# ECE 595, Section 10 <br> Numerical Simulations <br> Lecture 16: Applications of the Beam Propagation Method <br> <br> Prof. Peter Bermel <br> <br> Prof. Peter Bermel <br> February 15, 2013 

## Outline

- Recap from Wednesday
- BPM Mode Solver
- Vectorial BPM Applications:
- Waveguide
- Photonic Crystal Fiber


## BPM Mode Solver

- Can extend BPM method to solve for modes, by propagating in the imaginary direction
- First, drop all derivatives in BPM equation:

$$
[K]\left\{h_{t, l}\right\}=-\gamma^{2}[M]\left\{h_{t, l}\right\}
$$

- Second, write down next step in $z$ :

$$
\left\{h_{t, l}\right\}_{k+1}=\frac{-2 \gamma-0.5 \Delta z k_{o}^{2}\left(n_{e f f, \ell}^{2}-n_{o}^{2}\right)}{-2 \gamma+0.5 \Delta z k_{o}^{2}\left(n_{e f f, \ell}^{2}-n_{o}^{2}\right)}\left\{h_{t, l}\right\}_{k}
$$

- Third, substitute special value of $\Delta z$ :

$$
\Delta z \approx j \frac{4 n_{o}}{\left(n_{o f, t e x}^{2}-n_{b}^{2}\right) k_{o}}
$$

## BPM Mode Solver

- Since $\Delta z$ initially unknown, assume largest index possible, and decrease it as needed
- Will eventually converge to correct answer and effective refractive index:

$$
n_{e f f, \ell, k}^{2}=\frac{\left\{h_{t}\right\}_{k}^{*}[K]_{k}\left\{h_{t}\right\}_{k}}{k_{o}^{2}\left\{h_{t}\right\}_{k}^{*}[M]_{k}\left\{h_{t}\right\}_{k}}
$$

- Can use Gram-Schmidt normalization procedure to find higher-order modes:

$$
\left\{h_{t}\right\}_{1, \text { new }}=\left\{h_{t}\right\}_{1}-\sum_{\ell=1}^{i-1} \frac{\left\{h_{t, l}\right\}^{*}[M]\left\{h_{t}\right\}_{1}}{\left\{h_{t, l}\right\}^{*}[M]\left\{h_{t, l}\right\}}\left\{h_{t, l}\right\}
$$

## VBPM on a Waveguide: Problem Description



- Cross section defined above; $\lambda=1.3 \mu \mathrm{~m}$
- Propagation along $z$ is semi-infinite
- Must grid space with first-order triangular elements in cross-sectional plane; choose PML to reduce reflections to $10^{-100}$
- Will vary $\Delta z$ for maximum effectiveness


## VBPM on a Waveguide



- Fundamental mode is calculated accurately with 12,800 first-order triangular elements


## VBPM on a Waveguide



- Propagation step size in $Z$, known as $\Delta Z$, should equal transverse dimensions for best accuracy


## VBPM on a Waveguide: Longitudinal Imaginary Propagation



- With optimal step size, can solve the fundamental mode of both polarizations in a pretty modest number of steps!


## VBPM on a Waveguide: Accuracy



- Accuracy of calculation of waveguide coupling length as a function of mesh divisions $N$


## VBPM on a Waveguide



- Accuracy of coupling length as a function of $\Delta Z$ saturates below one wavelength


## VBPM on a Photonic Crystal Fiber



- Originally conceived of by P.J. Russell
- Confines light to core without total internal reflection!


## VBPM on a PhC Fiber



- Effective index vs. PhC period


## VBPM on a PhC Fiber



- $\mathrm{H}_{\mathrm{y}}$ field distributions for the fundamental TE modes


## VBPM on a PhC Fiber



- Confinement loss decreases sharply as period $\Lambda$ increases


## VBPM on a PhC Fiber



- Variation of the effective mode area with PhC period $\Lambda$


## VBPM on a PhC Fiber



- Effective index increases modestly with increasing period $\Lambda$, indicating increased mode confinement


## VBPM on a PhC Fiber



- Calculated dispersion relation (effective index versus wavelength) for a PhC Fiber


## VBPM on a PhC Fiber



- Obtained dispersion $D=d^{2} k / d \omega^{2}$ from earlier data
- Note modest changes in parameters flip sign of $D$


## Next Class

- Is on Monday, Feb. 18
- Next time, we shall finish the applications of BPM, and possibly cover other FEM applications
- Recommended reading: Obayya, Chapter 3

