ECE 201, Section 3
Lecture 8

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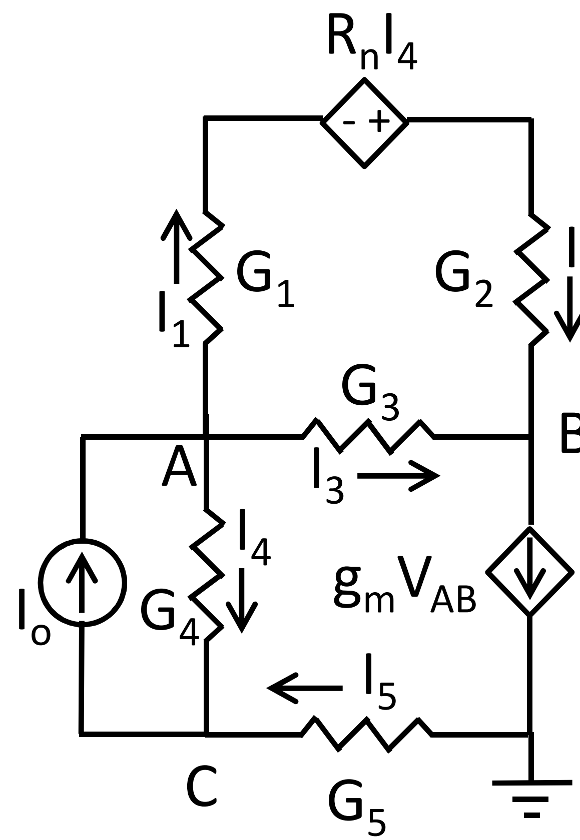
Nodal Analysis Recap

- For solving non-series-parallel circuits for networks of resistors and current sources (R-I_s-g_m networks)
- Uses KCL, V=IR, to formulate matrix for unknown voltages, written $MV = B$
- For 2 unknowns, use matrix inversion formula:

$$V = M^{-1}B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{\det(M)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

- For more than 2 unknowns:
  - Substitution/elimination methods
  - Adjoint method (calculate cofactor matrix, take transpose, divide by determinant)
  - Software techniques

Controlled Sources



- Using KCL:

$$I_o = I_1 + I_3 + I_4$$
$$I_o = I_4 + I_5$$
$$I_1 + I_3 = I_5 = g_m(V_a - V_b)$$

- Using KVL:

$$V_a - V_b = I_1(R_1 + R_2) + R_n I_4$$

- Using Ohm's Law:

$$V_a - V_b = (I_o - I_3 - I_4)R_{12} + R_n G_4(V_a - V_c)$$
$$I_o = G_4(V_a - V_c) - G_5 V_c$$
$$g_m(V_a - V_b) = -G_5 V_c$$

Controlled Sources

- Rearranging:

$$(G_{34}R_{12} + R_n G_4 + 1)V_a - (G_3 R_{12} + 1)V_b + (R_n - R_{12})G_4 V_c = I_o R_{12}$$

$$G_4 V_a - (G_4 + G_5)V_c = I_o$$

$$g_m V_a - g_m V_b + G_5 V_c = 0$$

- Which can be written as:

$$\begin{bmatrix} G_{34}R_{12} + R_n G_4 + 1 & -(G_3 R_{12} + 1) & (R_n - R_{12})G_4 \\ G_4 & 0 & -(G_4 + G_5) \\ g_m & -g_m & G_5 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} I_o R_{12} \\ I_o \\ 0 \end{bmatrix}$$

- Software easiest way to solve

Free Software

- SPICE on nanoHUB: https://nanohub.org/tools/spice3f4
- Falstad circuit simulator: http://www.falstad.com/circuit/index.html
- MATLAB, via: `whirlpool.ecn.purdue.edu`
- Octave (MATLAB substitute): http://octave.sourceforge.net

MATLAB code

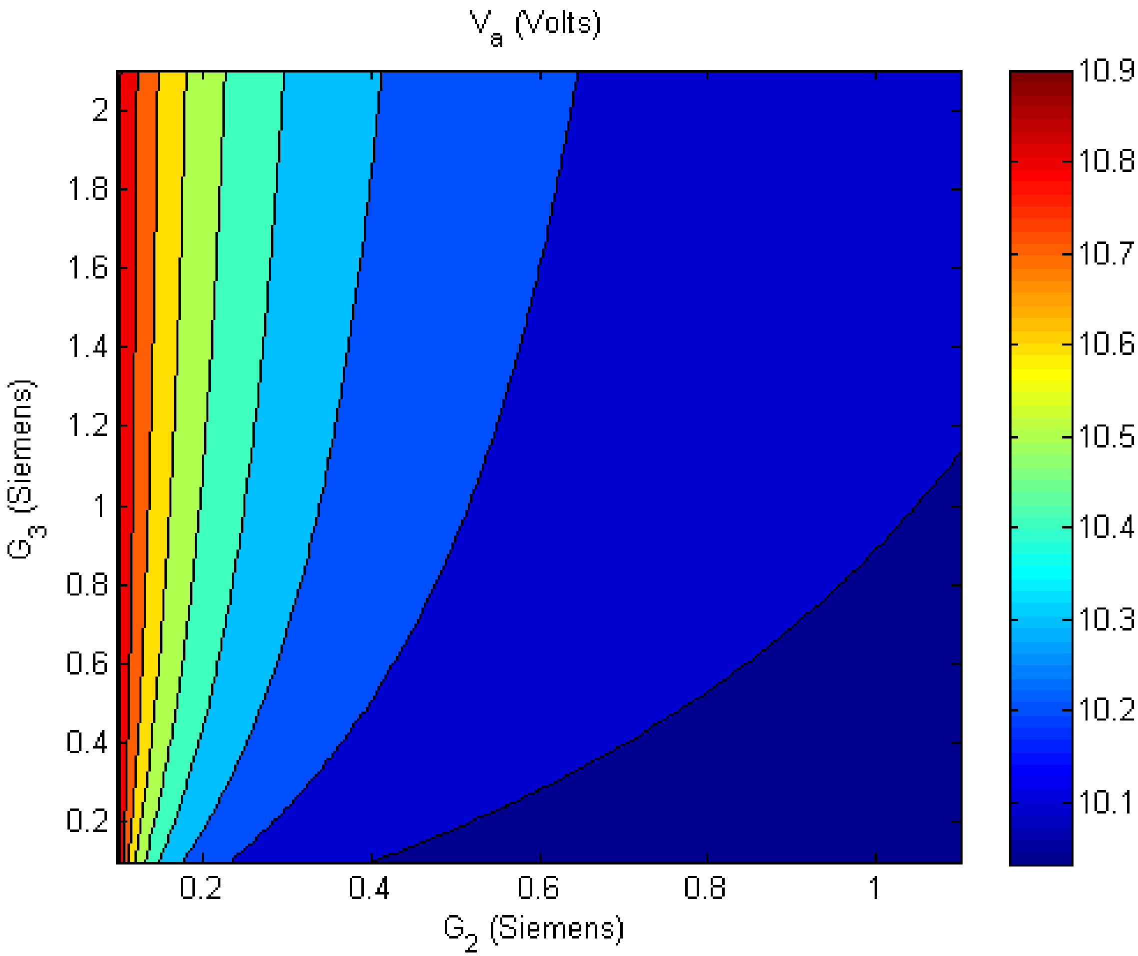
Editor - C:\Users\peter\Documents\classes\ece201-spring 2012\resistornetwork3.m

```
function [ V,I ] = resistornetwork3( G, Io )
%RESISTORNETWORK3 : calculates unknown voltages +
%   currents as a function of known resistor
%   conductances + ideal current supplied
R12=1/G(1)+1/G(2);
Rn=1/G(6);
gm=G(7);
M = [(G(3)+G(4))*R12+Rn*G(4)+1 -(G(3)*R12+1) (Rn-R12)*G(4); G(4) 0 -(G(4)+G(5)); gm -gm G(5)];
A = [Io*R12; Io; 0];
V = inv(M)*A;
upperdV=V(1)-V(2)-Rn*G(4)*(V(1)-V(3));
deltaV = [upperdV/(G(1)*R12) upperdV/(G(2)*R12) V(1)-V(2) V(1)-V(3) V(2)-V(3) 0 0];
I = G.*deltaV;
end
```

Calling the code

```
>> G=[0.7 0.2 0.3 0.5 0.8 2.0 0.1];
>> [V,I]=resistornetwork3(G,5)

V =

   10.3739
   11.5242
    0.1438


I =

  Columns 1 through 6

   -0.5768   -0.5768   -0.3451    5.1150    9.1044
```

Contour Plot of V_a



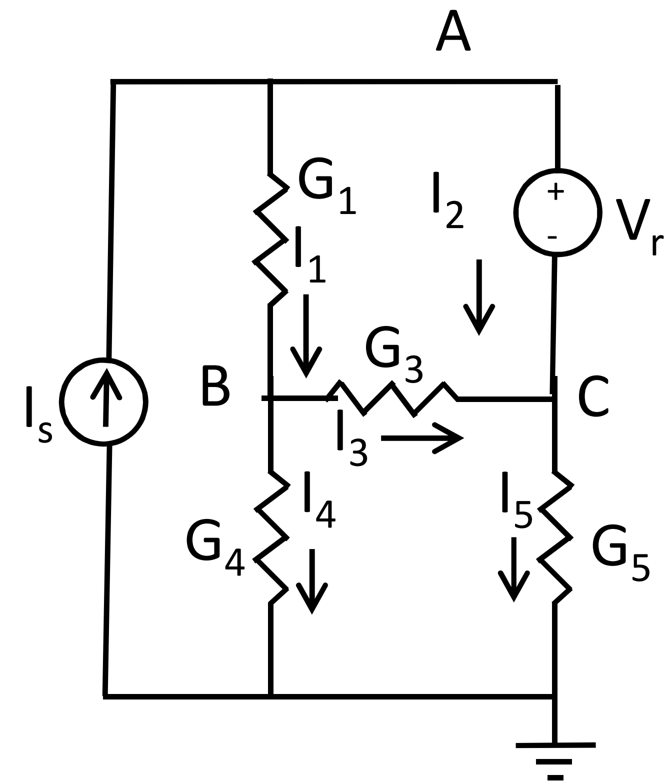
$G=(0.5,\ G_2,\ G_3,\ 0.5,\ 0.8,\ 2.0,\ 1.0)$

Circuit Analysis Techniques for Floating Voltage Sources

- Nodal Analysis
  - Shift ground to base of floating voltage
- Modified nodal analysis (MNA)
  - Associate an effective current with each floating voltage source
- Loop analysis
  - Also known as 'mesh analysis'
  - Alternative to MNA
  - Use KVL to calculate total voltage drop around each closed loop in a circuit
  - Reduces number of currents

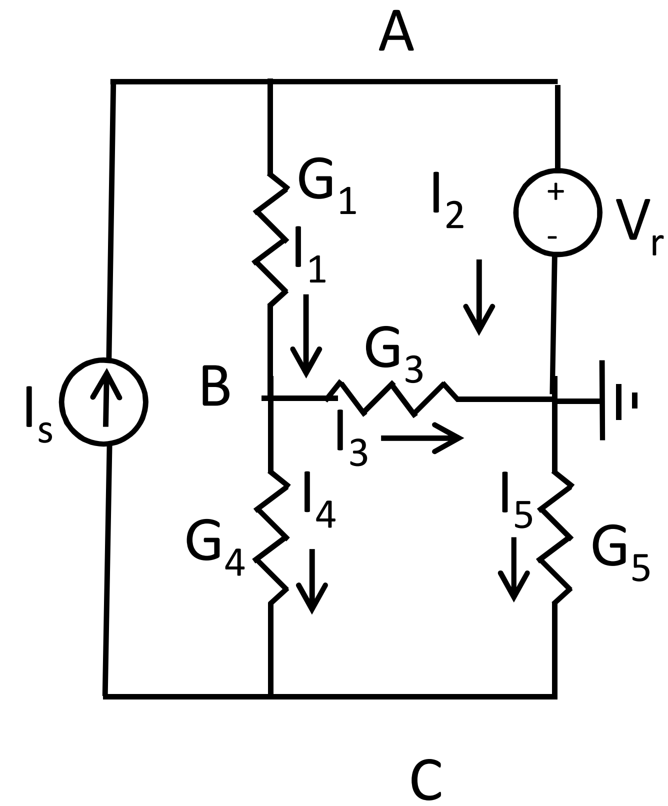
Floating Voltages: Nodal Analysis

- How to solve with KCL exclusively?



Floating Voltages: Nodal Analysis

- How to solve with KCL exclusively?
- For one floating voltage – move the ground
- Can use on next homework



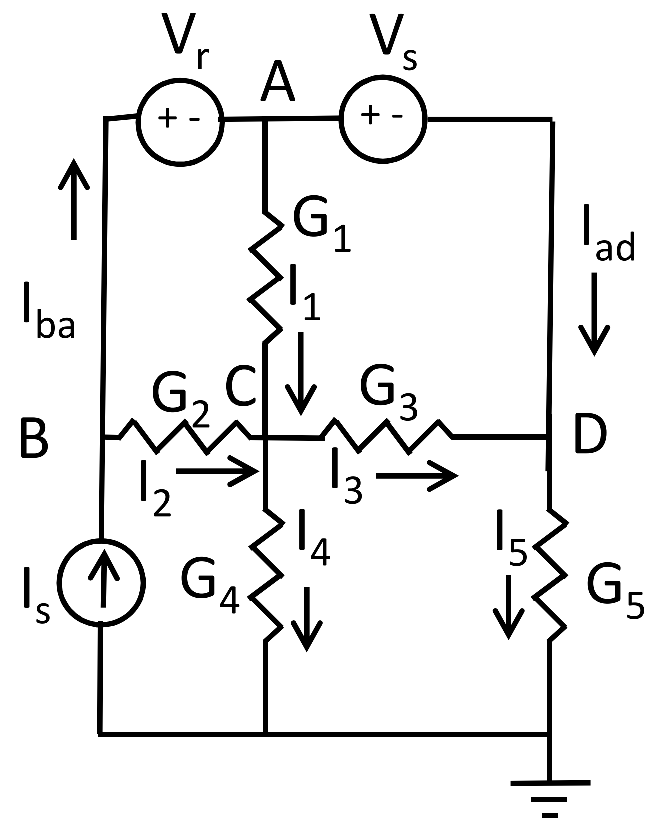
MNA Example

- Using KCL:

$$I_s = I_4 + I_5$$
$$I_s = I_2 + I_{ba}$$
$$I_1 + I_2 = I_3 + I_4$$
$$I_{ba} = I_1 + I_{ad}$$
$$I_{ad} + I_3 = I_5$$

- Using Ohm's Law:

$$I_s = G_4 V_c + G_5 V_d$$
$$I_s = G_2 V_{bc} + I_{ba}$$
$$G_1 V_{ac} + G_2 V_{bc} = G_3 V_{cd} + G_4 V_c$$
$$I_{ba} = G_1 V_{ac} + I_{ad}$$
$$I_{ad} + G_3 V_{cd} = G_5 V_d$$



MNA Example

- Grouping by each unknown (V_a, V_b, V_c, V_d,I_{ba},I_{ad}), with sources on right:

$$G_4V_c + G_5V_d = I_s$$

$$G_2V_b - G_2V_c + I_{ba} = I_s$$

$$G_1V_a + G_2V_b - (G_1 + G_2 + G_3 + G_4)V_c - G_3V_d = 0$$

$$G_1V_a - G_1V_c + I_{ad} - I_{ba} = 0$$

$$G_3V_c - (G_3 + G_5)V_d + I_{ad} = 0$$

$$V_b - V_a = V_r$$

$$V_a - V_d = V_s$$

MNA Example

$$\begin{bmatrix} 0 & G_2 & -G_2 & 0 & 1 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 & 0 \\ G_1 & G_2 & -\Sigma G_i & -G_3 & 0 & 0 \\ G_1 & 0 & -G_1 & 0 & -1 & 1 \\ 0 & 0 & G_3 & -G_{35} & 0 & 1 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \\ V_d \\ I_{ba} \\ I_{ad} \end{bmatrix} = \begin{bmatrix} I_s \\ V_r \\ V_s \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Known as a “sparse matrix”

MATLAB Code

Editor - C:\Users\peter\Documents\classes\ece201-spring 2012\resistormna.m

File Edit Text Go Cell Tools Debug Desktop Window Help

```
function [ V,I ] = resistormna( G, Is, Vr, Vs )
%RESISTORMNA : calculates unknown voltages +
%   currents as a function of known resistor
%   conductances + ideal currents + voltages supplied
%   using modified nodal analysis on a sparse matrix
G1234=G(1)+G(2)+G(3)+G(4);
% first line was: 0 0 G(4) G(5) 0 0
M = [0 G(2) -G(2) 0 1 0;-1 1 0 0 0 0;1 0 0 -1 0 0;G(1) G(2) -G1234 -G(3) 0 0;G(1) 0 -G(1) 0 -1
A = [Is Vr Vs 0 0 0]'
V = inv(M)*A;
deltaV = [V(1)-V(3) V(2)-V(3) V(3)-V(4) V(3) V(4)];
I = G.*deltaV;
%Check vector
check = [Is-G(2)*(V(2)-V(3))-V(5) Vr+V(1)-V(2) Vs+V(4)-V(1) G(1)*V(1)+G(2)*V(2)-G1234*V(3)-G(3)
end
```

Calling MATLAB Code

```
>> G=rand(1,5)

G =

    0.1419    0.4218    0.9157    0.7922    0.9595

>> [V,I] = resistormna(G,2,4,6)
Current check OK

V =

    6.0870
   10.0870
    2.2180
    0.0870
   -1.3189
   -1.8678


I =

    0.5490    3.3189    1.9514    1.7571    0.0835
```

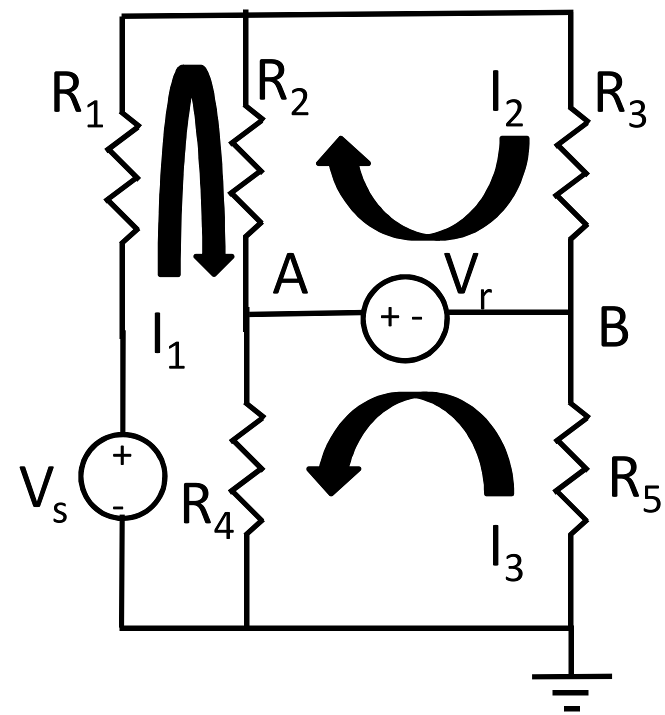
Loop Analysis

- Applying KVL around each loop:

$$R_4(I_1 + I_3) + R_2(I_1 - I_2) + R_1 I_1 = V_s$$

$$R_2(I_2 - I_1) + R_3 I_2 = V_r$$

$$R_4(I_3 + I_1) + R_5 I_3 = V_r$$



Loop Analysis

- Rearranging as a matrix equation:

$$\begin{bmatrix} R_4 & 0 & R_4 + R_5 \\ -R_2 & R_2 + R_3 & 0 \\ R_{124} & -R_2 & R_4 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_r \\ V_r \\ V_s \end{bmatrix}$$

MATLAB Code

```
function [ V,I ] = resistorloop( R, Vr, Vs )
%RESISTORLOOP : calculates unknown voltages +
%   currents as a function of known resistances + voltages supplied
%   using loop analysis on a sparse matrix
R124=R(1)+R(2)+R(4);
M = [R(4) 0 R(4)+R(5);-R(2) R(2)+R(3) 0;R124 -R(2) R(4)];
A = [Vr Vr Vs]';
I = inv(M)*A;
deltaI = [I(1) I(1)-I(2) I(2) I(1)-I(3) -I(3)];
V = deltaI.*R;
```

Calling MATLAB Code

```
>> R=rand(1,5)

R =

    0.7577    0.7431    0.3922    0.6555    0.1712

>> [V,I]=resistorloop(R,4,6)

V =

    3.5881   -1.4025    2.5975    2.3932   -0.1856


I =

    4.7352
    6.6225
    1.0841
```

Homework

- HW #7 due today by 4:30 pm in EE 325B
- HW #8 due Monday: DeCarlo & Lin, Chapter 3:
  – Problem 14 [Correction: Relabel "(c)" → "(b)", "(d)" → "(c)", and "(e)" → "(d)".]
  – Problem 26 (requires Octave/MATLAB)
  – Problem 34