

ECE 414 – Spring 2016

Homework #1

Out: 01.12

Due: 01.14

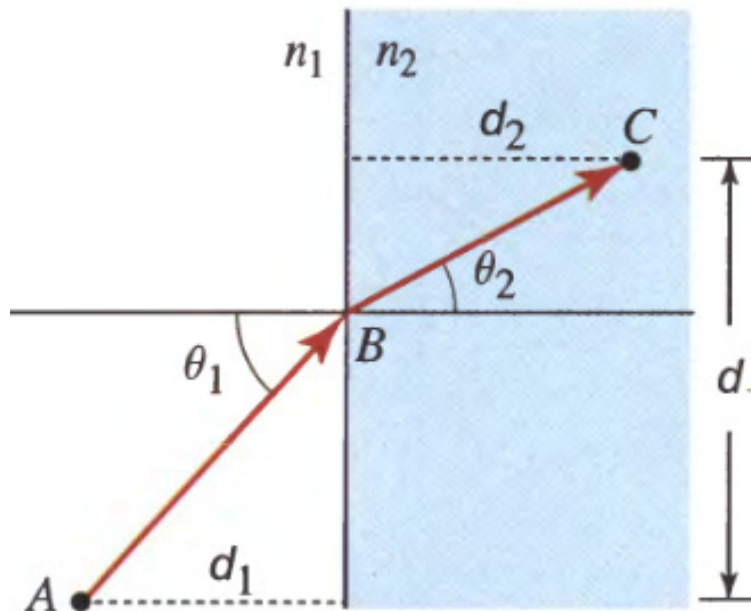
1. (Exercise 1.1-1) Proof of Snell's Law: Snell's law can be proven through application of Fermat's principle (i.e., that light rays travel paths taking the least time). Referring to the diagram below, this is equivalent to minimizing the optical path length $n_1 \overline{AB} + n_2 \overline{BC}$ between points A and C. We therefore have the following optimization problem: Minimize:

$$n_1 d_1 \sec \theta_1 + n_2 d_2 \sec \theta_2$$

with respect to angles θ_1 and θ_2 , subject to the condition

$$d_1 \tan \theta_1 + d_2 \tan \theta_2 = d$$

Show that the solution of this constrained minimization problem yields Snell's law.



2. (Exercise 1.2-1) Image Formation by a Spherical Mirror: Show that within the paraxial (small-angle) approximation, rays originating from a point $P_1 = (y_1, z_1)$ are reflected to a point $P_2 = (y_2, z_2)$, where z_1 and z_2 satisfy the relations:

$$\frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{f}$$

and

$$y_2 = -y_1 z_2 / z_1$$

(see figure below). This means that rays from each point in the plane $z = z_1$ meet at a single corresponding point in the plane $z = z_2$, so that the mirror acts as an image-forming system with magnification $-z_2/z_1$. Negative magnification means that the image is inverted.

