Steps for Solving Formulas with Quadratic Equations:

1. identify the variable you are trying to isolate
2. remove parentheses and combine like terms (if any)
3. eliminate fractions (if any) by multiplying by the denominators
4. isolate the specified variable by using the techniques for solving quadratic equations
    a. if it is possible to isolate the variable by extracting square roots then take the square root of both sides of the equation

Example 1: Solve the following formula for the specified variable. If there is more than one solution, separate your answers with commas. If there are no real solutions, enter NO SOLUTION.

a. $a_c = \frac{v^2}{r}$; solve for v

(Centripetal Acceleration formula; a is centripetal acceleration, v is velocity, and r is the radius)

<u>Example 2:</u> Solve the following formula for the specified variable. If there is more than one solution, separate your answers with commas. If there are no real solutions, enter NO SOLUTION.

b. $S = 4\pi r^2$; solve for r

(S is the surface area of a sphere and r is the radius)

$$\boldsymbol{S = 4\pi r^2}$$

Start by isolating the perfect square r^2 by dividing both sides by 4π

$$\boldsymbol{\frac{S}{4\pi} = r^2}$$

Next, solve for r by taking the square root of both sides of the equation

$$\boldsymbol{\pm\sqrt{\frac{S}{4\pi}} = r}$$

Keep in mind that we must simplify the radical completely

$$\boldsymbol{\pm\frac{1}{2}\sqrt{\frac{S}{\pi}} = r}$$

However rationalizing the denominator is not necessary

$$\boldsymbol{r = \frac{1}{2}\sqrt{\frac{S}{\pi}}}$$

<u>You do not need to rationalize the denominators on these problems, but you do need to simplify all radicals completely.</u>

Answers such as $r = \frac{1}{2}\sqrt{\frac{S}{\pi}}$, $\frac{\sqrt{4\pi S}}{4\pi}$ and $\frac{\sqrt{\pi S}}{2\pi}$ are all acceptable.

<u>Example 3:</u> Solve the following formula for the specified variable. If there is more than one solution, separate your answers with commas. If there are no real solutions, enter NO SOLUTION.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad ; \text{solve for } a$$

(equation for an ellipse; a, b, x, and y can be any real number)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$a^2b^2\left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right) = (1)a^2b^2$$

$$b^2x^2 + a^2y^2 = a^2b^2$$

$$b^2x^2 = a^2b^2 - a^2y^2$$

$$b^2x^2 = a^2(b^2 - y^2)$$

$$\frac{b^2x^2}{b^2 - y^2} = a^2$$

$$a = \pm\sqrt{\frac{b^2x^2}{b^2 - y^2}}$$

$$a = \pm\frac{\sqrt{b^2x^2}}{\sqrt{b^2 - y^2}}$$

$$a = \pm\frac{bx}{\sqrt{b^2 - y^2}}$$

Keep in mind that while we have a Product Rule for Radicals, and a Quotient Rule for Radicals, there is no sum or difference rules for radicals. That means we can break a product or a quotient into separate radicals, but we cannot do the same with sums or differences.

$$\boldsymbol{a = \frac{bx}{\sqrt{b^2 - y^2}}, -\frac{bx}{\sqrt{b^2 - y^2}}}$$

<u>Example 4:</u> Solve the following formula for the specified variable. If there is more than one solution, separate your answers with commas. If there are no real solutions, enter NO SOLUTION.

$$a^2 + b^2 - c^2 = 0 \qquad ; \text{solve for } c$$

(Pythagorean theorem; a, b, and c are side lengths of a right triangle)

Once again, as stated in Lesson 4, there is a Product Rule for Radicals and a Quotient Rule for Radicals, **<u>but there is no Sum Rule for Radicals or Difference Rule for Radicals:</u>**

- $\sqrt{x^2 + y^2} = (x^2 + y^2)^{\frac{1}{2}}$, **<u>BUT</u>** $\sqrt{x^2 + y^2} \neq x + y$
- $\sqrt{a^2 - b^2} = (a^2 - b^2)^{\frac{1}{2}}$, **<u>BUT</u>** $\sqrt{a^2 - b^2} \neq a - b$

When a sum or difference is raised to a power, the power is **<u>NOT</u>** distributive. It is often helpful to use real numbers to demonstrate this, as shown below:

- $\sqrt{3^2 + 4^2} \neq 3 + 4$

  $\sqrt{9 + 16} \neq 7$

  $\sqrt{25} \neq 7$

  $5 \neq 7$

- $\sqrt{5^2 - 4^2} \neq 5 - 4$

  $\sqrt{25 - 16} \neq 1$

  $\sqrt{9} \neq 1$

  $3 \neq 1$

Be sure to keep this idea in mind while solving the remaining problems in these notes, as well as when solving similar problems in the homework.

<u>Example 5:</u> Solve the following formulas for the specified variables. If there is more than one solution, separate your answers with commas. If there are no real solutions, enter NO SOLUTION. You do not need to rationalize your denominators.

a. $y = a(x - h)^2 + k$; solve for x

(Standard Equation of a Parabola; $a \neq 0$, $x, y, h,$ and k can be any real numbers)

b. $A = \pi(R^2 - r^2)$; solve for R

(A is the area of a circular ring, R is the outer radius and r is the inner radius)

c. $A = \pi(R^2 - r^2)$; solve for r

(A is the area of a circular ring, R is the outer radius and r is the inner radius)

Answers to Examples:

1. $v = \sqrt{a_c r}, -\sqrt{a_c r}$; *2.* $r = \sqrt{\frac{S}{4\pi}}$; *3.* $a = \frac{bx}{\sqrt{b^2-y^2}}, -\frac{bx}{\sqrt{b^2-y^2}}$;

4. $c = \sqrt{a^2 + b^2}$; *5a.* $x = h + \sqrt{\frac{y-k}{a}}$, $x = h - \sqrt{\frac{y-k}{a}}$;

5b. $R = \sqrt{\frac{A+\pi r^2}{\pi}}$; *5c.* $r = \sqrt{\frac{\pi R^2 - A}{\pi}}$;