An **angle** is defined as the set of points determined by two rays, or half-lines, l_1 and l_2 having the same end point O. An angle can also be considered as two finite line segments with a common point.

We call l_1 the **initial side**, l_2 the **terminal side**, and O the **vertex** of angle $\angle$AOB. The direction and number of rotations of l_1 makes before stopping at l_2 is not restricted.

The **standard position** of an angle is obtained by placing the vertex at the origin and letting the initial side l_1 coincide with the positive x-axis.

If l_1 is rotated in a counterclockwise direction to the terminal position l_2, then the angle is considered **positive**.

If l_1 is rotated in a clockwise direction, the angle is **negative**.

A **straight angle** is an angle whose sides lie on the same straight line but extend in opposite directions from its vertex.

A **right angle** is half of a straight angle and has measure 90°

An **acute angle** θ; $0° < \theta < 90°$

An **obtuse angle** θ; $90° < \theta < 180°$

Complementary angles α, β; $\alpha + \beta = 90°$
Supplementary angles α, β; $\alpha + \beta = 180°$

An angle is called a **quadrantal angle** if its terminal side lies on a coordinate axis.

Two units for measuring angles:

Degrees: A circle can be divided into 360 equal pieces. One complete revolution has measure 360 degrees, 360°

Radians: One complete revolution has measure 2π radians, 2π.

One radian is the measure of the central angle of a circle **subtended** by an arc equal in length to the radius of the circle.

360° = 2π radians and **180° = π radians**.

Radian measure do not use units.

When $\theta = 5$, it means $\theta = 5$ radians

When $\theta = 5°$, it means $\theta = 5$ degrees.

Convert the following angles from degrees into radians.

$\theta = 70°$

$\theta = 210°$

Convert the following angles from radians to degrees. Give the exact answer.

$\theta = \frac{2\pi}{3}$

$\theta = 1.58\pi$

If two angles have the same initial and terminal sides, they are **coterminal angles**.

List some coterminal angles for…

…$\theta = 55°$

…$\theta = -70°$

…$\theta = \frac{\pi}{8}$

For smaller units than degrees we have two choices:

For and angle that is between 150° and 151°…
Degrees Decimal, DD: example: 150.1234°

Degrees, Minutes, Seconds, DMS: example: 150° 7’ 24”
Divide one **degree** into 60 equal parts, **minutes** (')
Divide one **minute** into 60 equal parts, **seconds** (")

1° = 60’

1’ = 60”

1° = _____”

Convert the following angle to degrees, minutes, and seconds.
Round to the nearest second.

$\theta = 102.1459°$

Convert the following angles from degrees-minutes-seconds to decimals.
Round your answer to four decimal places.

$\theta = 15°\ 46'\ 32''$

Convert the following angles from radians to degrees.
For the decimal answers, round to four decimal places.
For the DMS answers, round to the nearest second.

$\theta = \frac{-14\pi}{13}$

Decimal: $\theta =$ DMS: $\theta =$

$\theta = 5$

Decimal: $\theta =$ DMS: $\theta =$

Complementary angles α, β; $\alpha + \beta = 90°$
Supplementary angles α, β; $\alpha + \beta = 180°$

Find an angle that is **Complement** and find the angle that is the **Supplement** to the given angles.

$\theta = 15.25°$

$\theta = 28°\ 41'\ 29''$

Definition of Reference Angle: Let θ be a nonquadrantal angle in standard position. The reference angle of θ is the acute angle θ_R that the terminal side of θ makes with the *x*-axis.

If θ is in QI, $\theta_R = \theta$
If θ is in QII, $\theta_R = 180° - \theta$ or $\pi - \theta$
If θ is in QIII, $\theta_R = \theta - 180°$ or $\theta - \pi$
If θ is in QIV, $\theta_R = 360° - \theta$ or $2\pi - \theta$

Find the reference angle θ_R.

$\theta = 132°$

$\theta = 311°$

$\theta = 236°$

$\theta = -120°$

Find the reference angle θ_R.

$\theta = \frac{13\pi}{4}$

$\theta = \frac{43\pi}{6}$

$\theta = \frac{7\pi}{3}$

$\theta = \frac{-5\pi}{6}$

Match the following angles to the quadrant or axis on which the terminal side of the angle lies.

$\theta = 360°$

$\theta = 215°$

$\theta = \frac{\pi}{3}$

$\theta = -2\pi$

$\theta = 750°$

$\theta = -85°$

$\theta = \frac{17\pi}{6}$

$\theta = \frac{-3\pi}{2}$