Nodal Analysis Recap

• For solving non-series-parallel circuits for networks of resistors and current sources (R-I_s-g_m networks)

• Uses KCL, V=IR, to formulate matrix for unknown voltages, written $MV = B$

• For 2 unknowns, use matrix inversion formula:

\[
V = M^{-1}B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{\det(M)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}
\]

• For more than 2 unknowns:
  – Substitution/elimination methods
  – Adjoint method (calculate cofactor matrix, take transpose, divide by determinant)
  – Software techniques
Controlled Sources

- **Using KCL:**
  \[ I_o = I_1 + I_3 + I_4 \]
  \[ I_o = I_4 + I_5 \]
  \[ I_1 + I_3 = I_5 = g_m(V_a - V_b) \]

- **Using KVL:**
  \[ V_a - V_b = I_1 (R_1 + R_2) + R_n I_4 \]

- **Using Ohm’s Law:**
  \[ V_a - V_b = (I_o - I_3 - I_4)R_{12} + R_n G_4 (V_a - V_c) \]
  \[ I_o = G_4 (V_a - V_c) - G_5 V_c \]
  \[ g_m (V_a - V_b) = -G_5 V_c \]
Controlled Sources

• Rearranging:
\[
(G_{34}R_{12} + R_n G_4 + 1)V_a - (G_3R_{12} + 1)V_b + (R_n - R_{12})G_4 V_c = I_o R_{12}
\]
\[
G_4 V_a - (G_4 + G_5)V_c = I_o
\]
\[
g_m V_a - g_m V_b + G_5 V_c = 0
\]

• Which can be written as:
\[
\begin{bmatrix}
G_{34}R_{12} + R_n G_4 + 1 & -(G_3R_{12} + 1) & (R_n - R_{12})G_4 \\
G_4 & 0 & -(G_4 + G_5) \\
g_m & -g_m & G_5
\end{bmatrix}
\begin{bmatrix}
V_a \\
V_b \\
V_c
\end{bmatrix}
= \begin{bmatrix}
I_o R_{12} \\
I_o \\
0
\end{bmatrix}
\]

• Software easiest way to solve
Free Software

• SPICE on nanoHUB: https://nanohub.org/tools/spice3f4
• MATLAB, via: whirlpool.ecn.purdue.edu
• Octave (MATLAB substitute): http://octave.sourceforge.net
function [ V, I ] = resistornetwork3( G, Io )

% RESISTORNWORK3 calculates unknown voltages +
% currents as a function of known resistor
% conductances + ideal current supplied

R12 = 1/G(1) + 1/G(2);
Rn = 1/G(6);
gm = G(7);
M = [(G(3) + G(4)) * R12 + Rn * G(4) + 1 - (G(3) * R12 + 1) * (Rn - R12) * G(4); G(4) 0 - (G(4) + G(5)); gm - gm G(5)];
A = [Io * R12; Io; 0];
V = inv(M) * A;
upperdV = V(1) - V(2) - Rn * G(4) * (V(1) - V(3));
deltaV = [upperdV / (G(1) * R12) upperdV / (G(2) * R12) V(1) - V(2) V(1) - V(3) V(2) - V(3) 0 0];
I = G .* deltaV;
end
Calling the code

\[ \begin{align*}
\text{for } G &= [0.7 \ 0.2 \ 0.3 \ 0.5 \ 0.8 \ 2.0 \ 0.1]; \\
\text{[V,I]} &= \text{resistornetwork3}(G,5) \\

V &= \\
10.3739 & 11.5242 & 0.1438 \\

I &= \\
\text{Columns 1 through 6} \\
-0.5768 & -0.5768 & -0.3451 & 5.1150 & 9.1044
\end{align*} \]
Contour Plot of $V_a$

$G=(0.5, G_2, G_3, 0.5, 0.8, 2.0, 1.0)$
Circuit Analysis Techniques for Floating Voltage Sources

• Nodal Analysis
  – Shift ground to base of floating voltage
• Modified nodal analysis (MNA)
  – Associate an effective current with each floating voltage source
• Loop analysis
  – Also known as ‘mesh analysis’
  – Alternative to MNA
  – Use KVL to calculate total voltage drop around each closed loop in a circuit
  – Reduces number of currents
Floating Voltages: Nodal Analysis

- How to solve with KCL exclusively?
Floating Voltages: Nodal Analysis

- How to solve with KCL exclusively?
- For one floating voltage – move the ground
- Can use on next homework
MNA Example

- Using KCL:
  \[ I_s = I_4 + I_5 \]
  \[ I_s = I_2 + I_{ba} \]
  \[ I_1 + I_2 = I_3 + I_4 \]
  \[ I_{ba} = I_1 + I_{ad} \]
  \[ I_{ad} + I_3 = I_5 \]

- Using Ohm’s Law:
  \[ I_s = G_4 V_c + G_5 V_d \]
  \[ I_s = G_2 V_{bc} + I_{ba} \]
  \[ G_1 V_{ac} + G_2 V_{bc} = G_3 V_{cd} + G_4 V_c \]
  \[ I_{ba} = G_1 V_{ac} + I_{ad} \]
  \[ I_{ad} + G_3 V_{cd} = G_5 V_d \]
MNA Example

- Grouping by each unknown \((V_a, V_b, V_c, V_d, I_{ba}, I_{ad})\), with sources on right:
  
  \[
  G_4 V_c + G_5 V_d = I_s \\
  G_2 V_b - G_2 V_c + I_{ba} = I_s \\
  G_1 V_a + G_2 V_b - (G_1 + G_2 + G_3 + G_4) V_c - G_3 V_d = 0 \\
  G_1 V_a - G_1 V_c + I_{ad} - I_{ba} = 0 \\
  G_3 V_c - (G_3 + G_5) V_d + I_{ad} = 0 \\
  V_b - V_a = V_r \\
  V_a - V_d = V_s
  \]
MNA Example

\[
\begin{bmatrix}
0 & G_2 & -G_2 & 0 & 1 & 0 \\
-1 & 1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & -1 & 0 & 0 \\
G_1 & G_2 & -\Sigma G_i & -G_3 & 0 & 0 \\
G_1 & 0 & -G_1 & 0 & -1 & 1 \\
0 & 0 & G_3 & -G_{35} & 0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
V_a \\
V_b \\
V_c \\
V_d \\
I_{ba} \\
I_{ad} \\
\end{bmatrix}
=
\begin{bmatrix}
I_s \\
V_r \\
V_s \\
0 \\
0 \\
0 \\
\end{bmatrix}
\]

Known as a “sparse matrix”
function [ V, I ] = resistormna( G, Is, Vr, Vs )

% RESISTORMNA : calculates unknown voltages +
% conductances + ideal currents + voltages supplied
% using modified nodal analysis on a sparse matrix

G1234 = G(1) + G(2) + G(3) + G(4);
% first line was: 0 0 G(4) G(5) 0 0
M = [0 G(2) -G(2) 0 1 0; -1 1 0 0 0 0; 1 0 0 -1 0 0; G(1) G(2) -G1234 -G(3) 0 0; G(1) 0 -G(1) 0 -1]
A = [Is Vr Vs 0 0 0]
V = inv(M) * A;

deltaV = [V(1) - V(3) V(2) - V(3) V(3) - V(4) V(3) V(4)];
I = G .* deltaV;

% Check vector
check = [Is - G(2) * (V(2) - V(3)) - V(5) Vr + V(1) - V(2) Vs + V(4) - V(1) G(1) * V(1) + G(2) * V(2) - G1234 * V(3) - G(3)];
end
Calling MATLAB Code

>> G=rand(1,5)

G =

    0.1419    0.4218    0.9157    0.7922    0.9595

>> [V,I] = resistormna(G,2,4,6)
Current check OK

V =

    6.0870
   10.0870
    2.2180
    0.0870
   -1.3189
   -1.8678

I =

    0.5490    3.3189    1.9514    1.7571    0.0835
Loop Analysis

• Applying KVL around each loop:

\[ R_4(I_1 + I_3) + R_2(I_1 - I_2) + R_1 I_1 = V_s \]
\[ R_2(I_2 - I_1) + R_3 I_2 = V_r \]
\[ R_4(I_3 + I_1) + R_5 I_3 = V_r \]
Loop Analysis

• Rearranging as a matrix equation:

\[
\begin{bmatrix}
R_4 & 0 & R_4 + R_5 \\
-R_2 & R_2 + R_3 & 0 \\
R_{124} & -R_2 & R_4
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2 \\
I_3
\end{bmatrix}
= \begin{bmatrix}
V_r \\
V_r \\
V_s
\end{bmatrix}
\]
MATLAB Code

function [ V, I ] = resistorloop( R, Vr, Vs )
    % RESISTORLOOP : calculates unknown voltages +
    % currents as a function of known resistances + voltages supplied
    % using loop analysis on a sparse matrix
    R124 = R(1) + R(2) + R(4);
    M = [ R(4) 0 R(4) + R(5); -R(2) R(2) + R(3) 0; R124 -R(2) R(4) ];
    A = [ Vr Vr Vs ]';
    I = inv(M) * A;
    deltaI = [ I(1) I(1) - I(2) I(2) I(1) - I(3) - I(3) ];
    V = deltaI .* R;

Calling MATLAB Code

>> R=rand(1,5)

R =

0.7577    0.7431    0.3922    0.6555    0.1712

>> [V,I]=resistorloop(R,4,6)

V =

3.5881   -1.4025    2.5975    2.3932   -0.1856

I =

4.7352
6.6225
1.0841
Homework

• HW #7 due today by 4:30 pm in EE 325B
• HW #8 due Monday: DeCarlo & Lin, Chapter 3:
  – Problem 14 [Correction: Relabel “(c)” → “(b)”, “(d)” → “(c)”, and “(e)” → “(d)”.
  – Problem 26 (requires Octave/MATLAB)
  – Problem 34