ECE 201, Section 3
Lecture 9

Prof. Peter Bermel

September 10, 2012

Exam #1: Thursday, Sep. 20

- Time: 6:30-7:30 pm
- Place: WTHR 200
- Review session: Mon., Sep. 17 from 6:30-7:30 pm (MATH 175)
- Posted 3 practice exams + solutions

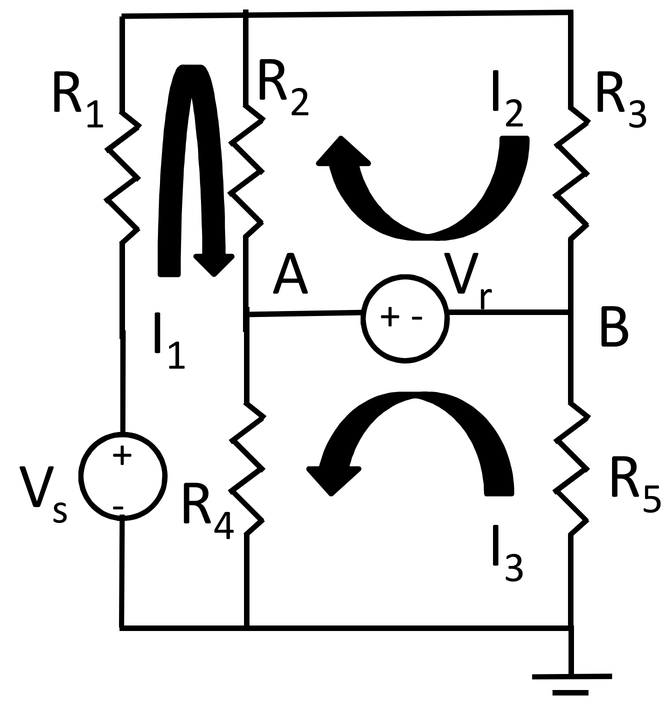
Loop Analysis

- Applying KVL around each loop:

$$R_4(I_1 + I_3) + R_2(I_1 - I_2) + R_1 I_1 = V_s$$

$$R_2(I_2 - I_1) + R_3 I_2 = V_r$$

$$R_4(I_3 + I_1) + R_5 I_3 = V_r$$



Loop Analysis

- Rearranging as a matrix equation:

$$\begin{bmatrix} R_4 & 0 & R_4 + R_5 \\ -R_2 & R_2 + R_3 & 0 \\ R_{124} & -R_2 & R_4 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_r \\ V_r \\ V_s \end{bmatrix}$$

MATLAB Code

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```
function [ V,I ] = resistorloop( R, Vr, Vs )
%RESISTORLOOP : calculates unknown voltages +
%   currents as a function of known resistances + voltages supplied
%   using loop analysis on a sparse matrix
R124=R(1)+R(2)+R(4);
M = [R(4) 0 R(4)+R(5);-R(2) R(2)+R(3) 0;R124 -R(2) R(4)];
A = [Vr Vr Vs]';
I = inv(M)*A;
deltaI = [I(1) I(1)-I(2) I(2) I(1)-I(3) -I(3)];
V = deltaI.*R;
```

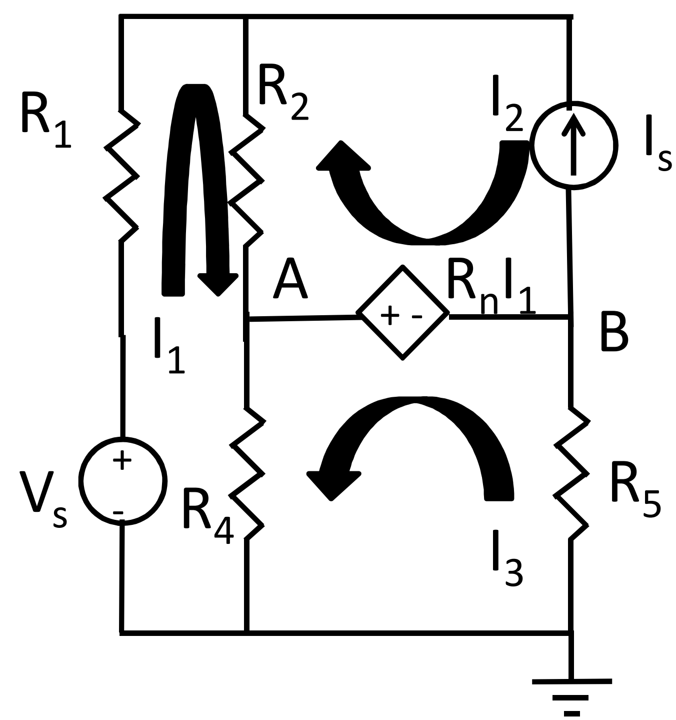
Calling MATLAB Code

```
>> R=rand(1,5)

R =

    0.7577    0.7431    0.3922    0.6555    0.1712

>> [V,I]=resistorloop(R,4,6)

V =

    3.5881   -1.4025    2.5975    2.3932   -0.1856


I =

    4.7352
    6.6225
    1.0841
```

Loop Analysis

- Applying KVL around each loop:

$$R_4(I_1 + I_3) + R_2(I_1 - I_2) + R_1 I_1 = V_s$$
$$R_2(I_2 - I_1) - v_1 = R_n I_1$$
$$R_4(I_3 + I_1) + R_5 I_3 = R_n I_1$$
$$I_2 = -I_s$$



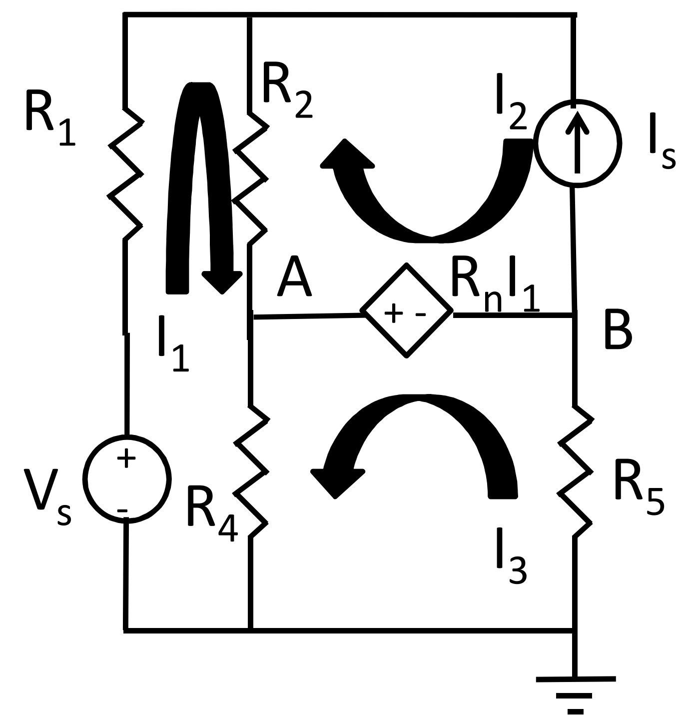
Loop Analysis

- Combining KVL + constraints:

$$[R_1 + R_2 + R_4]I_1 + R_4 I_3 = V_s - R_2 I_s$$

$$(R_2 + R_n)I_1 + v_1 = -R_2 I_s$$

$$(R_4 - R_n)I_1 + (R_4 + R_5)I_3 = 0$$



Loop Analysis

- Rearranging as a matrix equation:

$$\begin{bmatrix} R_{124} & R_4 & 0 \\ R_2 + R_n & 0 & 1 \\ R_4 - R_n & R_4 + R_5 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_3 \\ v_1 \end{bmatrix} = \begin{bmatrix} V_s - R_2 I_s \\ -R_2 I_s \\ 0 \end{bmatrix}$$

Loop Analysis

- Power delivered at each circuit element:
  - Resistors: can use $P=I^2R$
  - Voltage sources: can use $P=IV$
  - Current sources: use $P=IV$
- Note that powers should sum to zero, due to energy conservation

MATLAB Code

Editor - C:\Users\peter\Documents\classes\ece201-spring 2012\resistorloop2.m

```
function [ V,I,P ] = resistorloop2( R, Vs, Is )
%RESISTORLOOP2 : calculates unknown voltages +
%   currents as a function of known resistances + voltage and current supplied
%   using loop analysis on a sparse matrix
Rn=R(3);
R124=R(1)+R(2)+R(4);
M = [R124 R(4) 0;R(2)+Rn 0 1;R(4)-Rn R(4)+R(5) 0];
A = [Vs-R(2)*Is -R(2)*Is 0]';
I = inv(M)*A;
I2=-Is;
I3=I(2);
deltaI = [I(1) I(1)-I2 I(1) I(1)+I3 -I3];
V = deltaI.*R;
P=[I(1)^2*R(1) (I2-I(1))^2*R(2) Is*I(3) -Rn*I(1)*(I2+I3) -Vs*I(1) R(4)*(I(1)+I3)^2 I3^2*R(5)];
sum(P);
I(2)=I2;I(3)=I3;
end
```

Calling MATLAB Code

```
>> R=10.0*rand(1,5);
>> [V,I,P]=resistorloop2(R,10,-5)

V =

   10.4154  -11.3549    6.8484   10.9395    4.0911


I =

    3.0599
    5.0000
   -1.6037


P =

   31.8699   22.0298  -22.5325  -23.2588  -30.5988   15.9293    6.5611

>> sum(P)

ans =

  3.1974e-014
```

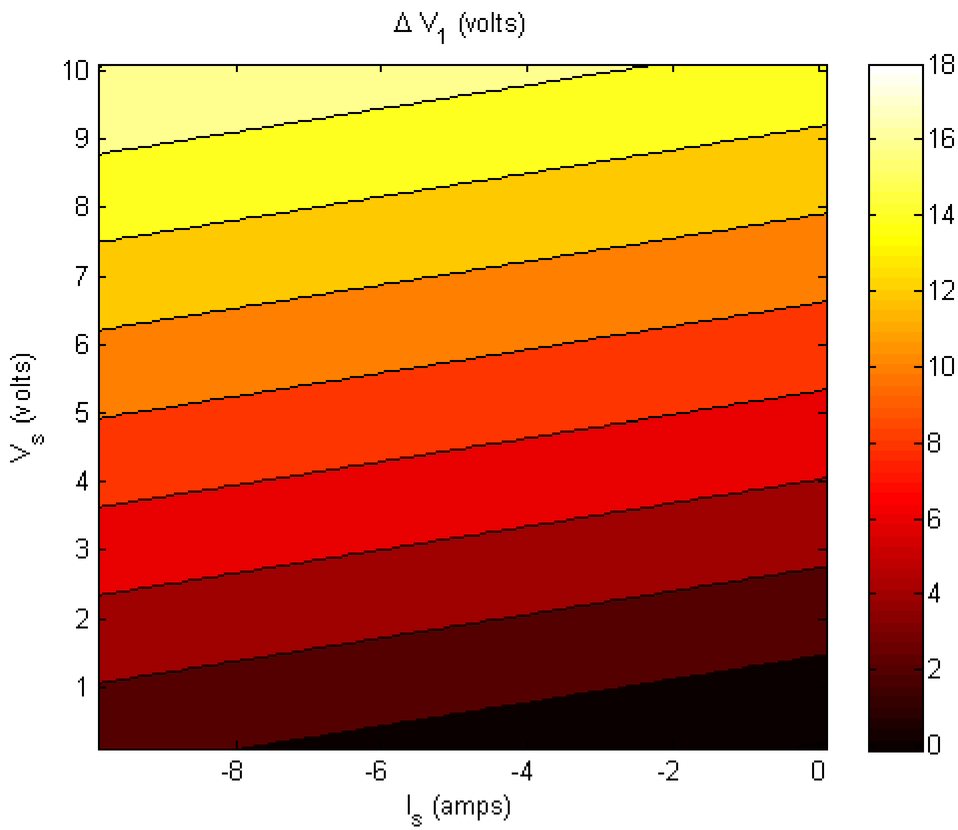
Linearity Theorem

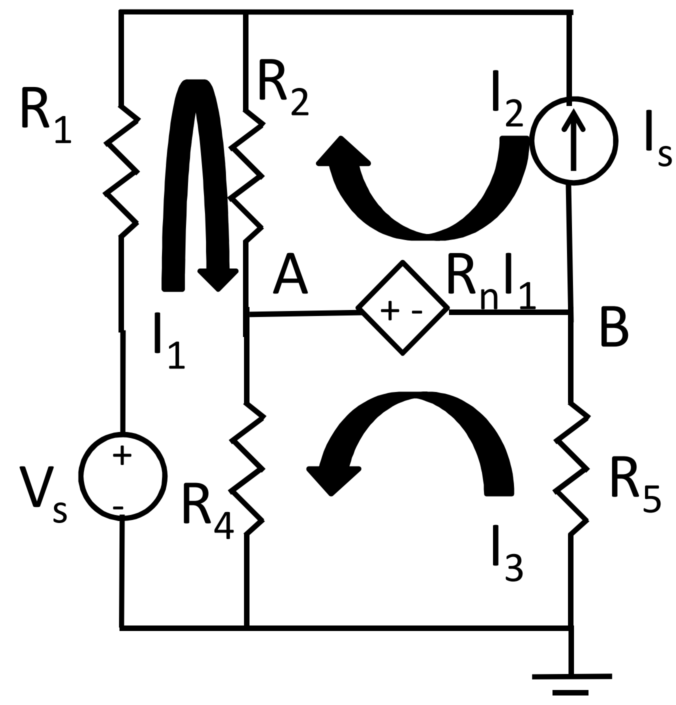
- For linear resistive circuits, output voltages and currents are a linear combination of independent sources, i.e.:

$$V_A = \sum_{k=1}^{N} [\alpha_k V_k + \beta_k I_k]$$

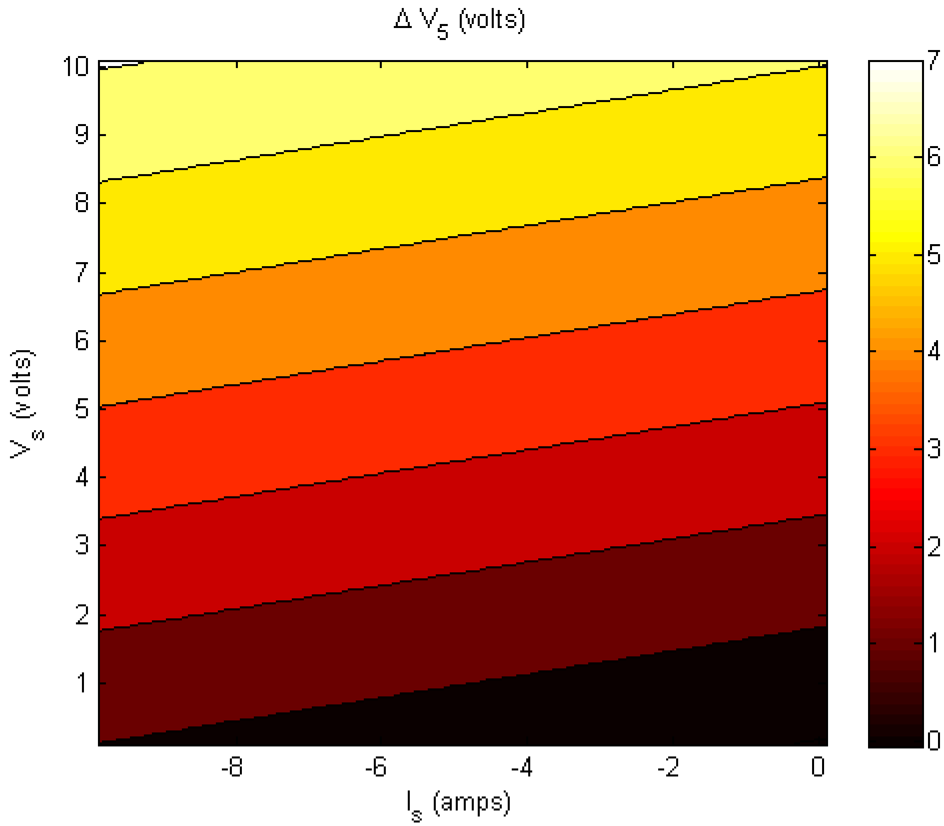
$$I_A = \sum_{k=1}^{N} [\alpha_k V_k + \beta_k I_k]$$

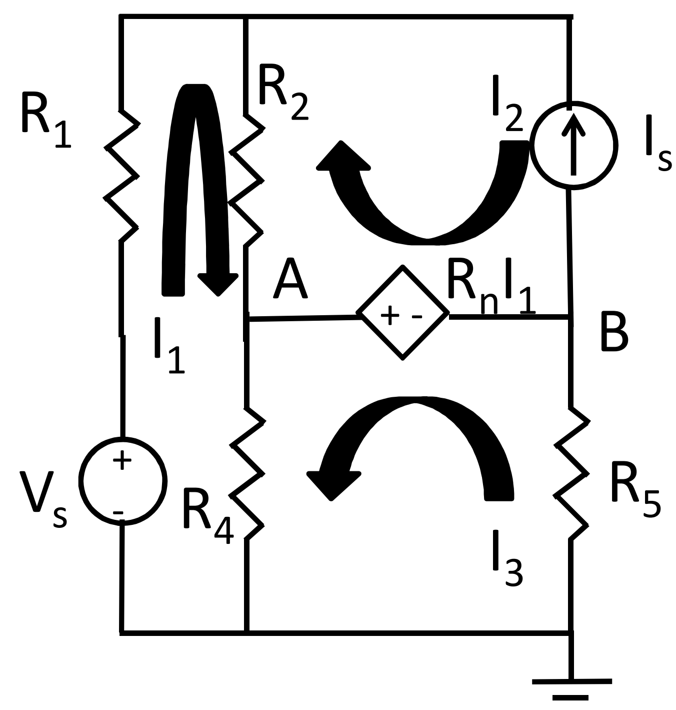
Linearity Example: 3-Loop Circuit



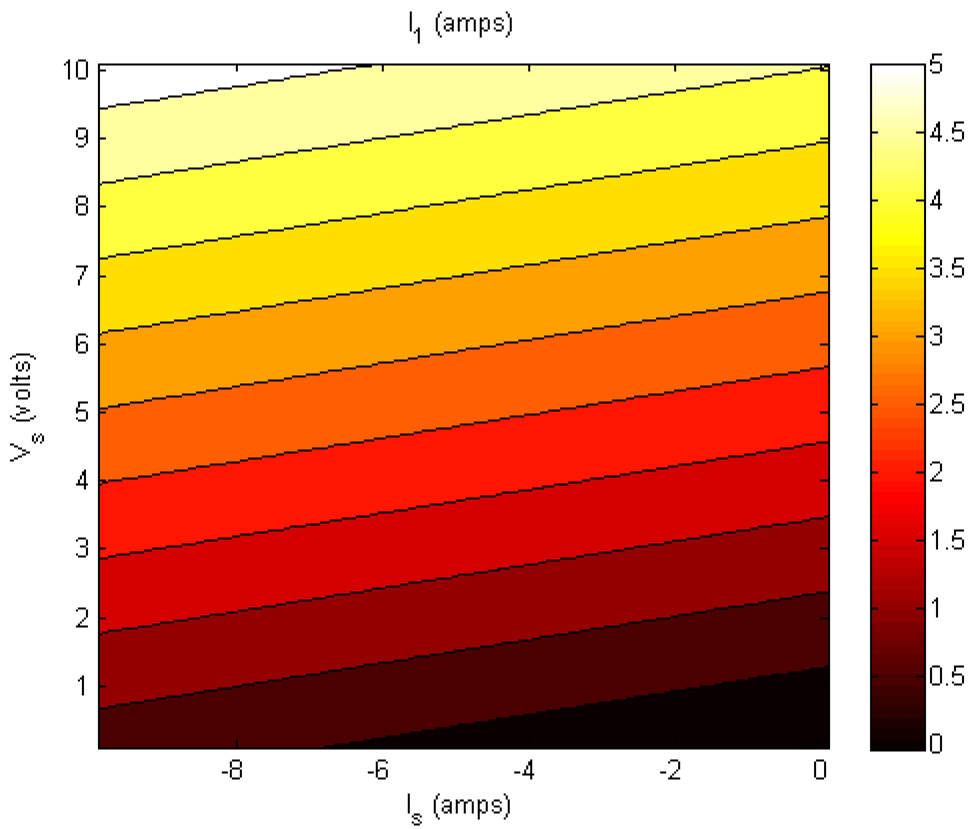


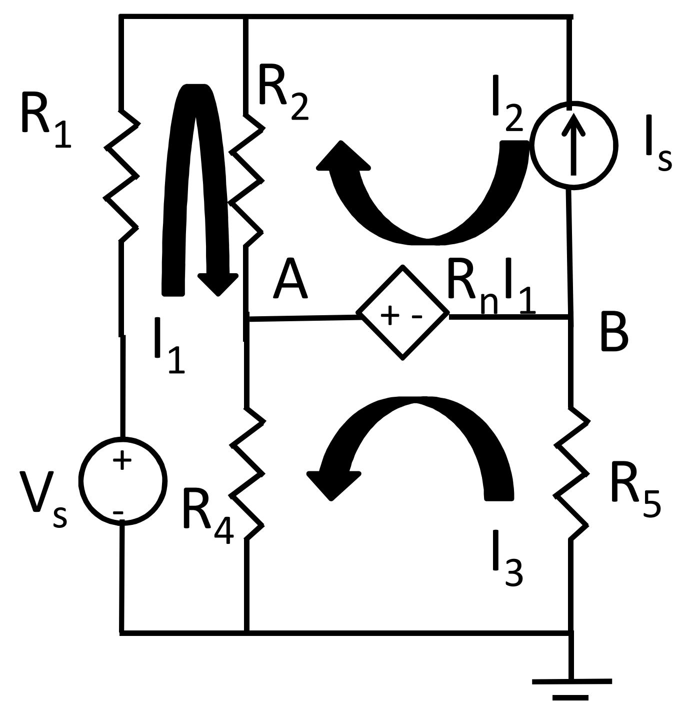
Linearity Example: 3-Loop Circuit





Linearity Example: 3-Loop Circuit





Superposition Property

- Superposition property: total output of linear circuit is sum of contributions from each independent source
- Special case of linearity property
- Applies to current and voltage but not power (in DC circuits)

Proportionality Property

- Proportionality property: multiplying independent source amplitude by α changes corresponding output term by factor of α
- Again, applies to current and voltage but not power (in DC circuits)
- Will lead to some interesting tricks next time!

Homework

- HW #8 due today by 4:30 pm in EE 325B
- HW #9 due Wednesday: DeCarlo & Lin, Chapter 3:
  – Problem 39
  – Problem 41
  – Problem 43