ECE 595, Section 10
Numerical Simulations
Lecture 29: Eigenmode Layered Computations (CAMFR)

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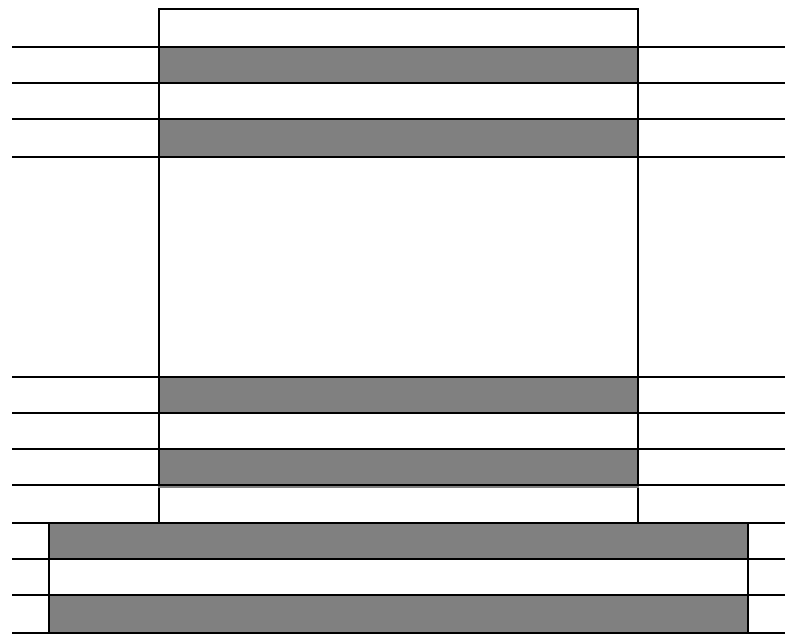
March 25, 2013

Recap from Friday: S-Matrices

- S-matrix method in plane wave basis
  - Calculate field eigenvectors in plane-wave basis
  - Calculate interface s-matrices and layer s-matrices
  - Compose S-matrices iteratively
- S4 simulation tool:
  - User interface
  - Lua commands
  - Results for example problems: multilayer stack; 1D grating; 2D Tikhodeev example

CAMFR: Rationale

- Many problems consist of layers with varying widths
- Examples:
  - LED stack
  - Rod-hole photonic crystal
- Natural form of solutions is semi-analytic, in terms of eigenmodes



P. Bienstman, "Rigorous and efficient modeling of wavelength scale photonic components," Ph.D. Thesis, University of Ghent (2001).

CAMFR: Basic Strategy

- Break up structure into layers
- Calculate eigenmodes in each layer (of four types)
- Apply Lorentz reciprocity to match BC's
- Propagate within layers using S-matrix method
- Apply inputs to calculate physical outputs

CAMFR: Eigenmode Decomposition

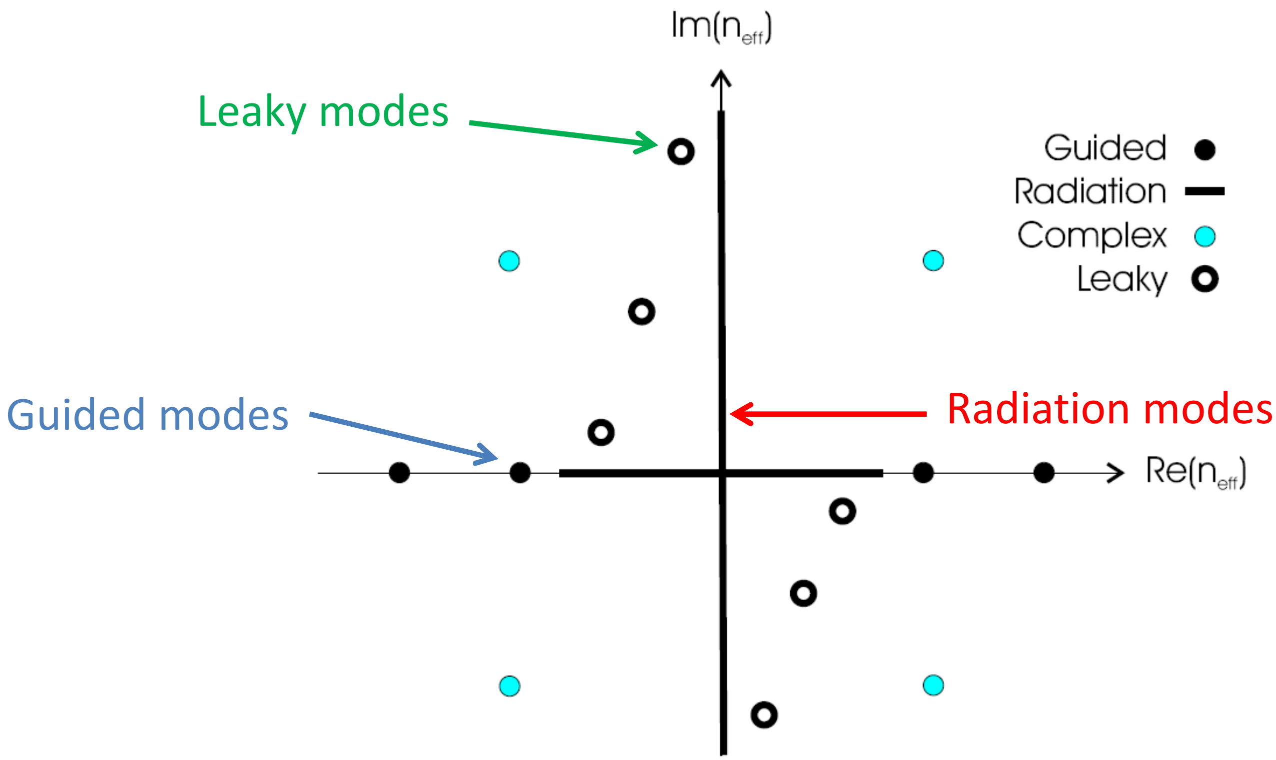
- This stage resembles BPM
- Begin with the Helmholtz equation:

$$[\nabla_t^2 + \epsilon\mu\omega^2]\psi = \beta^2\psi$$

- Where ψ represents *E*-field or *H*-field, and β is the eigenvalue (wavevector along *z*)
- Write 3D solutions in this form for each layer:

$$\begin{pmatrix} E(r) \\ H(r) \end{pmatrix} = \sum_k A_k e^{-j\beta_k z} \begin{pmatrix} E(r_t) \\ H(r_t) \end{pmatrix}$$

CAMFR: Eigenmode Decomposition



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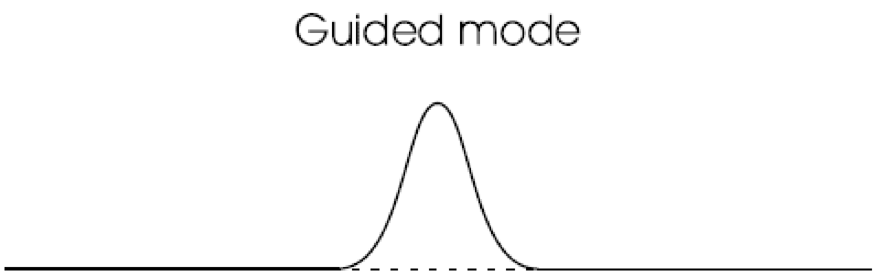
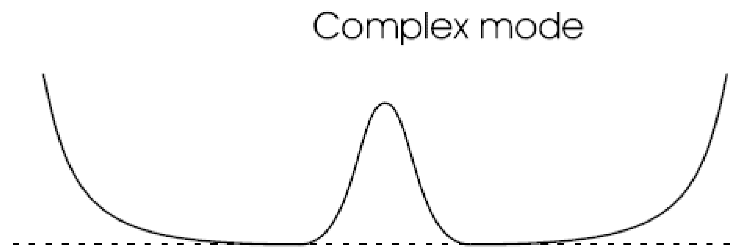
Can express eigenvalues in terms of $\mathrm{Re}\, n_{eff}$ and $\mathrm{Im}\, n_{eff}$

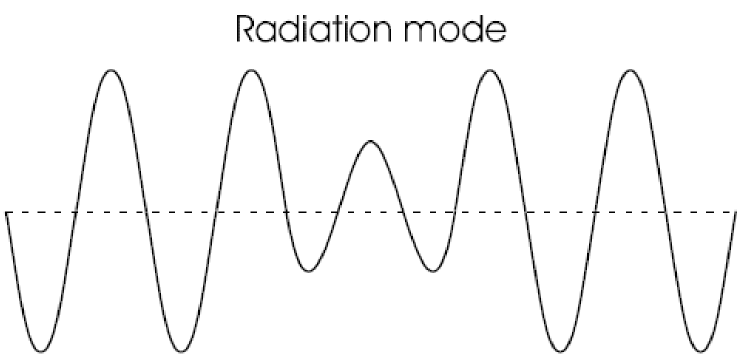
Eigenmode Classification



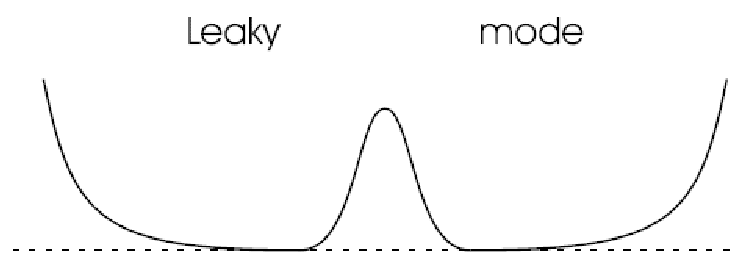
$\mathrm{Im}\,\beta = 0$; discrete



$\mathrm{Im}\,\beta \neq 0$; $\mathrm{Re}\,\beta \neq 0$; discrete complex-conjugate pairs



$\mathrm{Re}\,\beta = 0$ or $\mathrm{Im}\,\beta = 0$; continuous



$\mathrm{Im}\,\beta \neq 0$; $\mathrm{Re}\,\beta \neq 0$; discrete

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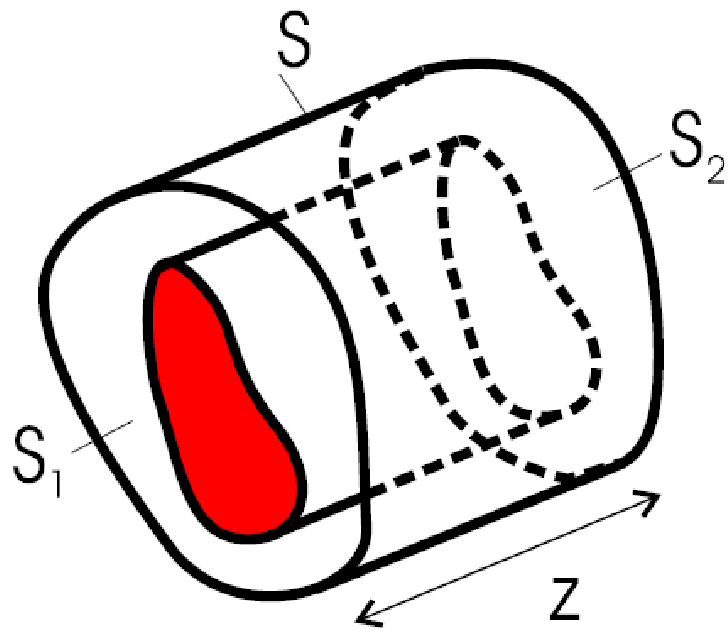
Lorentz Reciprocity



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- Evaluate Maxwell's equations across boundary using this surface

Lorentz Reciprocity

- Starting with Maxwell's equations:

$$\begin{aligned} \nabla \times \mathbf{E}_1 &= -j\omega\mu\mathbf{H}_1 & \nabla \times \mathbf{E}_2 &= -j\omega\mu\mathbf{H}_2 \\ \nabla \times \mathbf{H}_1 &= \mathbf{J}_1 + j\omega\varepsilon\mathbf{E}_1 & \nabla \times \mathbf{H}_2 &= \mathbf{J}_2 + j\omega\varepsilon\mathbf{E}_2 \end{aligned}$$

- Can form the expression:

$$\nabla \cdot (\mathbf{E}_1 \times \mathbf{H}_2 - \mathbf{E}_2 \times \mathbf{H}_1) = \mathbf{J}_1 \cdot \mathbf{E}_2 - \mathbf{J}_2 \cdot \mathbf{E}_1$$

- Integrating over *V* and using Gauss' theorem:

$$\int\int_S (\mathbf{E}_1 \times \mathbf{H}_2 - \mathbf{E}_2 \times \mathbf{H}_1) \cdot d\mathbf{S} = \int\int\int_V (\mathbf{J}_1 \cdot \mathbf{E}_2 - \mathbf{J}_2 \cdot \mathbf{E}_1) dV$$

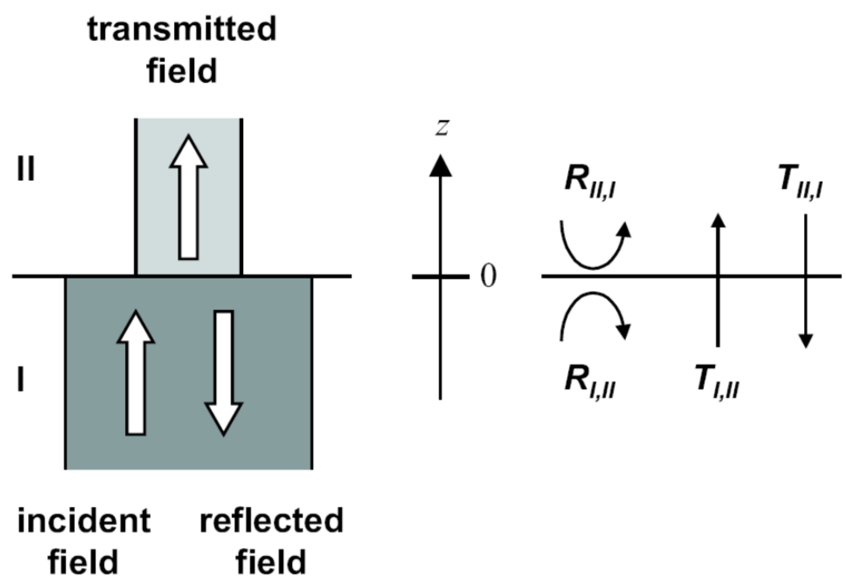
Lorentz Reciprocity

- Lorentz Reciprocity theorem becomes:

$$\int\int_S \frac{\partial}{\partial z}\left(\mathbf{E}_1 \times \mathbf{H}_2 - \mathbf{E}_2 \times \mathbf{H}_1\right) \cdot \mathbf{u_z} dS = \int\int_S (\mathbf{J}_1 \cdot \mathbf{E}_2 - \mathbf{J}_2 \cdot \mathbf{E}_1) dS$$

- For *z*-invariant media:

$$\int\int_S (\mathbf{E}_{m,t} \times \mathbf{H}_{n,t}) \cdot \mathbf{u}_z dS = 0$$



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Boundary Conditions

- Assuming:

$$\mathbf{E}_{p,t}^{I} + \sum_j R_{j,p}\mathbf{E}_{j,t}^{I} \quad = \quad \sum_j T_{j,p}\mathbf{E}_{j,t}^{II}$$

- Defining overlap between modes to be:

$$\langle \mathbf{E}_m, \mathbf{H}_n \rangle \equiv \int\int_s (\mathbf{E}_m \times \mathbf{H}_n) \cdot \mathbf{u}_z dS$$

- We obtain the transmission coefficient:

$$\sum_j \left[\left\langle \mathbf{E}_i^{I}, \mathbf{H}_j^{II} \right\rangle + \left\langle \mathbf{E}_j^{II}, \mathbf{H}_i^{I} \right\rangle \right] T_{j,p} = 2\delta_{ip} \left\langle \mathbf{E}_p^{I}, \mathbf{H}_p^{I} \right\rangle$$

- And reflection coefficient:

$$R_{i,p} = \frac{1}{2\left\langle \mathbf{E}_i^{I}, \mathbf{H}_i^{I} \right\rangle} \sum_j \left[\left\langle \mathbf{E}_j^{II}, \mathbf{H}_i^{I} \right\rangle - \left\langle \mathbf{E}_i^{I}, \mathbf{H}_j^{II} \right\rangle \right] T_{j,p}$$

S-Matrix Solution

- Now we can employ the standard S-matrix scheme from Li ’96:

$$
\begin{aligned}
\mathbf{T}_{1,p+1} &= \mathbf{t}_{p,p+1} \cdot (\mathbf{I} - \mathbf{R}_{p,1} \cdot \mathbf{r}_{p,p+1})^{-1} \cdot \mathbf{T}_{1,p} \\
\mathbf{R}_{p+1,1} &= \mathbf{t}_{p,p+1} \cdot (\mathbf{I} - \mathbf{R}_{p,1} \cdot \mathbf{r}_{p,p+1})^{-1} \cdot \mathbf{R}_{p,1} \cdot \mathbf{t}_{p+1,p} + \mathbf{r}_{p+1,p} \\
\mathbf{R}_{1,p+1} &= \mathbf{T}_{p,1} \cdot (\mathbf{I} - \mathbf{r}_{p,p+1} \cdot \mathbf{R}_{p,1})^{-1} \cdot \mathbf{r}_{p,p+1} \cdot \mathbf{T}_{1,p} + \mathbf{R}_{1,p} \\
\mathbf{T}_{p+1,1} &= \mathbf{T}_{p,1} \cdot (\mathbf{I} - \mathbf{r}_{p,p+1} \cdot \mathbf{R}_{p,1})^{-1} \cdot \mathbf{t}_{p+1,p}
\end{aligned}
$$

- We can compose the S-matrix starting from the identity matrix until we include all layers

Periodic Eigenproblems

- Periodic layered structures will:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \cdot \begin{bmatrix} F \\ B \end{bmatrix} = e^{-jk_z p} \cdot \begin{bmatrix} F \\ B \end{bmatrix}$$

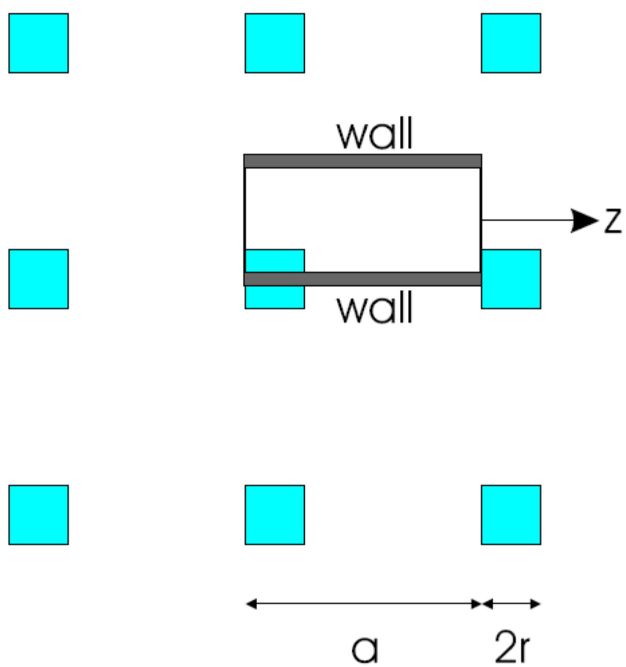
- Since T-matrix is nearly singular, use SVD:

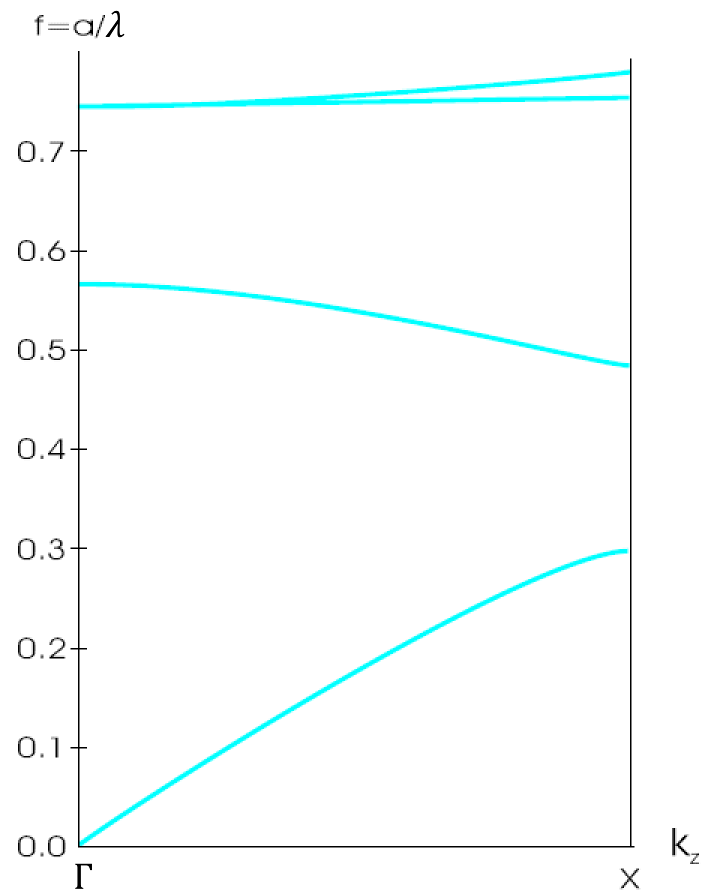
$$\mathbf{A} = \mathbf{U} \cdot \Sigma \cdot \mathbf{V}^H$$

- Where ***U*** and ***V*** are unitary; Σ diagonal. Then:

$$\mathbf{A}^{-1} = \mathbf{V} \cdot diag\left(\frac{1}{\sigma_i}\right) \cdot \mathbf{U}^H$$

CAMFR: 2D Photonic Crystals





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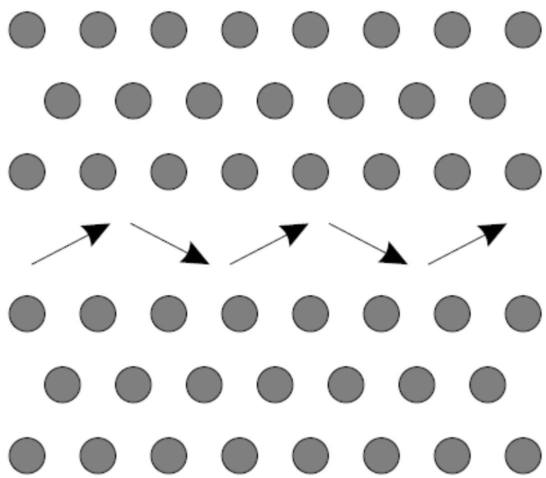
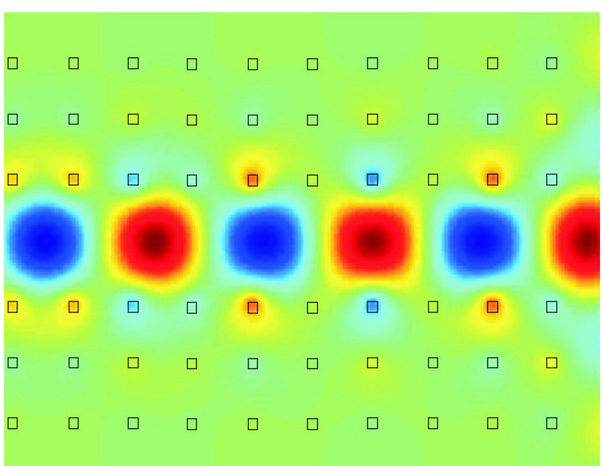
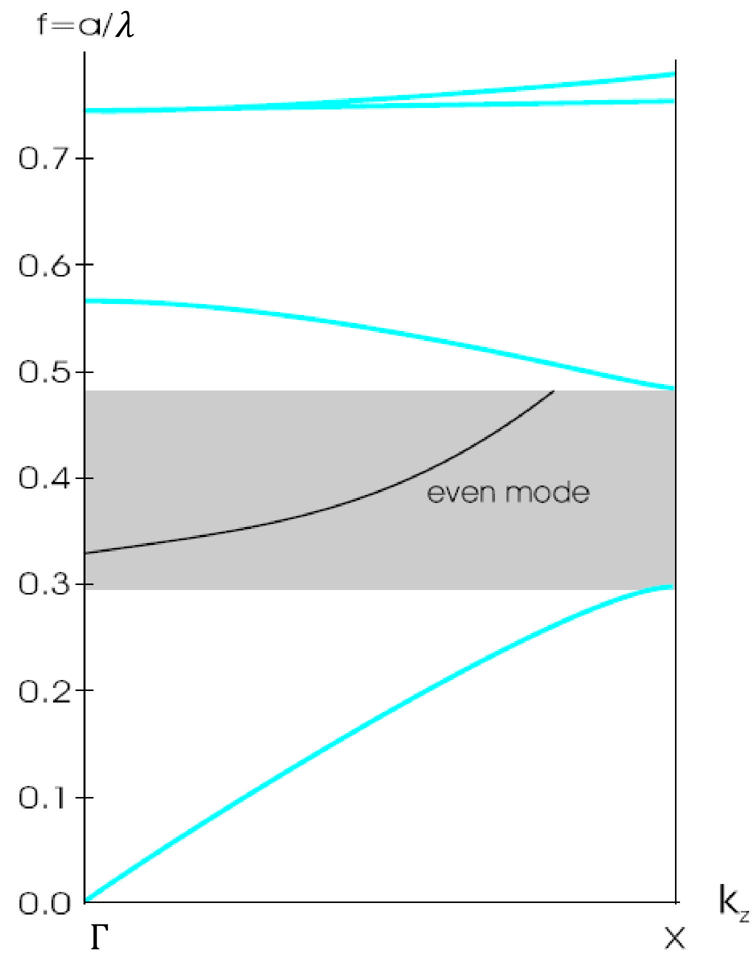
CAMFR: 2D PhC Waveguide







Next Class

- Is on Wednesday, March 27
- Will discuss CAMFR interface: http://camfr.sourceforge.net