ECE 201, Section 3
Lecture 7

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Nodal Analysis Recap

- For solving non-series-parallel circuits
- Uses KCL, V=IR, matrix algebra to calculate unknown voltages and currents
- Example of matrix for two unknowns:

$$\begin{bmatrix} G_1 + G_3 + G_4 & -G_3 \\ -G_3 & G_2 + G_3 + G_5 \end{bmatrix} \begin{bmatrix} V_b \\ V_c \end{bmatrix} = \begin{bmatrix} G_1 V_a \\ G_2 V_a \end{bmatrix}$$

- Solve with matrix inversion formula:

$$M^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{\det(M)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Nodal Analysis Recap

- Using matrix inversion formula, we obtain:

$$\begin{bmatrix} V_b \\ V_c \end{bmatrix} = \frac{1}{(G_1 + G_3 + G_4)(G_2 + G_3 + G_5) - {G_3}^2} \begin{bmatrix} G_2 + G_3 + G_5 & G_3 \\ G_3 & G_1 + G_3 + G_4 \end{bmatrix} \begin{bmatrix} G_1 V_a \\ G_2 V_a \end{bmatrix}$$

- Introducing the shorthand notation:

$$G_{235} = G_2 + G_3 + G_5$$
$$G_{134} = G_1 + G_3 + G_4$$

- Yields the solutions:

$$V_b = \left[\frac{G_{235} G_1 + G_2 G_3}{G_{134} G_{235} - G_3^2} \right] V_a$$
$$V_c = \left[\frac{G_{134} G_2 + G_1 G_3}{G_{134} G_{235} - G_3^2} \right] V_a$$

Nodal Analysis: # of Variables

- Should always be able to solve problems with 2 unknowns using matrix inversion formula
- What about more than 2 unknowns?
  - Substitution/elimination methods
  - Adjoint method (calculate cofactor matrix, take transpose, divide by determinant)
  - Software techniques

Substitution Method

- Formulate N equations with N unknowns
- Rewrite 1 equation in terms of 1 unknown, then **substitute** into the other equations, and iterate
- Example:

$$(G_1 + G_3 + G_4)V_b - G_3V_c = G_1V_a$$
$$-G_3V_b + (G_2 + G_3 + G_5)V_c = G_2V_a$$

- Rewrite:

$$V_c = (G_1 + G_3 + G_4)V_b/G_3 - G_1V_a/G_3$$

- Substitute:

$$-G_3V_b + (G_2 + G_3 + G_5)[(G_1 + G_3 + G_4)V_b/G_3 - G_1V_a/G_3] = G_2V_a$$

- Simplify:

$$\left[G_{235}G_{134} - G_3^2\right]V_b = [G_2G_3 + G_1G_{235}]V_a$$

Elimination Method

- Formulate N equations with N unknowns
- Add equations to each other to eliminate target variables
- For example, take:

$$(G_1 + G_3 + G_4)V_b - G_3V_c = G_1V_a$$
$$-G_3V_b + (G_2 + G_3 + G_5)V_c = G_2V_a$$

- Multiply each equation to eliminate V_c:

$$G_{235}\,[\;\; G_{134}V_b - G_3V_c = G_1V_a]$$
$$\underline{+\;\; G_3[-G_3V_b + G_{235}V_c = G_2V_a]}$$
$$[G_{235}G_{134} - G_3^2\,]V_b = [G_2G_3 + G_1G_{235}]V_a$$

Free, Web-Enabled Software

- SPICE on nanoHUB: https://nanohub.org/tools/spice3f4
- Falstad circuit simulator: http://www.falstad.com/circuit/index.html

Current Source Example

- Using KCL and KVL:

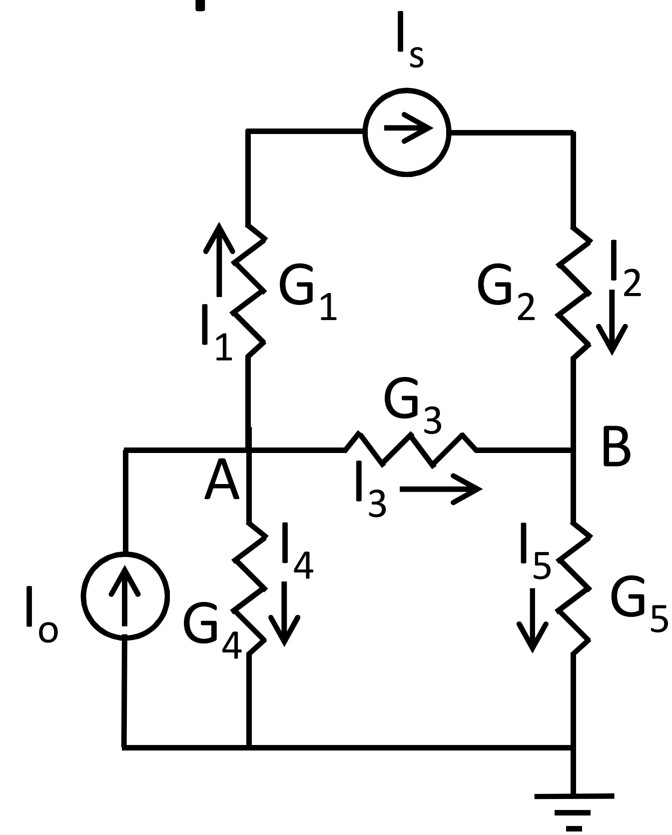
$$I_o = I_4 + I_5$$

$$I_5 = I_2 + I_3 = I_s + I_3$$

- Using Ohm's Law:

$$I_o = G_4 V_a + G_5 V_b$$

$$G_5 V_b = I_s + G_3 (V_a - V_b)$$



Current Source Example

- Combining & grouping terms:

$$-G_3 V_a + (G_3 + G_5) V_b = I_s$$
$$G_4 V_a + G_5 V_b = I_o$$

- In matrix form:

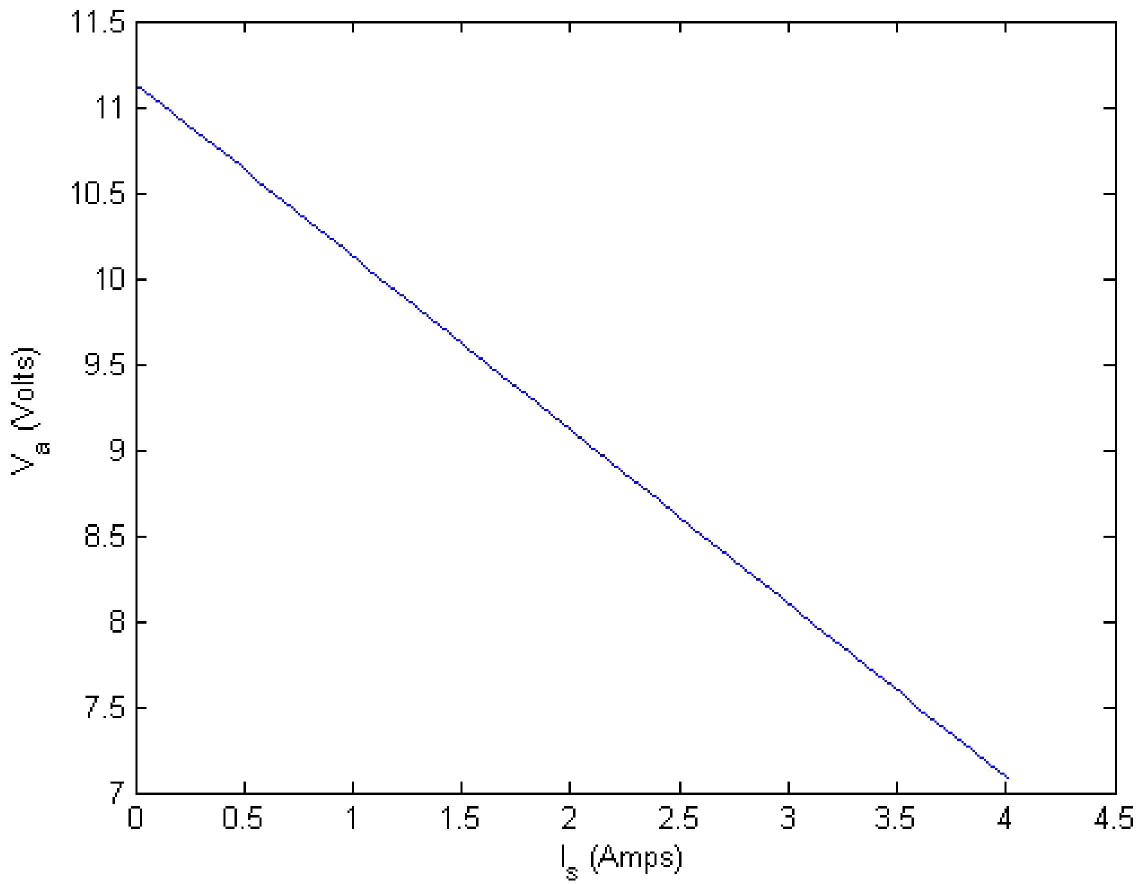
$$\begin{bmatrix} -G_3 & G_3 + G_5 \\ G_4 & G_5 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \end{bmatrix} = \begin{bmatrix} I_s \\ I_o \end{bmatrix}$$

- Solution:

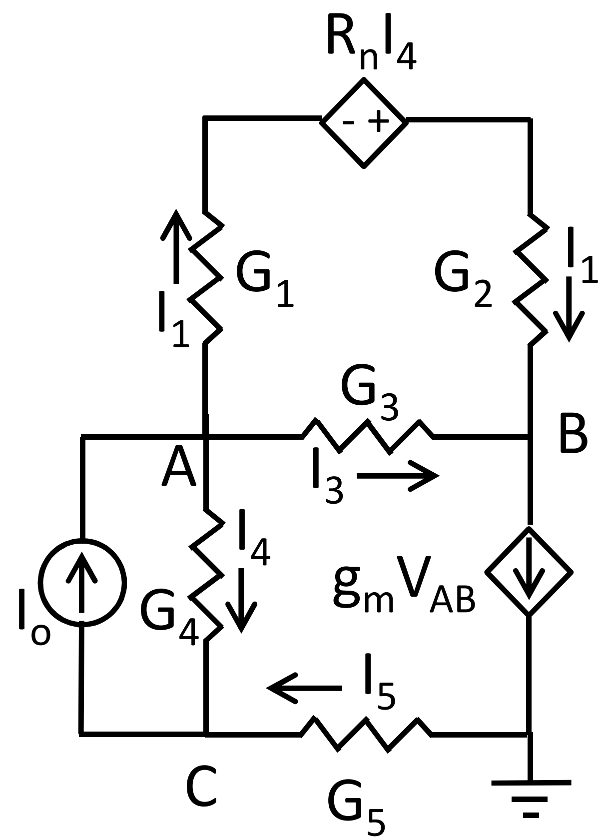
$$\begin{bmatrix} V_a \\ V_b \end{bmatrix} = \frac{1}{G_3 G_5 + G_4 G_{35}} \begin{bmatrix} -G_5 & G_3 + G_5 \\ G_4 & G_3 \end{bmatrix} \begin{bmatrix} I_s \\ I_o \end{bmatrix}$$

- Why doesn't solution depend on G_1 or G_2?

Voltage vs. Source Current



Controlled Sources



- Using KCL:

$$I_o = I_1 + I_3 + I_4$$
$$I_o = I_4 + I_5$$
$$I_1 + I_3 = I_5 = g_m(V_a - V_b)$$

- Using KVL:

$$V_a - V_b = I_1(R_1 + R_2) + R_n I_4$$

- Using Ohm's Law:

$$V_a - V_b = (I_o - I_3 - I_4)R_{12} + R_n G_4 (V_a - V_c)$$
$$I_o = G_4(V_a - V_c) - G_5 V_c$$
$$g_m(V_a - V_b) = -G_5 V_c$$

Controlled Sources

- Rearranging:

$$(G_{34}R_{12} + R_nG_4 + 1)V_a - (G_3R_{12} + 1)V_b + (R_n - R_{12})G_4V_c = I_oR_{12}$$

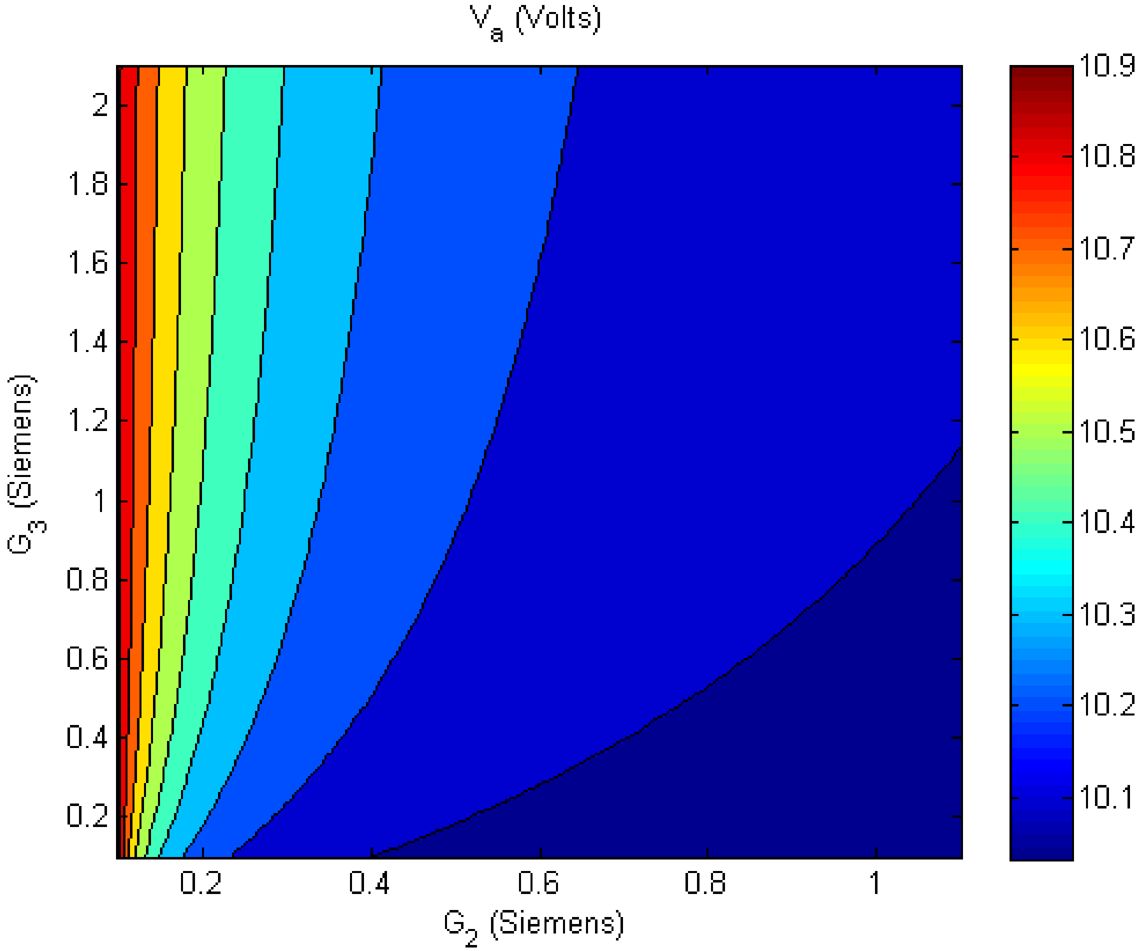
$$G_4V_a - (G_4 + G_5)V_c = I_o$$

$$g_mV_a - g_mV_b + G_5V_c = 0$$

- Which can be written as:

$$\begin{bmatrix} G_{34}R_{12} + R_nG_4 + 1 & -(G_3R_{12} + 1) & (R_n - R_{12})G_4 \\ G_4 & 0 & -(G_4 + G_5) \\ g_m & -g_m & G_5 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} I_oR_{12} \\ I_o \\ 0 \end{bmatrix}$$

Contour Plot of V_a



$G=(0.5,\ G_2,\ G_3,\ 0.5,\ 0.8,\ 2.0,\ 1.0)$

Homework

- HW #5 solution posted
- HW #6 due today by 4:30 pm in EE 325B
- HW #7 due Friday: DeCarlo & Lin, **Chapter 3**:
  - Problem 3
  - Problem 5
  - Problem 8(a),(b), and (c)