The only application of rational equations that we will cover in this class is **shared work problems**, which means two people (or machines, or objects) working together on a job to share the work.

Shared Work Problems:

- write an equation that represents the rate at which two people do a job separately, set equal to the rate that they would work together
  - person A does a job in 2 hours, so their rate is $\frac{1}{2}$ the job per hour
  - person B does the same job in 3 hours, so their rate is $\frac{1}{3}$ of the job per hour
  - if it's unknown how long it would take persons A and B do the job together, assign a variable (x hours); so their rate together would be $\frac{1}{x}$ of the job per hour

$$\begin{matrix}\text{rate}\\ \text{one}\end{matrix} + \begin{matrix}\text{rate}\\ \text{two}\end{matrix} = \begin{matrix}\text{rate}\\ \text{together}\end{matrix}$$

$$\frac{1}{} + \frac{1}{} = \frac{1}{}$$

- make sure the units of time are consistent
  - if one person does a job in 45 minutes, and another person does the same job in 2 hours, convert both rates to either minutes or hours
    - in terms of minutes, the equation would be $\frac{1}{45} + \frac{1}{120} = \frac{1}{x}$
- make sure your answer makes sense
  - if one person does a job in 45 minutes, and another person does the same job in 2 hours, it shouldn't take both people working together more than 45 minutes to do the job
    - the equation $\frac{1}{45} + \frac{1}{120} = \frac{1}{x}$ results in $x = \frac{360}{11}$, which is approximately 33 minutes, so each person saves time by working together

<u>Example 1:</u> It takes a boy 90 minutes to mow the lawn, but his sister can mow it in an hour. How long (in minutes) would it take them to mow the lawn if they worked together using two lawn mowers?

(hint: if you don't know, assign a variable)

$$\begin{array}{c}\text{rate}\\ \text{one}\end{array} + \begin{array}{c}\text{rate}\\ \text{two}\end{array} = \begin{array}{c}\text{rate}\\ \text{together}\end{array}$$

$$\frac{1}{} + \frac{1}{} = \frac{1}{}$$

Example 2: Working together, Bill and Tom painted a fence in 4 hours. If Tom has painted the same fence before by himself in 7 hours, how long (in hours) would it take Bill on his own? Round your answer to one decimal place.

If Tom has painted the fence on his own in 7 hours, his rate is $\frac{1}{7}$ of the job per hour. If Bill has never painted the fence on his own, we don't know how long it will take him, so we can say it would take him x hours. Therefore his rate would be $\frac{1}{x}$ of the job per hour. Working together, Bill and Tom can paint the fence in 4 hours, so their rate is $\frac{1}{4}$ of the job per hour. Since Tom's rate plus Bill's rate equals their rate together, we have the following equation:

$$\text{Tom's rate} + \text{Bill's rate} = \text{rate together}$$

$$\frac{1}{7} + \frac{1}{x} = \frac{1}{4}$$

To eliminate the fractions, we can multiply both sides of the equation by 7, x, and 4. This is the same as multiplying by the least common multiply, which is $28x$.

$$28x\left(\frac{1}{7} + \frac{1}{x}\right) = \left(\frac{1}{4}\right)28x$$

$$4x + 28 = 7x$$

$$28 = 3x$$

$$x = \frac{28}{3}$$

Since the directions say to round answers to one decimal place, we can express the answer as 9.3 hours. So if Bill painted the fence on his own, we would expect it to take him about **9.3 hours.**

Example 3: On average, Mary can do her homework assignments in 100 minutes. It takes Frank about 2 hours to complete a given assignment. How long (in minutes) will it take the two of them working together to complete an assignment (round to the one decimal place, if necessary)?

Mary completes her homework assignment in 100 minutes, so working at a constant rate she completes $\frac{1}{100}$ of the assignment per minute.

Frank completes his homework assignment in 2 hours, which is 120 minutes. Again, assuming Frank works at a constant rate he completes $\frac{1}{120}$ of the assignment per minute.

Since we don't know how long it will take the two of them working together, I will use a variable to express that quantity (x minutes). And again, assuming they work at a constant rate they will complete $\frac{1}{x}$ of the assignment per minute.

$$\frac{1}{100}+\frac{1}{120}=\frac{1}{x}$$

$$100\cdot 120\cdot x\left(\frac{1}{100}+\frac{1}{120}\right)=\frac{1}{x}\cdot 100\cdot 120\cdot x$$

$$\frac{100\cdot 120\cdot x}{100}+\frac{100\cdot 120\cdot x}{120}=\frac{100\cdot 120\cdot x}{x}$$

$$\frac{\cancel{100}\cdot 120\cdot x}{\cancel{100}}+\frac{100\cdot \cancel{120}\cdot x}{\cancel{120}}=\frac{100\cdot 120\cdot \cancel{x}}{\cancel{x}}$$

$$120x+100x=12{,}000$$

$$220x=12{,}000$$

$$x=54.\overline{54}$$

I expressed Frank's rate in terms of minutes so it would be consistent with the way I expressed Mary's rate. Whenever you have different units of time, you need to convert one to make them consistent.

Since Mary's rate plus Frank's rate equals their rate together, we got the equation $\frac{1}{100}+\frac{1}{120}=\frac{1}{x}$

To solve that equation, I multiplied both sides of the equation by $100, 120,$ and x to clear the fractions, then solved for x.

So working together it would take Mary and Frank about $\mathbf{54.5}$ minutes to complete the assignment.

<u>Example 4:</u> Two pipes can be used to fill a swimming pool. When the first pipe is closed, the second pipe can fill the pool in 9 hours. When the second pipe is closed, the first pipe can fill the pool in 7 hours. How long (in hours) will it take to fill the pool if both pipes are open? Round your answer to two decimal places.

$$\begin{matrix}\text{rate}\\ \text{one}\end{matrix} + \begin{matrix}\text{rate}\\ \text{two}\end{matrix} = \begin{matrix}\text{rate}\\ \text{together}\end{matrix}$$

$$\frac{1}{} + \frac{1}{} = \frac{1}{}$$

Answers to Exercises:

1. 36 minutes *; 2.* 9.3 hours *; 3.* 54.5 minutes *; 4.* 3.94 hours