ECE 695
Numerical Simulations
Lecture 22: Cavity Modeling Framework (CAMFR)

Prof. Peter Bermel

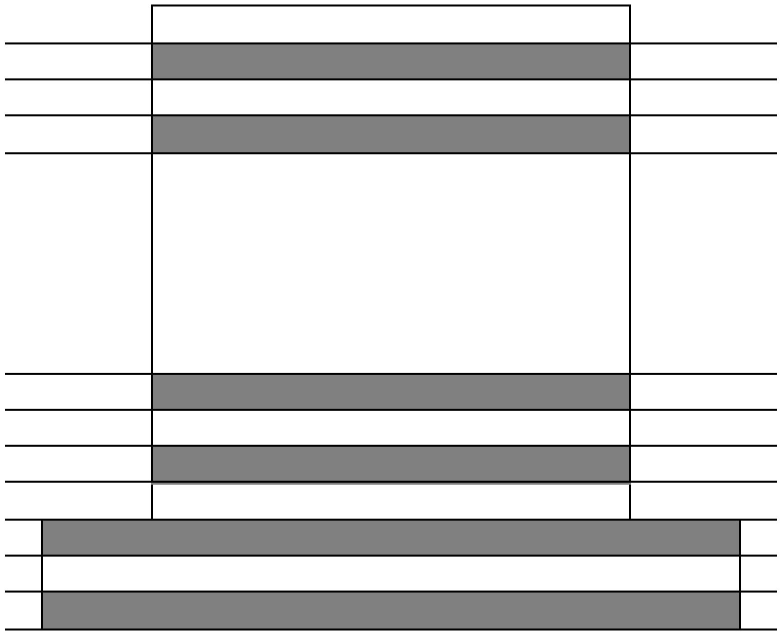
March 6, 2017

Outline

- CAMFR
  - Rationale
  - Problem formulation
  - Examples

CAMFR: Rationale

- Many problems consist of layers with varying widths
- Examples:
  - LED stack
  - Rod-hole photonic crystal
- Natural form of solutions is semi-analytic, in terms of eigenmodes



P. Bienstman, “Rigorous and efficient modeling of wavelength scale photonic components,” Ph.D. Thesis, University of Ghent (2001).

CAMFR: Basic Strategy

- Break up structure into layers
- Calculate eigenmodes in each layer (of four types)
- Apply Lorentz reciprocity to match BC's
- Propagate within layers using S-matrix method
- Apply inputs to calculate physical outputs

CAMFR: Eigenmode Decomposition

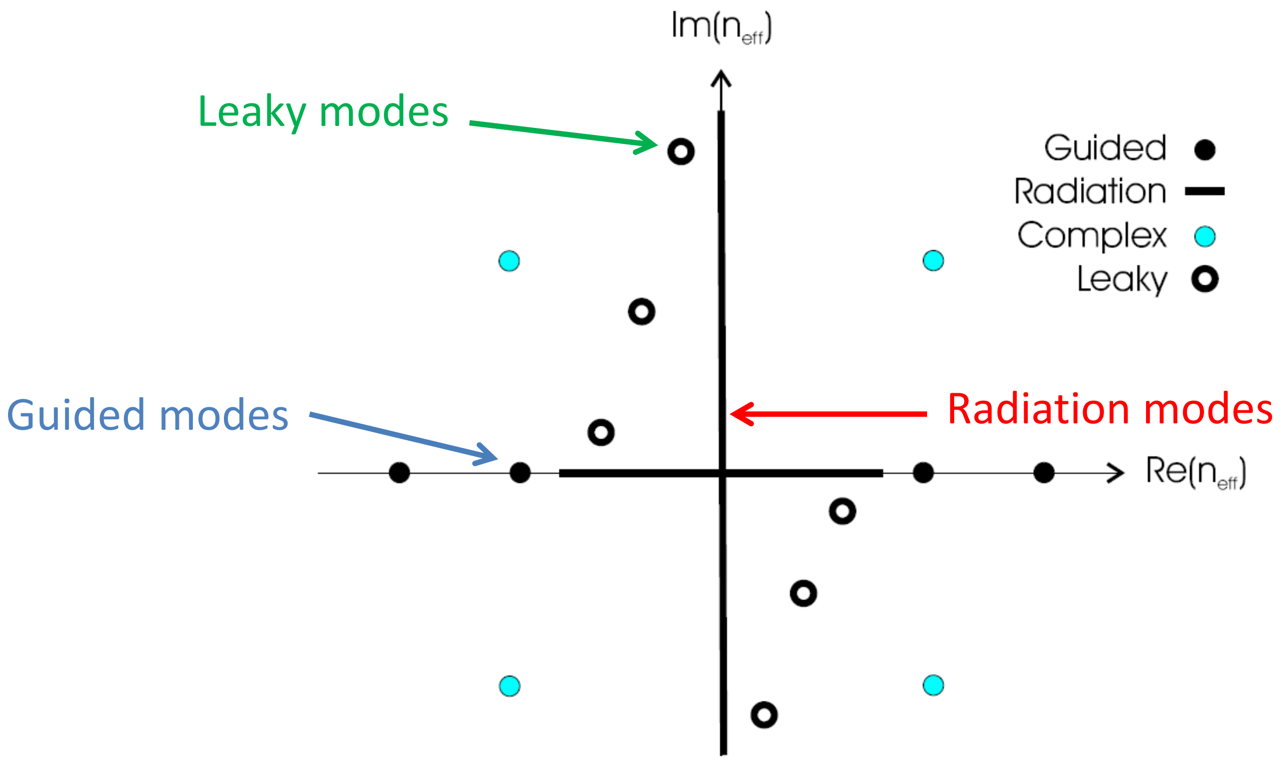
- This stage resembles BPM
- Begin with the Helmholtz equation:

$$[\nabla_t^2 + \epsilon\mu\omega^2]\psi = \beta^2\psi$$

- Where ψ represents *E*-field or *H*-field, and β is the eigenvalue (wavevector along *z*)
- Write 3D solutions in this form for each layer:

$$\begin{pmatrix} E(r) \\ H(r) \end{pmatrix} = \sum_k A_k e^{-j\beta_k z} \begin{pmatrix} E(r_t) \\ H(r_t) \end{pmatrix}$$

CAMFR: Eigenmode Decomposition



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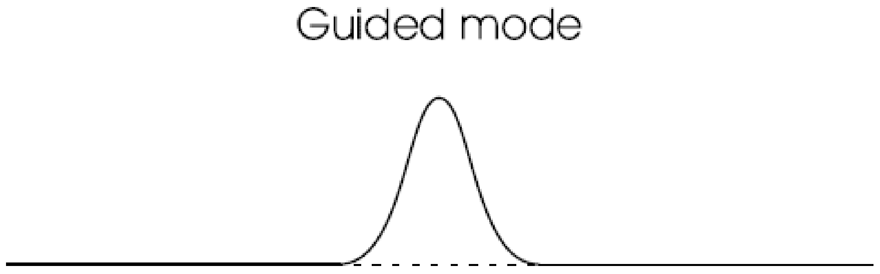
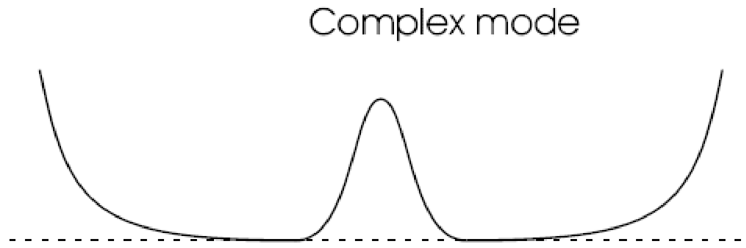
Can express eigenvalues in terms of $\mathrm{Re}\, n_{eff}$ and $\mathrm{Im}\, n_{eff}$

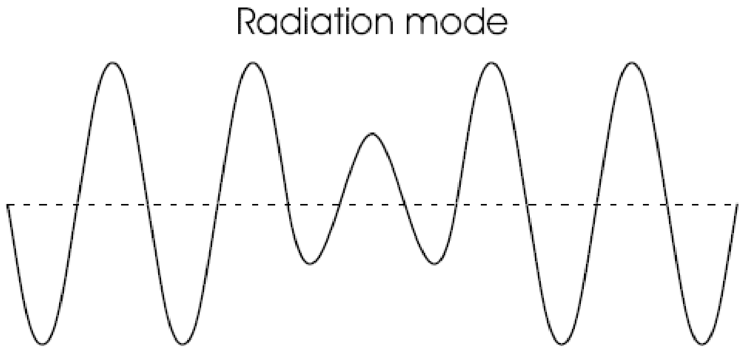
Eigenmode Classification



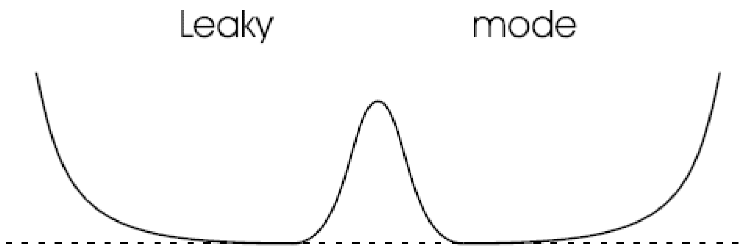
$\mathrm{Im}\,\beta = 0$; discrete



$\mathrm{Im}\,\beta \neq 0$; $\mathrm{Re}\,\beta \neq 0$; discrete complex-conjugate pairs



$\mathrm{Re}\,\beta = 0$ or $\mathrm{Im}\,\beta = 0$; continuous



$\mathrm{Im}\,\beta \neq 0$; $\mathrm{Re}\,\beta \neq 0$; discrete

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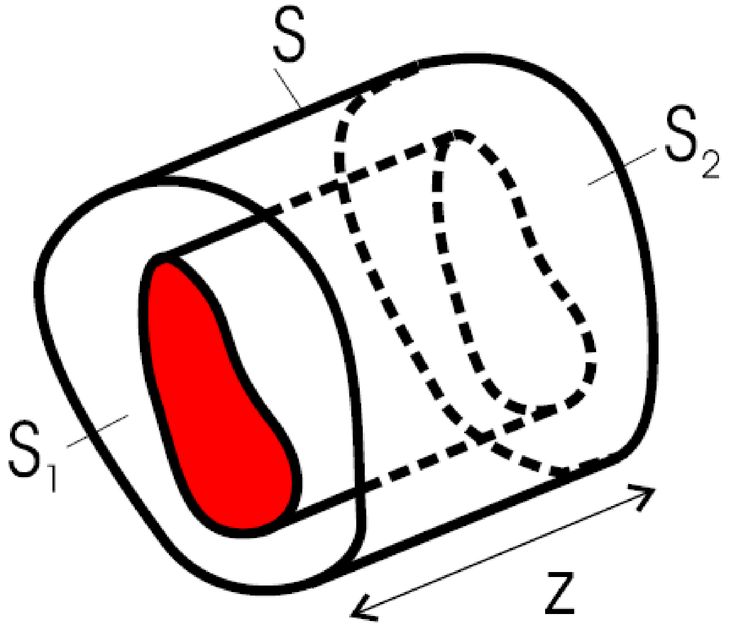
Lorentz Reciprocity



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- Evaluate Maxwell's equations across boundary using this surface

Lorentz Reciprocity

- Starting with Maxwell's equations:

$$\begin{aligned} \nabla \times \mathbf{E}_1 &= -j\omega\mu\mathbf{H}_1 \\ \nabla \times \mathbf{H}_1 &= \mathbf{J}_1 + j\omega\varepsilon\mathbf{E}_1 \end{aligned} \qquad \begin{aligned} \nabla \times \mathbf{E}_2 &= -j\omega\mu\mathbf{H}_2 \\ \nabla \times \mathbf{H}_2 &= \mathbf{J}_2 + j\omega\varepsilon\mathbf{E}_2 \end{aligned}$$

- Can form the expression:

$$\nabla \cdot (\mathbf{E}_1 \times \mathbf{H}_2 - \mathbf{E}_2 \times \mathbf{H}_1) = \mathbf{J}_1 \cdot \mathbf{E}_2 - \mathbf{J}_2 \cdot \mathbf{E}_1$$

- Integrating over *V* and using Gauss' theorem:

$$\int\int_S (\mathbf{E}_1 \times \mathbf{H}_2 - \mathbf{E}_2 \times \mathbf{H}_1) \cdot d\mathbf{S} = \int\int\int_V (\mathbf{J}_1 \cdot \mathbf{E}_2 - \mathbf{J}_2 \cdot \mathbf{E}_1) dV$$

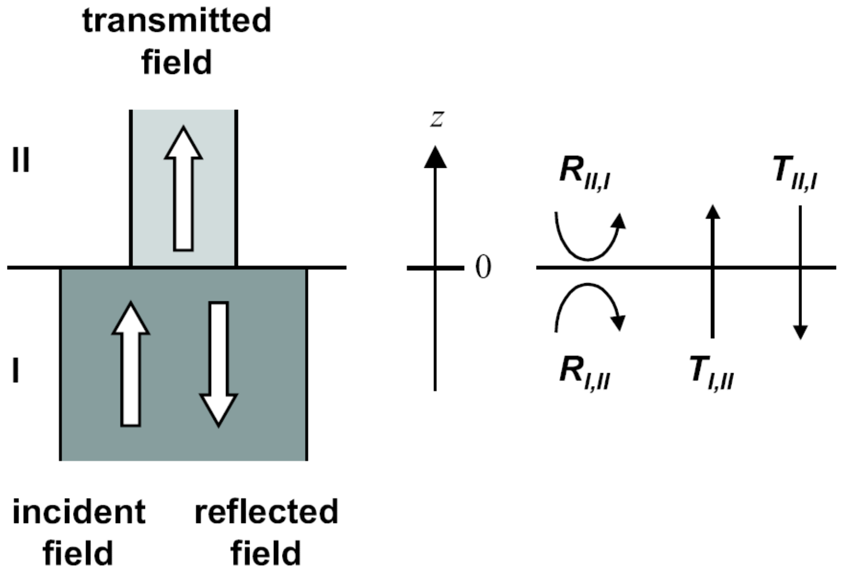
Lorentz Reciprocity

- Lorentz Reciprocity theorem becomes:

$$\int\int_S \frac{\partial}{\partial z}(\mathbf{E}_1 \times \mathbf{H}_2 - \mathbf{E}_2 \times \mathbf{H}_1) \cdot \mathbf{u_z} dS = \int\int_S (\mathbf{J}_1 \cdot \mathbf{E}_2 - \mathbf{J}_2 \cdot \mathbf{E}_1) dS$$

- For *z*-invariant media:

$$\int\int_S (\mathbf{E}_{m,t} \times \mathbf{H}_{n,t}) \cdot \mathbf{u}_z dS = 0$$



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Boundary Conditions

- Assuming:

$$\mathbf{E}^I_{p,t} + \sum_j R_{j,p}\mathbf{E}^I_{j,t} \quad = \quad \sum_j T_{j,p}\mathbf{E}^{II}_{j,t}$$

- Defining overlap between modes to be:

$$\langle \mathbf{E}_m, \mathbf{H}_n \rangle \equiv \int\int_s (\mathbf{E}_m \times \mathbf{H}_n) \cdot \mathbf{u}_z dS$$

- We obtain the transmission coefficient:

$$\sum_j \left[\langle \mathbf{E}^I_i, \mathbf{H}^{II}_j \rangle + \langle \mathbf{E}^{II}_j, \mathbf{H}^I_i \rangle \right] T_{j,p} = 2\delta_{ip} \langle \mathbf{E}^I_p, \mathbf{H}^I_p \rangle$$

- And reflection coefficient:

$$R_{i,p} = \frac{1}{2\langle \mathbf{E}^I_i, \mathbf{H}^I_i \rangle} \sum_j \left[\langle \mathbf{E}^{II}_j, \mathbf{H}^I_i \rangle - \langle \mathbf{E}^I_i, \mathbf{H}^{II}_j \rangle \right] T_{j,p}$$

S-Matrix Solution

- Now we can employ the standard S-matrix scheme from Li ’96:

$$
\begin{aligned}
\mathbf{T}_{1,p+1} &= \mathbf{t}_{p,p+1} \cdot (\mathbf{I} - \mathbf{R}_{p,1} \cdot \mathbf{r}_{p,p+1})^{-1} \cdot \mathbf{T}_{1,p} \\
\mathbf{R}_{p+1,1} &= \mathbf{t}_{p,p+1} \cdot (\mathbf{I} - \mathbf{R}_{p,1} \cdot \mathbf{r}_{p,p+1})^{-1} \cdot \mathbf{R}_{p,1} \cdot \mathbf{t}_{p+1,p} + \mathbf{r}_{p+1,p} \\
\mathbf{R}_{1,p+1} &= \mathbf{T}_{p,1} \cdot (\mathbf{I} - \mathbf{r}_{p,p+1} \cdot \mathbf{R}_{p,1})^{-1} \cdot \mathbf{r}_{p,p+1} \cdot \mathbf{T}_{1,p} + \mathbf{R}_{1,p} \\
\mathbf{T}_{p+1,1} &= \mathbf{T}_{p,1} \cdot (\mathbf{I} - \mathbf{r}_{p,p+1} \cdot \mathbf{R}_{p,1})^{-1} \cdot \mathbf{t}_{p+1,p}
\end{aligned}
$$

- We can compose the S-matrix starting from the identity matrix until we include all layers

Periodic Eigenproblems

- Periodic layered structures will:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \cdot \begin{bmatrix} F \\ B \end{bmatrix} = e^{-jk_z p} \cdot \begin{bmatrix} F \\ B \end{bmatrix}$$

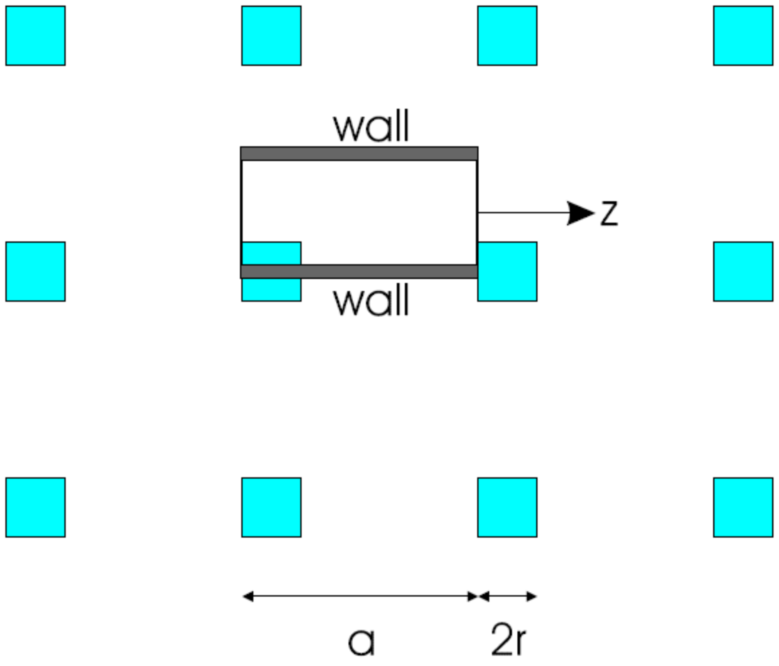
- Since T-matrix is nearly singular, use SVD:

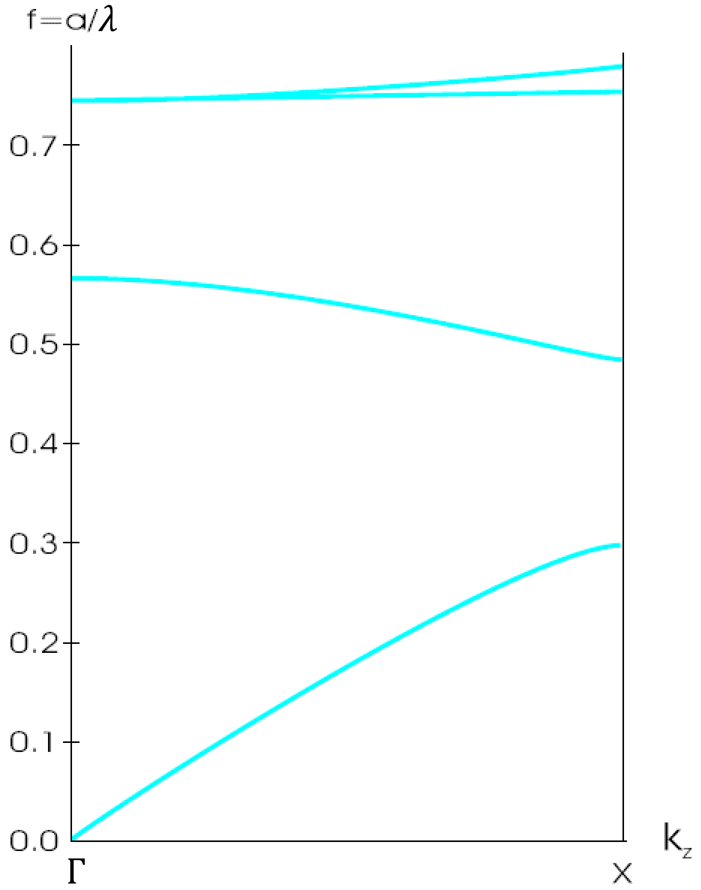
$$\mathbf{A} = \mathbf{U} \cdot \Sigma \cdot \mathbf{V}^H$$

- Where ***U*** and ***V*** are unitary; **Σ** diagonal. Then:

$$\mathbf{A}^{-1} = \mathbf{V} \cdot diag\left(\frac{1}{\sigma_i}\right) \cdot \mathbf{U}^H$$

CAMFR: 2D Photonic Crystals





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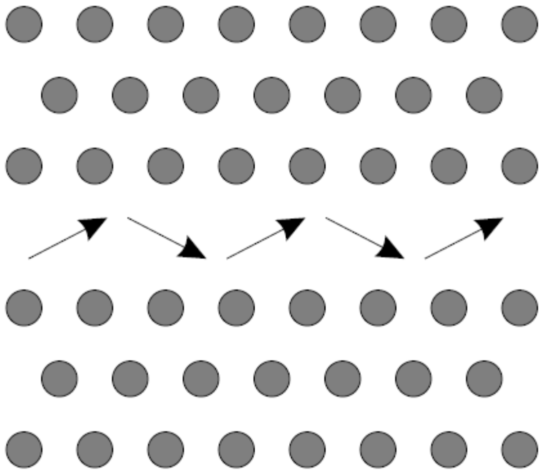
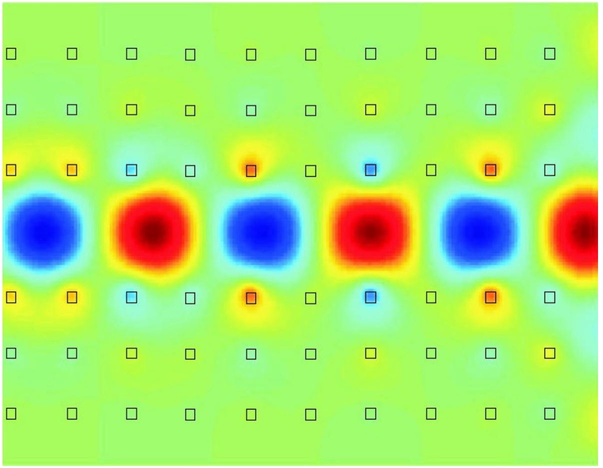
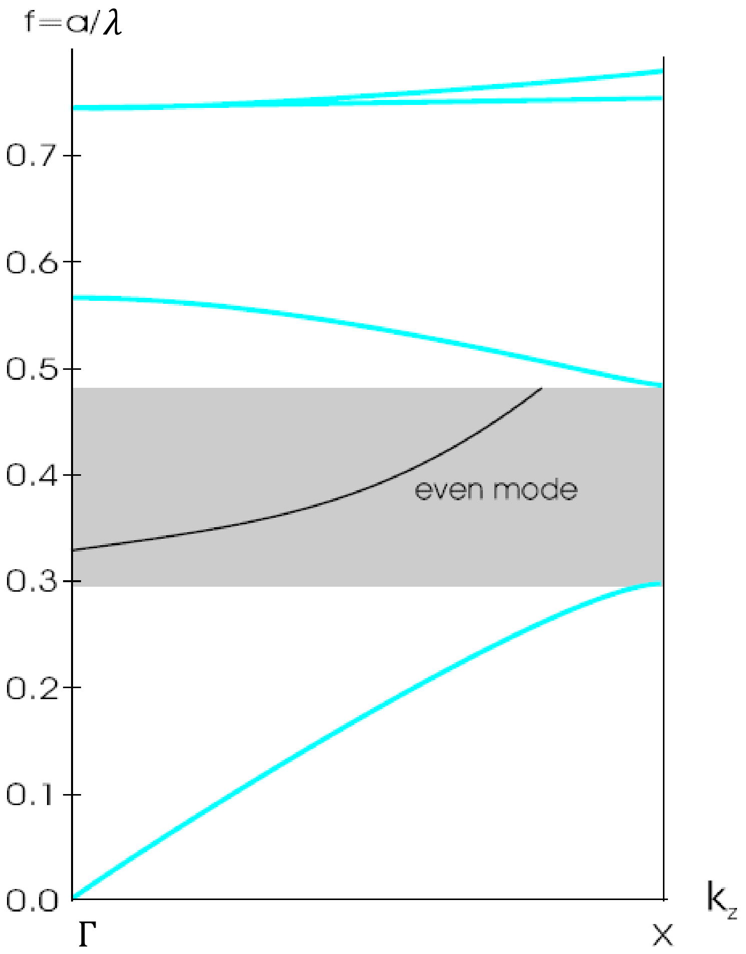
CAMFR: 2D PhC Waveguide





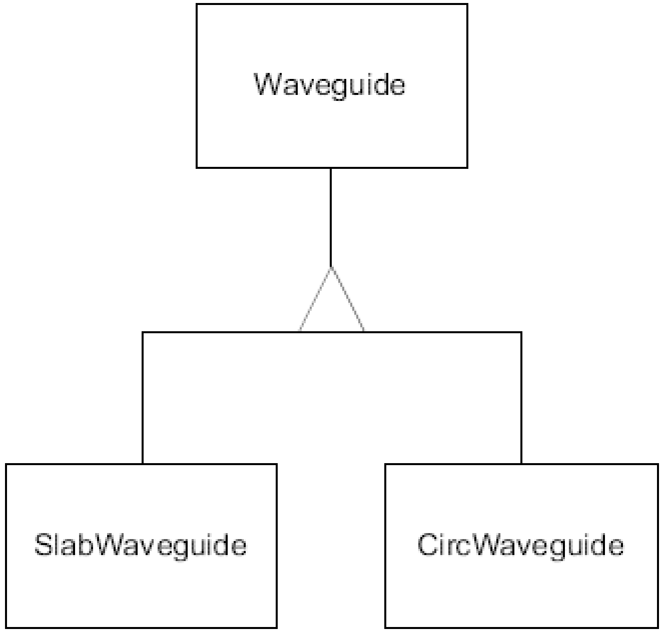


CAMFR Architecture

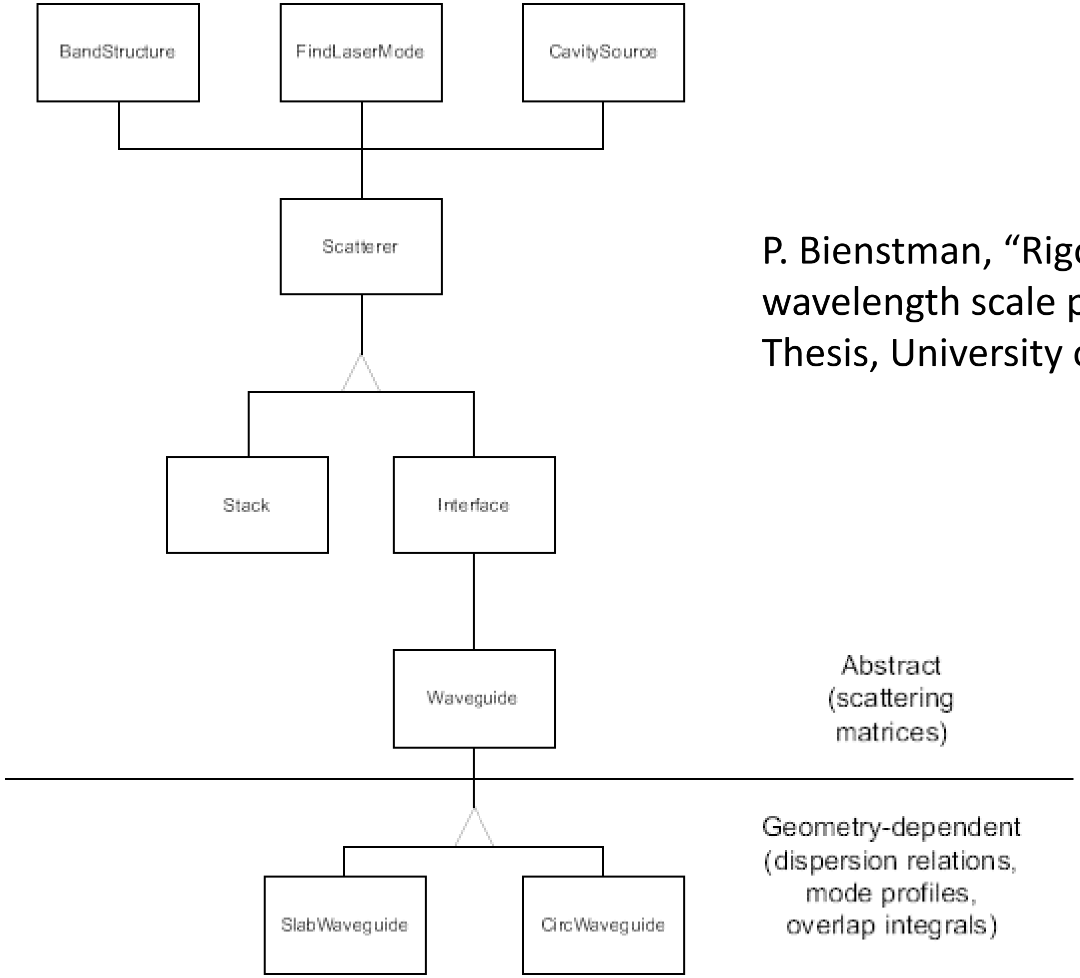
- Three key architectural elements:
  - User interface
  - Core logic
  - Low-level numerical routines
- Basic concept is to make each level independent yet interlocking with the others

CAMFR Architecture

- Built on object-oriented framework:
  - Abstract data types including slabs, waveguides
  - Encapsulated/reusable code
  - Polymorphism
- Implemented as Python library



CAMFR Architecture



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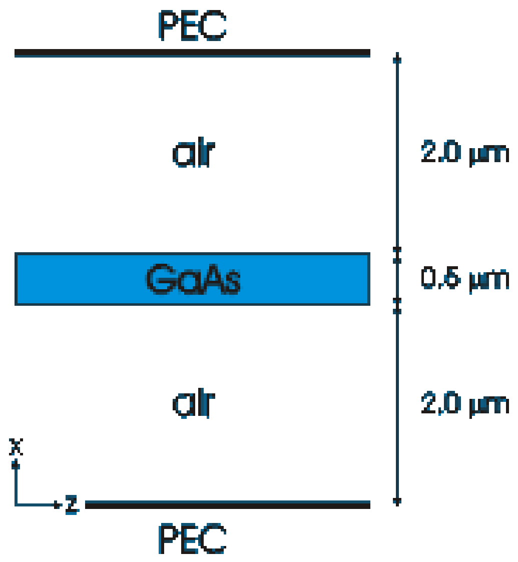
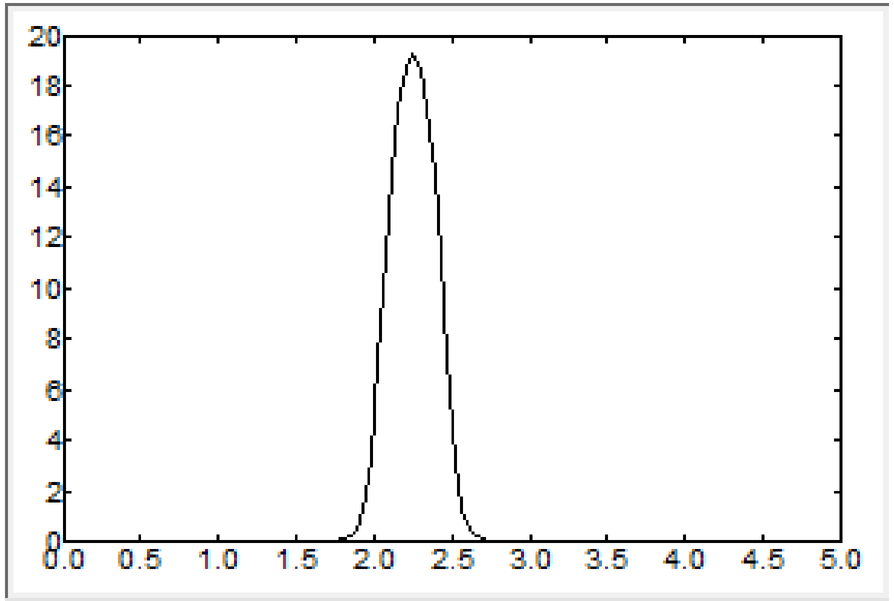
Code: 1D Waveguide

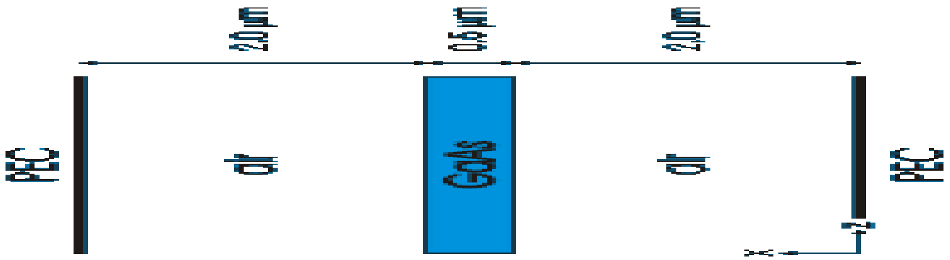
```
#!/usr/bin/env python
from camfr import *
set_lambda(1)
set_N(20)
set_polarisation(TE)
GaAs = Material(3.5)
air = Material(1.0)
slab = Slab(air(2) + GaAs(0.5) + air(2))
slab.calc()
....
slab.plot()
```



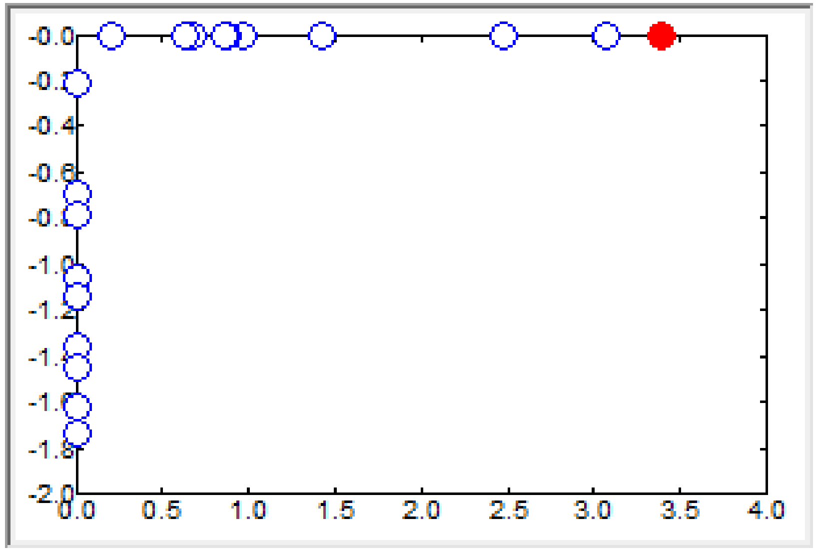
Results: 1D Waveguide

E-field spatial distribution

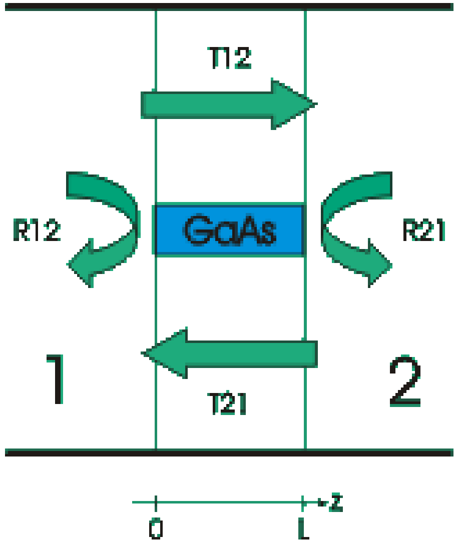




Effective index profile

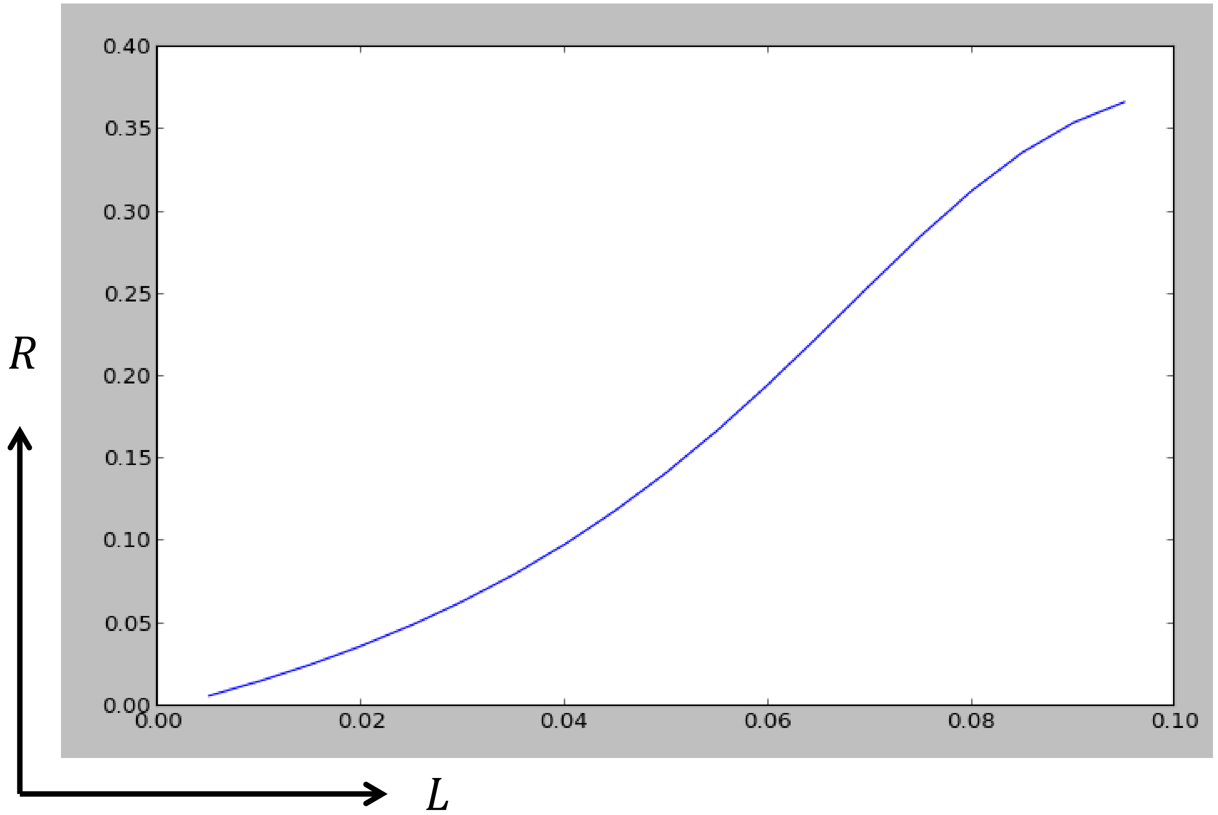


Code: 2D Waveguide




```
...
slab = Slab(air(2) + GaAs(0.5) + air(2))
space = Slab(air(4.5))
for L in arange(.005,.1,.005):
        stack = Stack(space(0) + slab(L) + space(0))
        stack.calc()
        print L, abs(stack.R12(0,0))
```

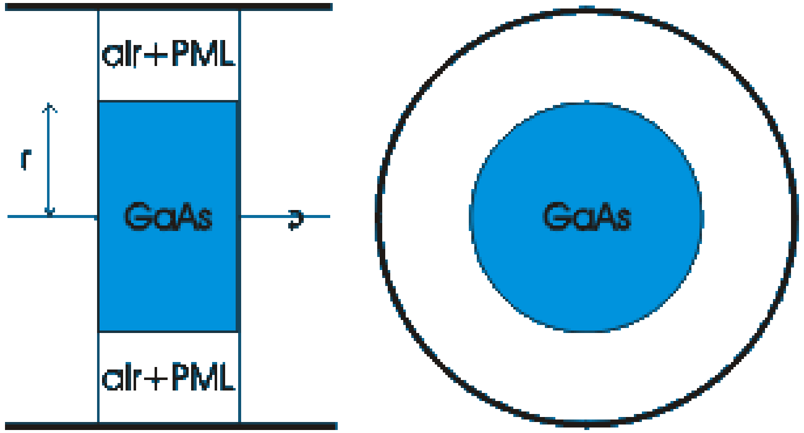
Results: 2D Waveguide



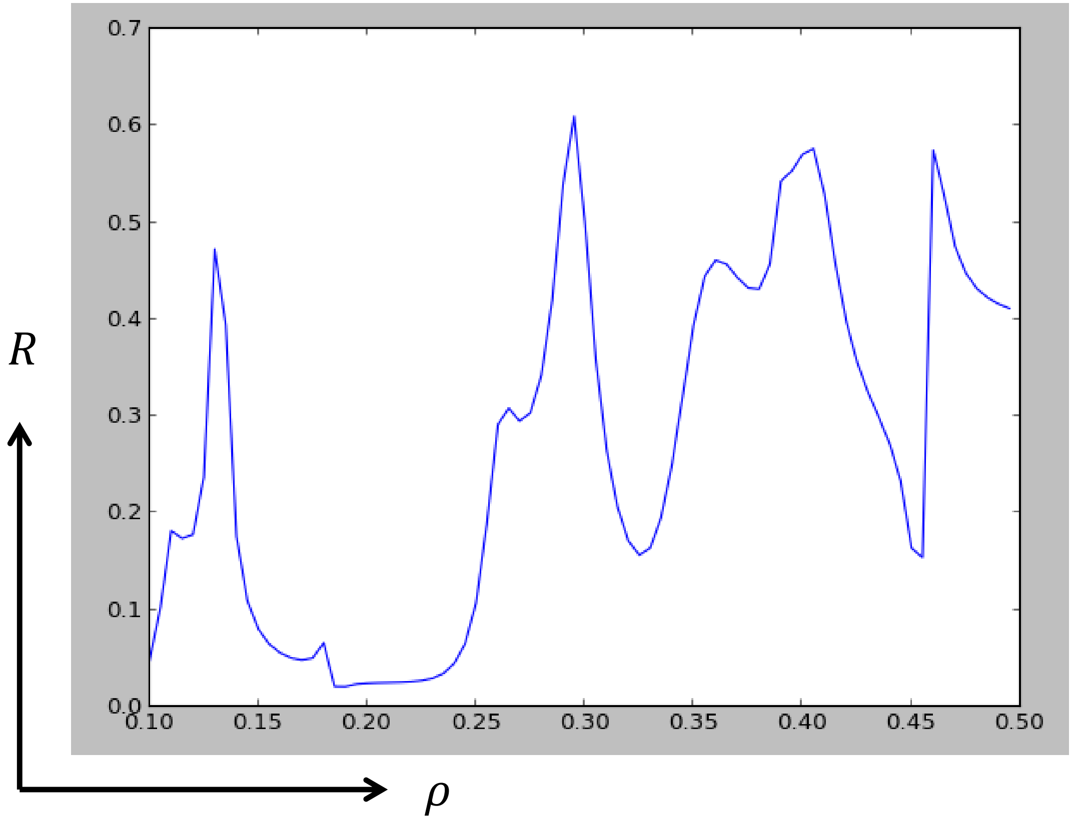
- Can see smooth increase from 0, with nonlinearities at larger *L*'s from interference

Code: Cylindrical Stack

```
…
set_circ_order(0)
set_polarisation(TE)
…
Set_circ_PML(-0.1)
Space = Circ(air(1))
for r in arange(.1,.5,.05):
        circ = Circ(GaAs(r) + air(1-r))
        stack = Stack(space(0) + circ(0.5) + space(0))
        stack.calc()
        print r, abs(stack.R12(0,0))
```



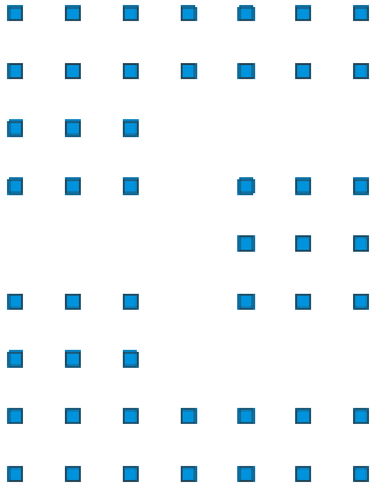
Results: Cylindrical Stack



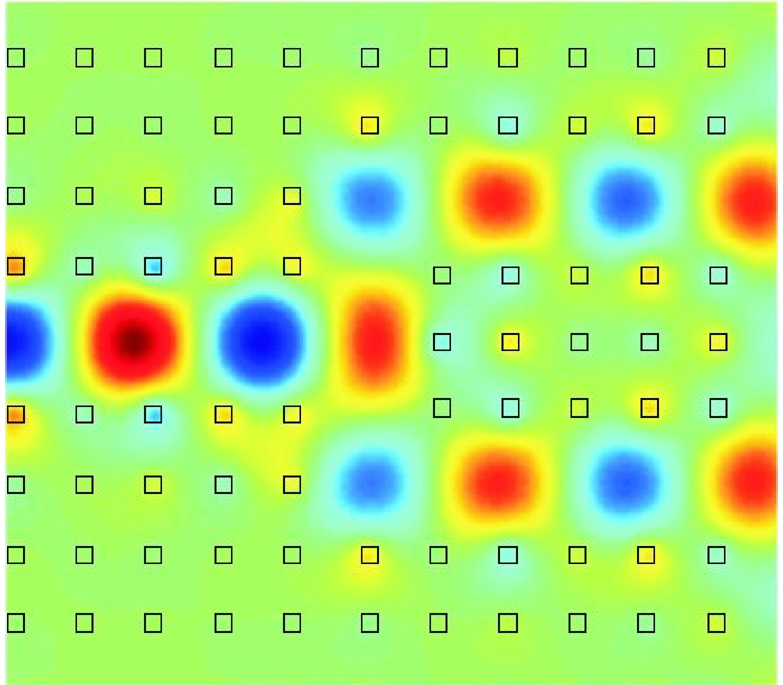
Can see heightened sensitivity to details of cylindrical geometry, with multiple reflection peaks

Code: Photonic Crystal Splitter

```
…
set_lower_wall(slab_H_wall)
periods = 3 # periods above outer waveguide
sections = 1 # intermediate 90 deg sections
no_rods = Slab(air(a-r+(sections+1+periods)*a+cl))
cen = Slab(air(a-r)+(sections+1+periods)*(GaAs(2*r) +
air(a-2*r))+air(cl) ) # Central waveguide
ver = Slab(air(a-r + (sections+1)*a) + periods*(GaAs(2*r)
+ air(a-2*r) )+air(cl) ) # Vertical section.
arm = Slab( GaAs(r) + air(a-2*r) + sections*(GaAs(2*r) +
air(a-2*r))+air(a)+periods*(GaAs(2*r) + air(a-
2*r))+air(cl) ) # Outer arms.
wg = BlochStack(cen(2*r) + no_rods(a-2*r))
wg.calc() # Find lowest order waveguide mode.
```



Results: Photonic Crystal Splitter



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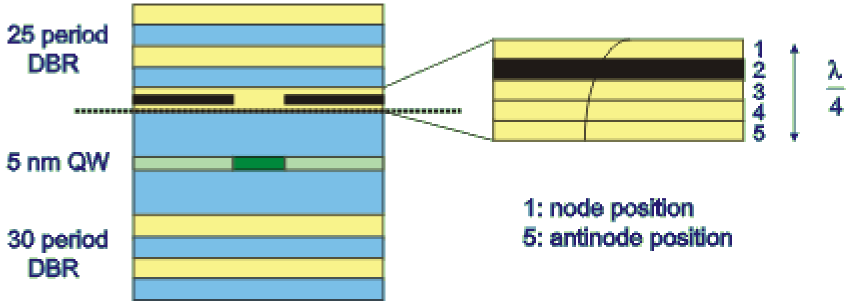
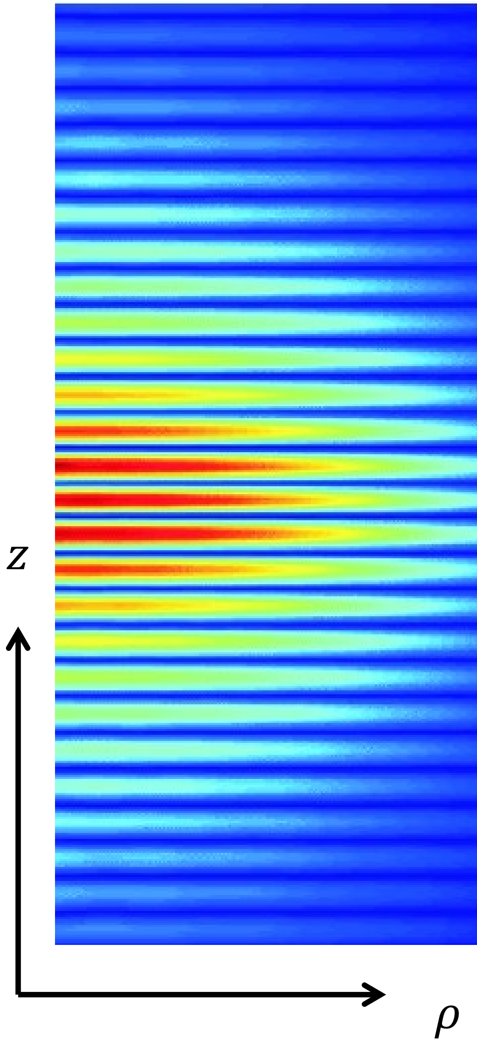
- Can demonstrate a low loss (3 dB) split within wavelength scale – compares favorably with index-guided fibers

Code: Vertical Cavity Surface Emitting Lasers (VCSELs)



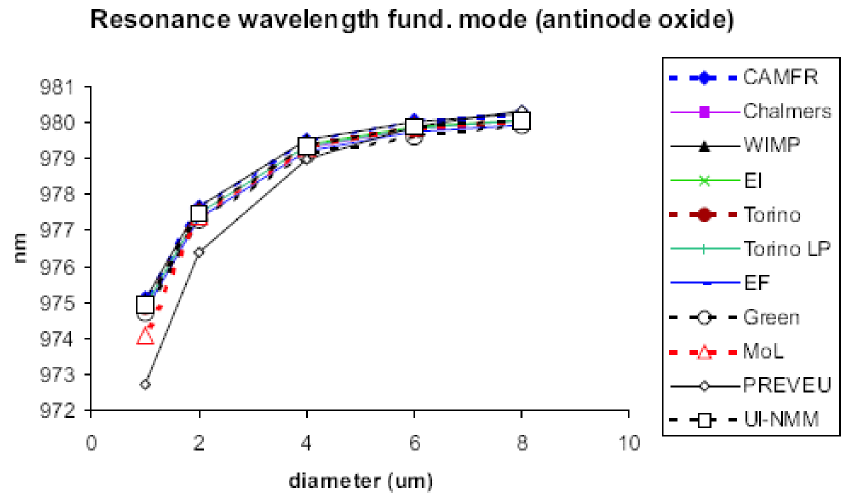

```
set_N(100)
set_circ_order(1)
set_circ_PML(-0.1)
GaAs=Circ (GaAs_m(r+d_cladding))
AlGaAs=Circ(AlGaAs_m(r+d_cladding))
…
top = Stack( (GaAs(0) + AlGaAs(x)) + ox(.2*d_AlGaAs) +
(AlGaAs(.8*d_AlGaAs - x) + GaAs(d_GaAs) +
24*(AlGaAs(d_AlGaAs) + GaAs(d_GaAs)) + air(0)) )
bottom = Stack(GaAs(.13659) + QW(.00500) \ + (GaAs(.13659)
+ 30*(AlGaAs(d_AlGaAs) + GaAs(d_GaAs) + GaAs(0))) )
cavity = Cavity(bottom, top)
cavity.find_mode(.980, .981)
```

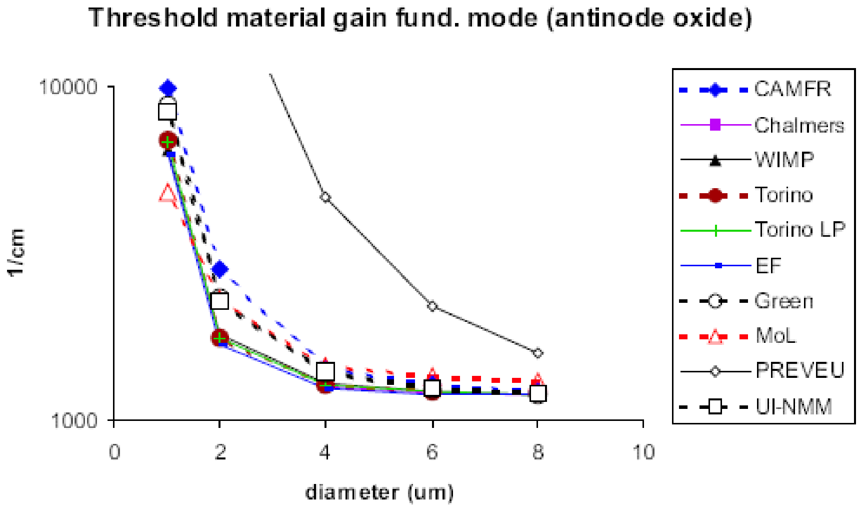
Results: VCSEL



- Field profile resulting from this design (in ρ-z plane)

Results: VCSEL





Next Class

- Is on Wednesday, March 8
- Will discuss CAMFR interface: http://camfr.sourceforge.net