Solving quadratic equations will be the main topic on Exam 3; we will be solving quadratic equations in Lessons 12, 13, 14, and 15. You will still need to understand the other topics that we've covered (such as rational equations from Lesson 11) and will cover (such as equations containing fraction exponents from Lesson 15), but solving quadratic equations will make-up about 75% of Exam 3, so please be prepared.

Quadratic equation:

- any equation that can be written in the form $ax^2 + bx + c = 0$
  - b and/or c can equal zero, but a cannot
  - if $a = 0$ the equation is **NOT** quadratic
- the degree of a quadratic equation is always 2
- the following are all examples of quadratic equations
  - $2x^2 - 3x - 5 = 0$
  - $-x^2 + 4x = 0$
  - $\frac{1}{2}x^2 + \frac{3}{4} = 0$
  - $-3x^2 = 0$

The first method we will use to solve quadratic equations is factoring. If we can take a quadratic equation, which is just a polynomial set equal to zero, and factor that polynomial, we can use the **Zero Factor Theorem** to solve it.

Zero Factor Theorem:

- the product of two or more factors is zero if and only if at least one of the factors is zero
  - $xy = 0$ if and only if $x = 0$ or $y = 0$
- we will use the Zero Factor Theorem to solve quadratic equations that are in factored form and set equal to zero
  - if $(x + 2)(x - 3) = 0$, then $x + 2 = 0$ and $x - 3 = 0$, which means $x = -2$ and $x = 3$
- this Theorem does **NOT** work for any other numbers but zero
  - if you have an equation in factored form such as $(x - 2)(x + 3) = 1$ or $(x + 5)(x - 1) = -3$, we **CANNOT** simply set each factor equal to the number to solve; the Zero Factor Theorem only works with zero, because the only way to get a product of zero is to multiply by zero

When an equation is factorable, we set it equal to zero so we can use the Zero Factor Theorem to solve it.

Steps for Solving an Equation by Factoring:

1. write the equation as a polynomial and set it equal to zero
2. factor the polynomial (review the Steps for Factoring if needed)
3. use Zero Factor Theorem to solve

Example 1: Solve the quadratic equation $15x^2 - 2 = 13x$ for x and enter exact answers only (no decimal approximations). If there is more than one solution, separate your answers with commas. If there are no real solutions, enter NO SOLUTION.

$$15x^2 - 2 = 13x$$

ac	***b***	**Think about the signs of the product and the sum.**

Quadratic equations are not the only type of equation that can be solved by factoring. Other polynomial equations such as $2x^4 - 168x^2 + 486 = 0$ (which we will see in a future lesson) that are not quadratic can still be solved by factoring.

<u>Example 2:</u> Solve the quadratic equation $x(3x - 25) = -8$ for x and enter exact answers only (no decimal approximations). If there is more than one solution, separate your answers with commas. If there are no real solutions, enter NO SOLUTION.

$$x(3x - 25) = -8$$

ac	***b***	**Think about the signs of the product and the sum.**

Example 3: Solve the following equations for x and enter exact answers only (no decimal approximations). If there is more than one solution, separate your answers with commas. If there are no real solutions, enter NO SOLUTION.

a. $x = \frac{15-4x^2}{17}$

b. $26x + 24 = 5x^2$

$$0 = 5x^2 - 26x - 24$$

ac	***b***	**Think about the signs of the product and the sum.**
-120	-26	
$1, -120$	-119	
$2, -60$	-58	
$3, -40$	-36	
$\mathbf{4, -30}$	$\mathbf{-26}$	

$$0 = 5x^2 + 4x - 30x - 24$$

$$0 = x(5x + 4) - 6(5x + 4)$$

$$0 = (5x + 4)(x - 6)$$

$$5x + 4 = 0 \quad ; \quad x - 6 = 0$$

$$\boldsymbol{x = -\frac{4}{5} \quad ; \quad x = 6}$$

c. $10 + 17x = 6x^2$

$0 = 6x^2 - 17x - 10$

$0 = 6x^2 + 3x - 20x - 10$

$0 = 3x(2x + 1) - 10(2x + 1)$

$0 = (2x + 1)(3x - 10)$

$0 = 2x + 1 \;\; ; \;\; 0 = 3x - 10$

$\boldsymbol{-\frac{1}{2} = x \;\; ; \;\; \frac{10}{3} = x}$

d. $(x - 1)(x - 2) = 6$

$x^2 - 3x + 2 = 6$

$x^2 - 3x - 4 = 0$

$x^2 + x - 4x - 4 = 0$

$x(x + 1) - 4(x + 1) = 0$

$(x + 1)(x - 4) = 0$

$x + 1 = 0 \;\; ; \;\; x - 4 = 0$

$\boldsymbol{x = -1 \;\; ; \;\; x = 4}$

e. $x^2 - 25 = 0$

f. $9x^2 - 16 = 0$

Again, this method of solving equations can be used to solve more than just quadratic equations, as we will see in future lessons.

Answers to Examples:

1. $x = -\frac{2}{15}, 1$ *; 2.* $x = \frac{1}{3}, 8$ *; 3a.* $x = -5, \frac{3}{4}$ *; 3b.* $x = -\frac{4}{5}, 6$ *;*
3c. $= -\frac{1}{2}, \frac{10}{3}$ *; 3d.* $x = -1, 4$ *; 3e.* $= -5, \; 5$ *; 3f.* $x = -\frac{4}{3}, \frac{4}{3}$ *;*