ECE 695 (Numerical Simulations) – Homework 7

Due March 24, 2017 at 4:30 pm
Email to pbermel@purdue.edu
Please write your programs in C/C++, MATLAB, or Python

1. Galerkin's Method Consider the following boundary value problem – the Helmholtz equation in 1D, where:

$$-\frac{\partial^2 E}{\partial x^2} = \left(\frac{n\omega}{c}\right)^2 E. \tag{1}$$

Let the field vanish at the boundaries, i.e., $E(0) = E(L) = 0$, and be unity at the midpoint, i.e., $E(L/2) = 1$.

1a. Set the basis function to be quadratic, i.e., $N_1(x) = x*(L-x)$ with a single degree of freedom (i.e., the overall magnitude), and use the basis function $N_1(x)$ as a testing function $M_1(x)$ to obtain an approximate solution. The above differential equation can be simplified if we consider integration by parts, or

$$\int_0^L M_1(x)E''(x)dx = M_1(x)E'(x)|_0^L - \int_0^L M_1'(x)E'(x)dx \tag{2}$$

1b. How close is this solution to the analytical solution, $w(x) = \sin(\pi * x/L)$?

2. Basic FEM

2a. Construct the FEM matrix problem associated with the 1D Helmholtz equation presented in the previous problem, using the Galerkin method with the Lagrangian basis functions over 2 elements shown below in Fig. 1.

2b. Solve this matrix problem to find the approximate field values associated with this FEM problem. How close is this to the analytical solution?

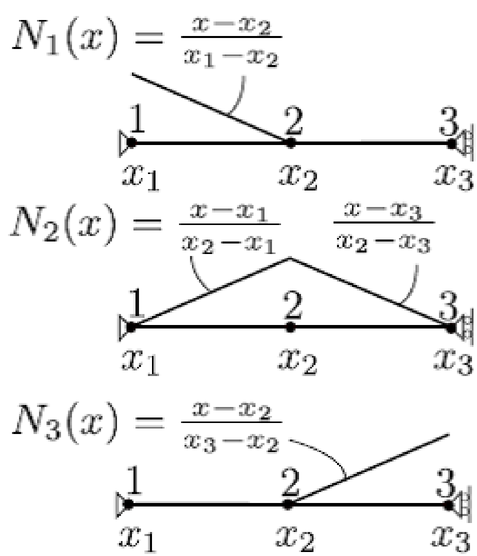

Figure 1: Linear basis functions used to solve the 1D Hemholtz equation in Problem 2