Recap from Friday

• Series resistors:

\[ R_{eq} = \sum R_l \]

\[ V_k = VR_k / R_{eq}; \text{ currents equal} \]

• Parallel resistors

\[ G_{eq} = \sum G_l \]

\[ I_k = IR_{eq} / R_k; \text{ voltages equal} \]

• Series-parallel circuits
  – Analyzed iteratively
**Dependent Sources**

- Can use current or voltage to control output current or voltage

<table>
<thead>
<tr>
<th>Control type</th>
<th>Output type</th>
<th>Voltage</th>
<th>Current</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Voltage</strong></td>
<td>VCCS $V = \mu_v x$</td>
<td>CCVS $V = IR$</td>
<td></td>
</tr>
<tr>
<td><strong>Current</strong></td>
<td>VCCS $I = gV$</td>
<td>CCCS $I = \beta I_x$</td>
<td></td>
</tr>
</tbody>
</table>
Dependent Sources Example

• What is the output voltage and current, gain, and total power dissipated?

\[ I = \frac{V}{5} \]

\[ 4 \Omega \]

\[ 12 \Omega \]

\[ 20 \text{ V} \]

\[ V_1 \]

\[ I = \frac{V_1}{5} \]

\[ 2 \Omega \]

\[ 4 \Omega \]
• Voltage division $\rightarrow V_1 = 15\,\text{V}$, $I_o = 3\,\text{A}$, $I_1 = 2\,\text{A}$, $I_2 = 1\,\text{A}$
• Gain $g = \frac{I_2 R_2}{20\,\text{V}} = 0.2$ (a 7 dB attenuator)
• Power dissipated $= \frac{400}{16} + 4 \times 3 = 37\,\text{W}$
Nodal Analysis

• General linear circuits aren’t simple combination of series and parallel circuits
• Instead, must apply KCL and Ohm’s law to solve for voltage at all unknown nodes
Nodal Analysis Example

• For these 5 resistors with a voltage source, solve for the voltages and currents everywhere:
Nodal Analysis Example

- Using KCL:
  \[ I_o = I_1 + I_2 \]
  \[ I_o = I_4 + I_5 \]
  \[ I_1 = I_3 + I_4 \]
  \[ I_5 = I_2 + I_3 \]

- Using Ohm’s Law:
  \[ G_1 (V_a - V_b) = G_3 (V_b - V_c) + G_4 V_b \]
  \[ G_5 V_c = G_2 (V_a - V_c) + G_3 (V_b - V_c) \]
  \[ G_1 (V_a - V_b) + G_2 (V_a - V_c) = G_4 V_b + G_5 V_c \]
Nodal Analysis Example

• Grouping by voltages:
  \[(G_1 + G_3 + G_4)V_b - G_3 V_c = G_1 V_a\]
  \[-G_3 V_b + (G_2 + G_3 + G_5)V_c = G_2 V_a\]
  \[(G_1 + G_4)V_b + (G_2 + G_5)V_c = (G_1 + G_2)V_a\]

• Third equation is sum of first two and can be eliminated; good cross-check

• Leaves 2 equations for 2 unknowns, \(V_b\) and \(V_c\)
Nodal Analysis Example

• Rewriting as matrix:

\[
\begin{bmatrix}
G_1 + G_3 + G_4 & -G_3 \\
-G_3 & G_2 + G_3 + G_5
\end{bmatrix}
\begin{bmatrix}
V_b \\
V_c
\end{bmatrix}
= 
\begin{bmatrix}
G_1 V_a \\
G_2 V_a
\end{bmatrix}
\]

• Use matrix inversion formula:

\[
M^{-1} = \begin{bmatrix}
a & b \\
c & d
\end{bmatrix}^{-1} = \frac{1}{\text{det}(M)} \begin{bmatrix}
d & -b \\
-c & a
\end{bmatrix}
\]

• Note \(\text{det}(M) = ad - bc\). Now check:

\[
MM^{-1} = \frac{1}{\text{det}(M)} \begin{bmatrix}
a & b \\
c & d
\end{bmatrix} \begin{bmatrix}
d & -b \\
-c & a
\end{bmatrix} = \begin{bmatrix}1 & 0 \\
0 & 1\end{bmatrix}
\]
Nodal Analysis Example

• Using matrix inversion formula, we obtain:

\[
\begin{bmatrix}
V_b \\
V_c
\end{bmatrix}
= \frac{1}{(G_1 + G_3 + G_4)(G_2 + G_3 + G_5) - G_3^2}
\begin{bmatrix}
G_2 + G_3 + G_5 & G_3 \\
G_3 & G_1 + G_3 + G_4
\end{bmatrix}
\begin{bmatrix}
G_1V_a \\
G_2V_a
\end{bmatrix}
\]

• If \( \vec{G} = (0.2, 0.4, 0.5, 0.1, 0.7) \), \( V_a = 5 \ V \), then:

\[
\begin{bmatrix}
V_b \\
V_c
\end{bmatrix}
= \frac{1}{1.03}
\begin{bmatrix}
1.6 & 0.5 \\
0.5 & 0.8
\end{bmatrix}
\begin{bmatrix}
0.2 \\
0.4
\end{bmatrix}
(5 \ V)
= \begin{bmatrix}
2.524 \ V \\
2.039 \ V
\end{bmatrix}
\]

And \( \vec{I} = (0.495, 1.185, 0.243, 0.252, 1.428) \)

Finally, check KCL is obeyed
Nodal Analysis

• Should always be able to solve problems with 2 unknowns using matrix inversion formula

• What about more than 2 unknowns?
  – Adjoint method (calculate cofactor matrix, take transpose, divide by determinant)
  – Software techniques
Free, Web-Enabled Software

• SPICE on nanoHUB: https://nanohub.org/tools/spice3f4
Homework

• HW #4 solution now posted
• HW #5 due today by 4:30 pm in EE 325B
• HW #6 due Wednesday: DeCarlo & Lin, Chapter 2:
  – Problem 46
  – Problem 62 [In place of $P_{\text{load}} = 100\ P_{\text{in}}$, let $P_{\text{load}} = 10\ P_{\text{in}}$.]
  – Problem 63