To determine the value of a function for a particular input, we replace the original input of the function (usually x) with the new input. For instance, to determine the value of the function $f(x) = \frac{x+2}{1-x}$ when $x = -1$, we would replace x (the original input) with -1 (the new input). This means not only replacing the x in parentheses with -1, but also replacing any x's in the expression that defines the function with -1's.

$$f(x) = \frac{x+2}{1-x}$$

$$f(-1) = \frac{-1+2}{1-(-1)}$$

$$f(-1) = \frac{1}{1+1}$$

$$f(-1) = \frac{1}{2}$$

This shows that for the function $f(x) = \frac{x+2}{1-x}$, when -1 is the input $(x = -1)$, $\frac{1}{2}$ is the output $\left(f(-1) = \frac{1}{2}\right)$.

The same idea is true when finding $f(0)$; simply replace all the x's in the function with 0's.

$$f(x) = \frac{x+2}{1-x}$$

$$f(0) = \frac{0+2}{1-0}$$

$$f(0) = \frac{2}{1}$$

$$f(0) = 2$$

One function value we would not be able to find would be $f(1)$. Keep in mind that any input of a function must be part of the domain of that function, and in this case, 1 is not part of the domain of f. We could not find the value of the function f when x is replaced with 1 because 1 would result in a denominator of zero.

$$f(x) = \frac{x+2}{1-x}$$

$$f(1) = \frac{1+2}{1-1}$$

$$f(1) = \frac{3}{0}$$

$f(1)$ is **UNDEFINED**

Had we simply found the domain of the function $f(x) = \frac{x+2}{1-x}$ first, before finding the function values, we would have already known that 1 would not be part of the domain, since the denominator cannot equal zero:

$$1 - x \neq 0$$

$$1 \neq x$$

$$\text{Domain of } f(x) = (-\infty, \mathbf{1}) \cup (\mathbf{1}, \infty)$$

Division by zero is not defined for any function, so that is why $f(1)$ is undefined, and why 1 is not part of the domain of f. This is one reason it is important to be able to find the domain of a function; so that we know what we can (and what we cannot) plug into the function for x.

When working with a function that contains a square root, ALWAYS state that the radicand must be greater than or equal to zero.

When working with a function that contains a fraction, ALWAYS state that the denominator can never be equal to zero.

<u>Example 1:</u> Given $g(x) = 2 - 5\sqrt{x-3}$, find the following. Enter exact answers only (no approximations). If the function is undefined for a particular input, write UNDEFINED.

a. Find the domain of g and list in interval notation

b. Find $g(4)$; *When x is replaced with* 4, *the output is ?*

c. Find $g(3)$; *When x is replaced with* 3, *the output is ?*

d. Find $g(2)$; *When x is replaced with* 2, *the output is ?*

<u>Keep in mind that when finding the domain of the function $g(x) = 2 - 5\sqrt{x-3}$, we only focused on the radicand $x - 3$ being non-negative $(x - 3 \geq 0)$. However when finding the function values, we used the entire expression $-5\sqrt{x-3}$:</u>

$$\boldsymbol{g(4) = 2 - 5\sqrt{4-3}}$$
$$\boldsymbol{g(3) = 2 - 5\sqrt{3-3}}$$

Example 2: Given $h(x) = 1 - 3\sqrt{\frac{7-x}{-5}}$, find the following. Enter exact answers only (no approximations). If the function is undefined for a particular input, write UNDEFINED.

$\frac{7-x}{-5} \geq 0$

$(-5)\left(\frac{7-x}{-5}\right) \geq (0)(-5)$

$7 - x \leq 0$

$7 \leq x$

Domain of h: $[7, \infty)$

Keep in mind that when you multiply or divide both sides of an inequality by a negative value, the inequality sign switches (greater than or equal to becomes less than or equal to, or vice versa). That is why $\frac{7-x}{-5} \geq 0$ became $7 - x \leq 0$, because we multiplied both sides of the equation by -5.

a. $h(2)$

$h(2) = 1 - 3\sqrt{\frac{7-2}{-5}}$

$h(2) = 1 - 3\sqrt{\frac{5}{-5}}$

$h(2) = 1 - 3\sqrt{-1}$

Since 2 is not part of the domain of the function, $h(2)$ is **UNDEFINED**.

b. $h(7)$

$h(7) = 1 - 3\sqrt{\frac{7-7}{-5}}$

$h(7) = 1 - 3\sqrt{\frac{0}{-5}}$

$h(7) = 1 - 3\sqrt{0}$

$h(7) = 1 - 3(0)$

$h(7) = 1 - 0$

$\boldsymbol{h(7) = 1}$

c. $h(27)$

$h(27) = 1 - 3\sqrt{\frac{7-27}{-5}}$

$h(27) = 1 - 3\sqrt{\frac{-20}{-5}}$

$h(27) = 1 - 3\sqrt{4}$

$h(27) = 1 - 3(2)$

$h(27) = 1 - 6$

$\boldsymbol{h(27) = -5}$

<u>Example 3:</u> Given $j(x) = \frac{x^2-4}{x^2+4}$, find the following. Enter exact answers only (no approximations). If the function is undefined for a particular input, write UNDEFINED.

$$x^2 + 4 \neq 0$$

$$x^2 \neq -4$$

$$x \neq \pm\sqrt{-4}$$

Notice that we have the inequality x cannot equal plus or minus the square root of -4. That cannot happen using real numbers, since the square root of -4 (or negative anything) does not exist with real numbers. So since the only restriction on x occurs using imaginary numbers, there are no real restrictions. And that means that x can be any real number, so the domain of this function is unrestricted.

Domain of j: $(-\infty, \infty)$

a. $j(-2)$

$j(-2) = \frac{(-2)^2-4}{(-2)^2+4}$

$j(-2) = \frac{4-4}{4+4}$

$j(-2) = \frac{0}{8}$

$\boldsymbol{j(-2) = 0}$

b. $j(0)$

$j(0) = \frac{0^2-4}{0^2+4}$

$j(0) = \frac{0-4}{0+4}$

$j(0) = \frac{-4}{4}$

$\boldsymbol{j(0) = -1}$

c. $j(2)$

$j(2) = \frac{2^2-4}{2^2+4}$

$j(2) = \frac{4-4}{4+4}$

$j(2) = \frac{0}{8}$

$\boldsymbol{j(2) = 0}$

Example 4: Given $k(x) = \frac{1}{x\sqrt{1+x}}$, find the following. Enter exact answers only (no approximations). If the function is undefined for a particular input, write UNDEFINED.

Domain of k:

$$x\sqrt{1+x} \neq 0 \quad \text{AND} \quad 1+x \geq 0$$

$$\left(x\sqrt{1+x}\right)^2 \neq (0)^2 \quad \text{AND} \quad x \geq -1$$

$$\left(x\sqrt{1+x}\right)\left(x\sqrt{1+x}\right) \neq 0 \quad \text{AND} \quad -1 \leq x$$

$$x^2(1+x) \neq 0 \quad \text{AND} \quad -1 \leq x$$

$$x^2 \neq 0 \quad \text{AND} \quad 1+x \neq 0 \quad \text{AND} \quad -1 \leq x$$

$$x \neq 0 \quad \text{AND} \quad x \neq -1 \quad \text{AND} \quad -1 \leq x$$

$$x \neq 0 \quad \text{AND} \quad -1 < x$$

$$-1 < x \qquad x \neq 0$$

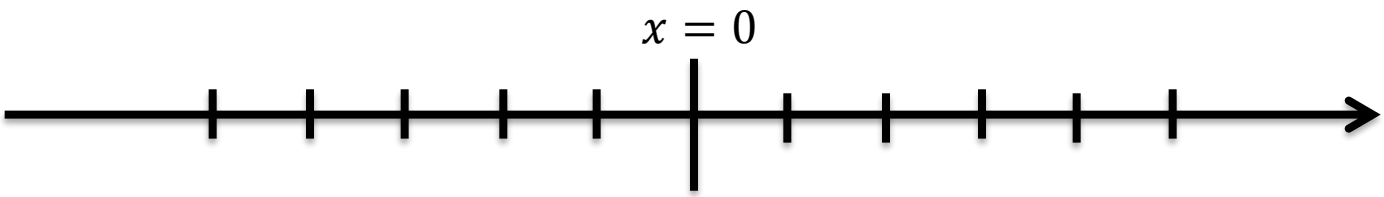
Notice that the two separate inequalities $x \neq -1$ and $-1 \leq x$ are combined to make one single inequality $-1 < x$. Since x cannot be equal to -1, we have to change the inequality $-1 \leq x$ to $-1 < x$ because plugging -1 in for x would make the function undefined.

$$\mathbf{(-1, 0) \cup (0, \infty)}$$

b. $k(-1)$ b. $k(0)$ c. $k(3)$

<u>Example 5:</u> Given $m(x) = \frac{\sqrt{x}}{2-\sqrt{x+1}}$, find the following. Enter exact answers only (no approximations). If the function is undefined for a particular input, write UNDEFINED.

<u>Domain of m</u>:



a. $m(2)$ b. $m(3)$ c. $m(4)$

Example 6: Given $(x) = \frac{x^2+4}{(x+2)\sqrt{x-2}}$, find the following. Enter exact answers only (no approximations). If the function is undefined for a particular input, write UNDEFINED.

Domain of n:

$$(x+2)\sqrt{x-2} \neq 0 \quad \text{AND} \quad x-2 \geq 0$$

$$\left((x+2)\sqrt{x-2}\right)^2 \neq (0)^2 \quad \text{AND} \quad x \geq 2$$

$$(x+2)^2\left(\sqrt{x-2}\right)^2 \neq 0 \quad \text{AND} \quad 2 \leq x$$

$$(x+2)^2(x-2) \neq 0 \quad \text{AND} \quad 2 \leq x$$

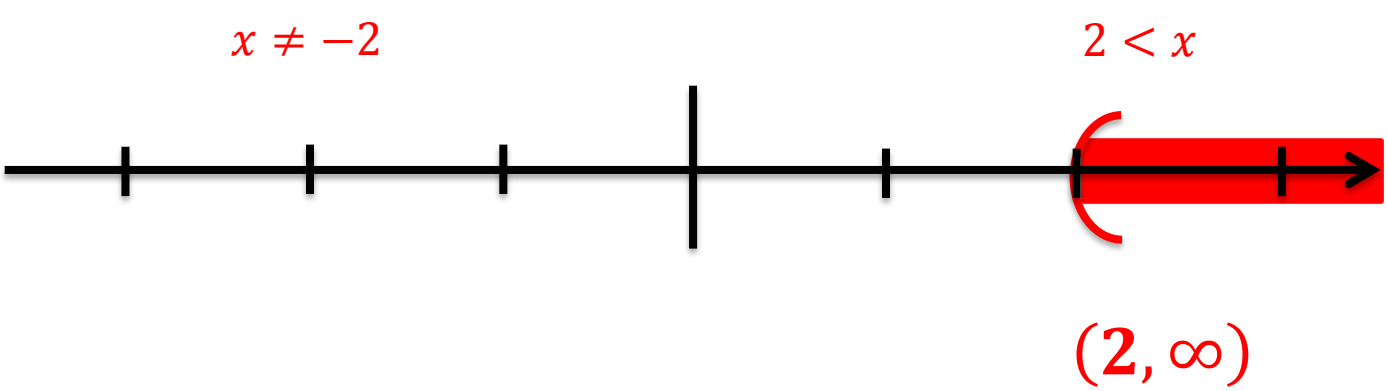
$$(x+2)^2 \neq 0 \quad \text{AND} \quad x-2 \neq 0 \quad \text{AND} \quad 2 \leq x$$

$$x+2 \neq 0 \quad \text{AND} \quad x \neq 2 \quad \text{AND} \quad 2 \leq x$$

$$x \neq -2 \quad \text{AND} \quad 2 < x$$

Once again notice that the two separate inequalities ($x \neq 2$ and $2 \leq x$) are combined to make one single inequality $2 < x$.

Also, if $2 < x$, then the third inequality $x \neq -2$ has no effect on the domain of the function, because -2 is not part $2 < x$.



a. $n(2)$ b. $n(3)$ c. $n(6)$

Answers to Exercises:

1a. Domain of h: $[3, \infty)$; *1b.* -3 ; *1c.* 2 ; *1d.* UNDEFINED ;

2. Domain of g: $(-\infty, 3]$; *2a.* 2 ; *2b.* 1 ; *2c.* UNDEFINED ;

3. Domain of j: $(-\infty, \infty)$; *3a.* 0 ; *3b.* -1 ; *3c.* 0 ;

4. Domain of k: $(-1, 0) \cup (0, \infty)$; *4a.* UNDEFINED ;

4b. UNDEFINED ; *4c.* $\frac{1}{6}$;

5. Domain of m: $[0, 3) \cup (3, \infty)$; *5a.* $\frac{\sqrt{2}}{2-\sqrt{3}}$; *5b.* UNDEFINED ;

5c. $\frac{2}{2-\sqrt{5}}$;

6. Domain of n: $(2, \infty)$; *6a.* UNDEFINED ; *6b.* $\frac{13}{5}$; *6c.* $\frac{5}{2}$;