ECE 595, Section 10
Numerical Simulations
Lecture 14: Beam Propagation Method

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Outline

- Recap from Friday
- Derivation of Beam Propagation Method
- Nonlinear Schrodinger equation
- Comparison of BPM Strategies
  - FFT
  - Uniform spatial grid
  - Finite element
- Perfectly Matched Layers

Recap from Friday

- Rationale for FFTW
- Planning DFTs
- Executing DFTs
  - Basic interface
  - Advanced interface
- BPM example

Recap: Beam Propagation

- Starting from the Helmholtz equation:

$$-\nabla^2\psi = \left(\frac{n\omega}{c}\right)^2 \psi$$

- One can assume a solution of the form:

$$\psi = \phi e^{-j\beta z}$$

- Where ϕ is slowly varying, which gives rise to:

$$-\nabla^2\phi + 2j\beta\hat{z}\cdot\nabla\phi = k_\perp^2\phi$$

Beam Propagation

- To simplify problem, drop second derivatives in z – now we can write as:

$$\frac{\partial \phi}{\partial z} = \frac{j}{2\beta} \nabla_{\perp}^{2} \phi + \frac{j k_{\perp}^{2}}{2\beta} \phi$$

- Can simplify by defining two operators:

$$U = \frac{j}{2\beta} \nabla_{\perp}^{2}$$

$$W = \frac{j k_{\perp}^{2}}{2\beta}$$

$$\frac{\partial \phi}{\partial z} = (U + W)\phi$$

Nonlinear Schrodinger Equation

- Can derive expressions suitable for understanding fibers with dispersion and Kerr nonlinearity:

$$U = -\frac{j\beta_2}{2}\frac{\partial^2}{\partial t^2}$$

$$W = -\alpha + \frac{j\kappa}{2}|\phi|^2$$

$$\frac{\partial\phi}{\partial z} = (U + W)\phi$$

Nonlinear Schrodinger Equation

- In the presence of nonlinearity, don't actually know the value of $W(z+h)$
- Can obtain the result iteratively
  - Use $W(z)$ to evaluate $W(z+h)$
  - Work backwards to refine guess for $\phi(z+h)$
- After a few iterations, generally reach a self-consistent solution

Beam Propagation

- For a small z-step of size *h*, we can formally write a solution:

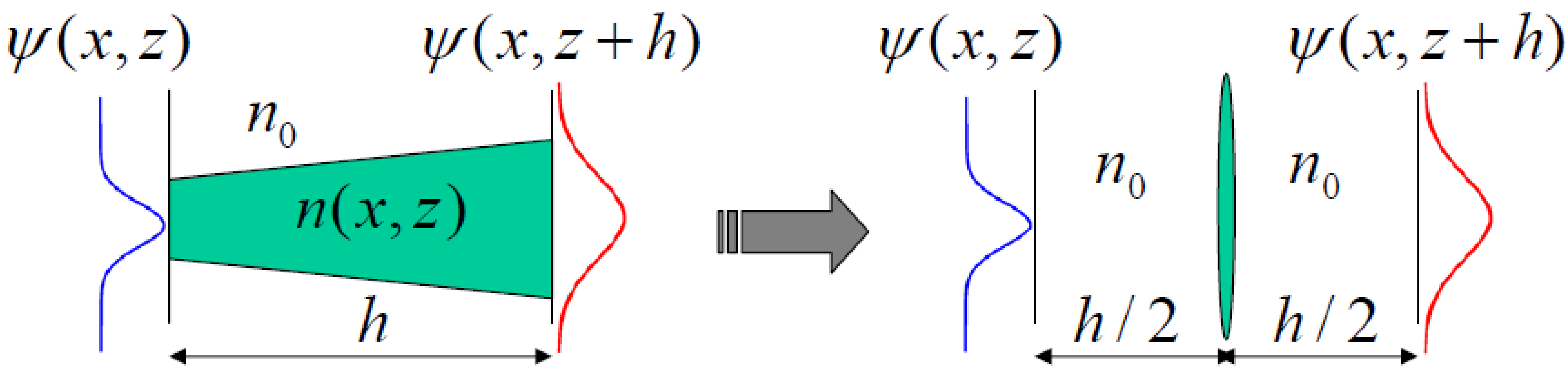
$$\phi(z+h) = e^{h(U+W)}\phi(z)$$

- If we know that U and W operators commute, we can rewrite as:

$$\phi(z+h) = e^{hU}e^{hW}\phi(z)$$
$$\phi(z+h) = e^{hU/2}e^{hW}e^{hU/2}\phi(z)$$

Beam Propagation

- Split-step method
  - Propagate half a step with the Laplacian
  - Propagate linear phase shift over the full distance
  - Propagate half a step with the Laplacian



BPM Strategies

- Most important decision is handling inhomogeneity well
- Possible strategies:
  - FFT
  - Uniform spatial grid
  - Finite-element method

FFT BPM

- Well-suited for diffraction step, where we can rephrase the operator as:

$$U = -\frac{j}{2\beta}(k + G)_{\perp}^{2}$$

- Can transform before and then back afterwards, via FFT

Uniform Spatial Grid BPM

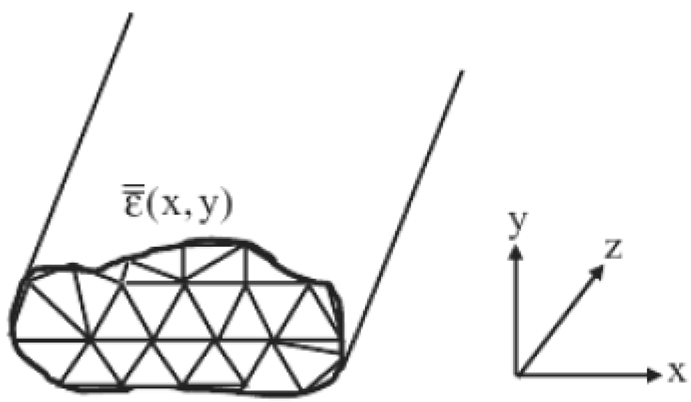
- Reformulate Laplacian in 2D with:

$$\nabla^2\phi \approx \frac{\phi_{i-N} + \phi_{i-1} - 4\phi_i + \phi_{i+1} + \phi_{i+N}}{h^2}$$

- Where *h* is the grid spacing

Finite Element BPM

- Finite element method consists of dividing a spatial domain in 1D, 2D or 3D into a mesh
- Mesh generally has D+1 vertices
- Solution can take various forms, but usually a tent function within each D+1-gon



Finite Element BPM

- In general, can formulate FE problems as:

$$Lu = b$$

  - *L* is the stiffness matrix, representing overlap between basis functions
  - *b* is the integral of given PDE with respect to basis
  - *u* is unknown
- Value of FEA comes from:
  - *Spatial flexibility*: can define each element to vary in size quite substantially
  - *Speed*: properly chosen basis functions have compact support, leading to a sparse matrix

Perfectly Matched Layers

- In order to prevent lateral reflections (e.g., from PEC boundaries), can introduce perfectly matched layers (PML)
- Several formulations (including split-field and uniaxial), but here we'll follow stretched coordinate PML
- Effected by the transformation:

$$\nabla \longrightarrow A \cdot \nabla$$

$$A = \begin{pmatrix} 1 - j\beta & 0 & 0 \\ 0 & 1 - j\beta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\beta = -\frac{3\lambda\rho^2}{4\pi n d^3} \ln R$$

Next Class

- Is on Wednesday, Feb. 13
- Will continue with beam propagation method
- Recommended reading: Obayya, Sections 2.7-2.8