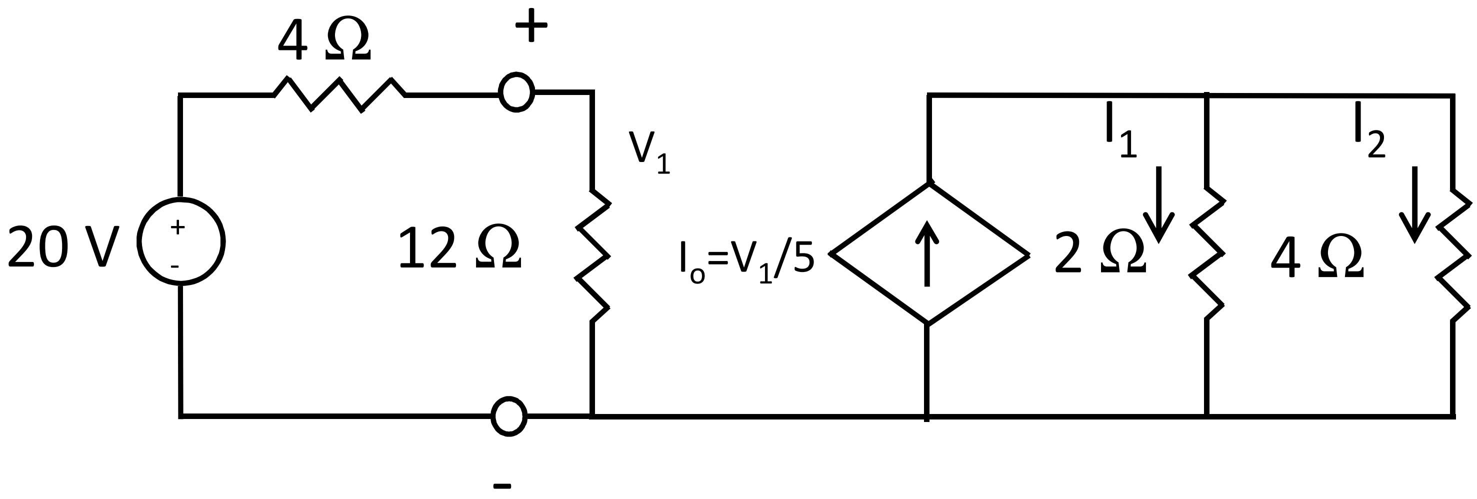
	Voltage	**Current**
Voltage	VCVS $V=\mu v_x$ 	CCVS $V=IR$ 
Current	VCCS $I=gV$ 	CCCS $I=\beta I_x$ 

Dependent Sources Example

- What is the output voltage and current, gain, and total power dissipated?



Dependent Sources Solution



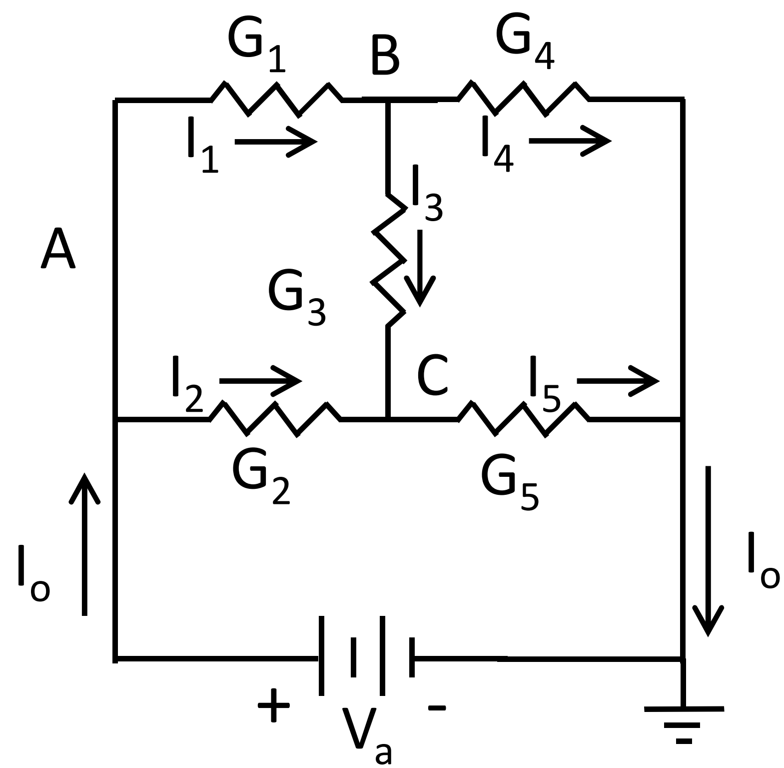
- Voltage division →V_1=15V, I_o=3A, I_1=2A, I_2=1A
- Gain g=I_2R_2/(20V)=0.2 (a 7 dB attenuator)
- Power dissipated=400/16+4*3=37 W

Nodal Analysis

- General linear circuits aren’t simple combination of series and parallel circuits
- Instead, must apply KCL and Ohm’s law to solve for voltage at all unknown nodes

Nodal Analysis Example

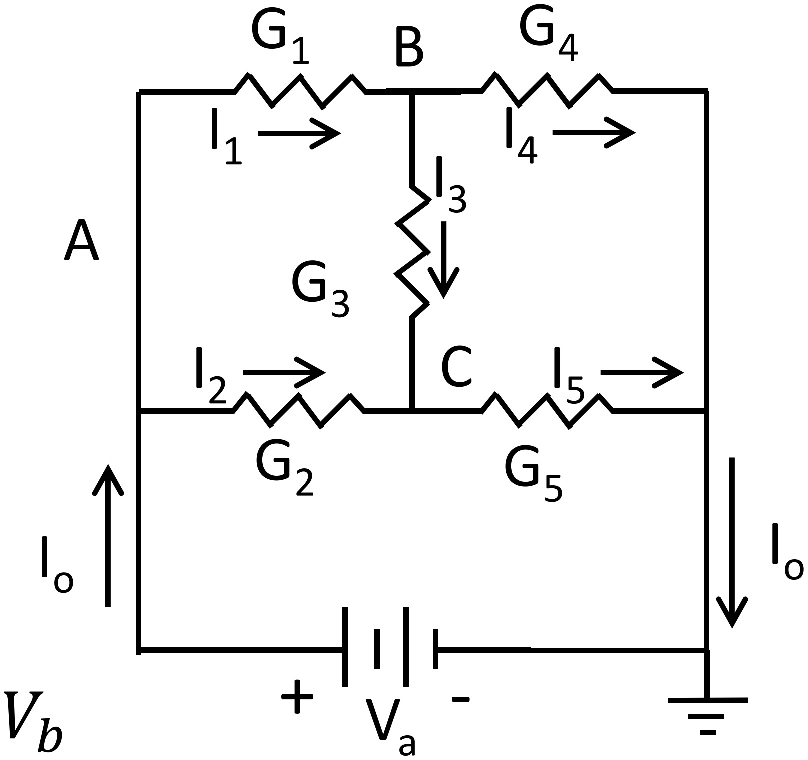
- For these 5 resistors with a voltage source, solve for the voltages and currents everywhere:



Nodal Analysis Example

- Using KCL:

$$I_o = I_1 + I_2$$
$$I_o = I_4 + I_5$$
$$I_1 = I_3 + I_4$$
$$I_5 = I_2 + I_3$$



- Using Ohm's Law:

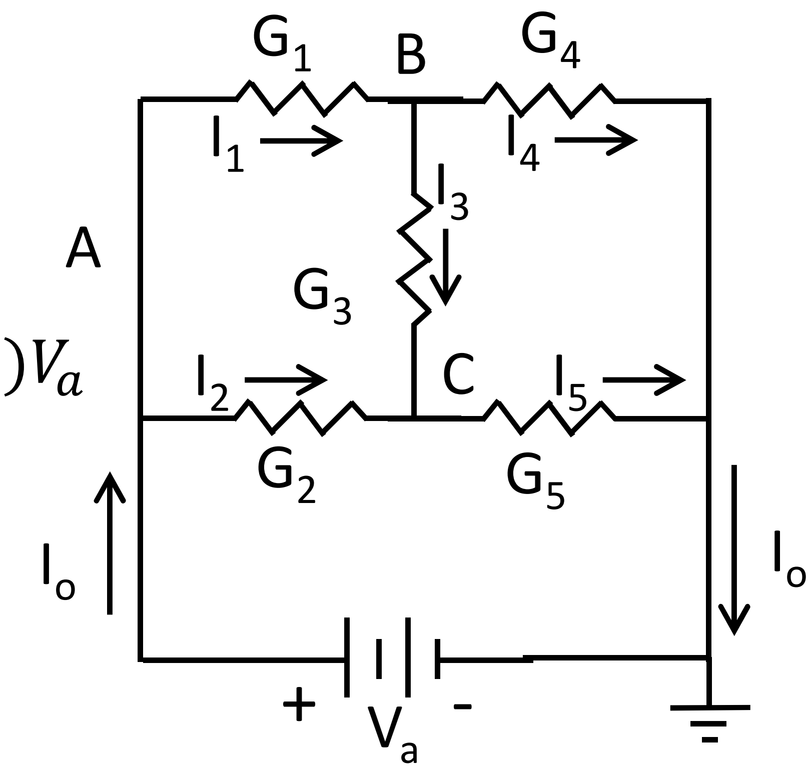
$$G_1(V_a - V_b) = G_3(V_b - V_c) + G_4 V_b$$
$$G_5 V_c = G_2(V_a - V_c) + G_3(V_b - V_c)$$
$$G_1(V_a - V_b) + G_2(V_a - V_c) = G_4 V_b + G_5 V_c$$

Nodal Analysis Example

- Grouping by voltages:

$$(G_1 + G_3 + G_4)V_b - G_3V_c = G_1V_a$$
$$-G_3V_b + (G_2 + G_3 + G_5)V_c = G_2V_a$$
$$(G_1 + G_4)V_b + (G_2 + G_5)V_c = (G_1 + G_2)V_a$$

- Third equation is sum of first two and can be eliminated; good cross-check
- Leaves 2 equations for 2 unknowns, V_b and V_c



Nodal Analysis Example

- Rewriting as matrix:

$$\begin{bmatrix} G_1 + G_3 + G_4 & -G_3 \\ -G_3 & G_2 + G_3 + G_5 \end{bmatrix} \begin{bmatrix} V_b \\ V_c \end{bmatrix} = \begin{bmatrix} G_1 V_a \\ G_2 V_a \end{bmatrix}$$

- Use matrix inversion formula:

$$M^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{\det(M)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

- Note $\det(M) = ad - bc$. Now check:

$$MM^{-1} = \frac{1}{\det(M)} \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Nodal Analysis Example

- Using matrix inversion formula, we obtain:

$$\begin{bmatrix} V_b \\ V_c \end{bmatrix} = \frac{1}{(G_1 + G_3 + G_4)(G_2 + G_3 + G_5) - {G_3}^2} \begin{bmatrix} G_2 + G_3 + G_5 & G_3 \\ G_3 & G_1 + G_3 + G_4 \end{bmatrix} \begin{bmatrix} G_1 V_a \\ G_2 V_a \end{bmatrix}$$

- If $\vec{G} = (0.2,0.4,0.5,0.1,0.7)$, $V_a = 5\ V$, then:

$$\begin{bmatrix} V_b \\ V_c \end{bmatrix} = \frac{1}{1.03} \begin{bmatrix} 1.6 & 0.5 \\ 0.5 & 0.8 \end{bmatrix} \begin{bmatrix} 0.2 \\ 0.4 \end{bmatrix} (5\ V) = \begin{bmatrix} 2.524\ V \\ 2.039\ V \end{bmatrix}$$

And $\vec{I} = (0.495, 1.185, 0.243, 0.252, 1.428)$

Finally, check KCL is obeyed

Nodal Analysis

- Should always be able to solve problems with 2 unknowns using matrix inversion formula
- What about more than 2 unknowns?
  - Adjoint method (calculate cofactor matrix, take transpose, divide by determinant)
  - Software techniques

Free, Web-Enabled Software

- SPICE on nanoHUB: https://nanohub.org/tools/spice3f4
- Falstad circuit simulator: http://www.falstad.com/circuit/index.html

Homework

- HW #4 solution now posted
- HW #5 due today by 4:30 pm in EE 325B
- HW #6 due Wednesday: DeCarlo & Lin, Chapter 2:
  - Problem 46
  - Problem 62 [In place of $P_{load} = 100\ P_{in}$, let $P_{load} = 10\ P_{in}$.]
  - Problem 63