ECE 595 (Numerical Simulations) – Homework 3

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Due February 20, 2013 at 4:30 pm
Please write your programs in C/C++ or MATLAB

1 Consider a series of numbers generated from the following recurrence relation:

$$z_{n+1} = f z_n + \sqrt{1-f^2} r_n \tag{1}$$

Where $z_1 = r_1$ and each r_n is an independent sampling from a random distribution of Gaussian variables with zero mean and unit variance (e.g., see Fig. 1). The series length $N = 1000$.

1a. Plot this series for $f = 0,\ 0.5,\ 0.75,\ 0.99,$ and 0.999. What happens as f increases, and what could be its physical relevance?

1b. Plot the autocorrelation function $g_2(k) = \frac{1}{N}\sum_{i=1}^{N} z_i z_{i+k}$ for each f. What is its average dependence on f for a fixed $k \neq 0$? Hint: focus on modest values of k.

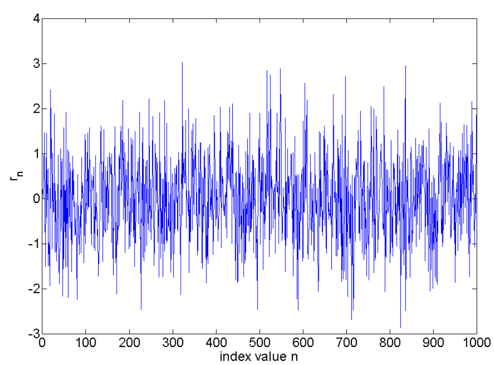

Figure 1: An example of random data r_n, used as an input to generate the recurrence relation z_n, where $N = 1000$.

2 Consider a checkerboard, consisting of alternating high and low dielectric squares, with each square measuring 20 x 20 pixels, and 10 on each side (as shown in Fig. 2).

2a. Calculate the 2D Fourier transform of this checkerboard, and plot the amplitude of the Fourier components in 2D. How can you explain the features observed?

2b. While still in the Fourier domain, multiply your data by a 2D Gaussian:

$$H(k_x, k_y) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{\left(k_x^2 + k_y^2\right)}{\sigma^2}\right] \tag{2}$$

and perform an inverse Fourier transform, back to the original image space. What is different about the image now, and how does it vary with the standard deviation σ of the Gaussian? Create at least one plot for a value of σ where the structure is clearly different from the original value, but not trivial (i.e., not a field of white).

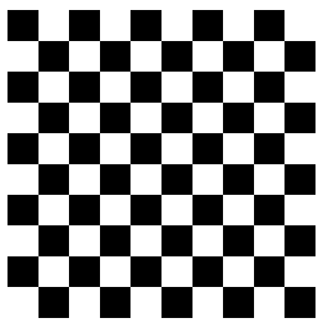

Figure 2: Checkerboard pattern for Problem 2, consisting of 5 black and 5 white squares on each side, with each square measuring 20 x 20 pixels. The total image dimensions are 200 x 200 pixels.