When an exponent n is a positive integer, such as $1, 2, 3, 4, \ldots$, exponential notation represents the product of repeated factors (the base times itself some number of times)

- $a^2 = a \cdot a$
  - the exponent of 2 indicates there are 2 factors of a
- $b^5 = b \cdot b \cdot b \cdot b \cdot b$
  - the exponent of 5 indicates there are 5 factors of b
- $x^n = x \cdot x \cdot \ldots \cdot x$
  - the exponent of n indicates there are n factors of x

What about when an exponent n is not a positive integer? In this section we'll look at exponents of zero and exponents that are negative integers.

One way to approach exponents of zero is to think about a term divided by itself; for instance, $\frac{x^2}{x^2} = 1$ because anything over itself is one. However, what happens if we simplified $\frac{x^2}{x^2}$ using the Quotient Rule that was discussed earlier?

$$\frac{x^2}{x^2} = x^{2-2} = x^0$$

This shows that $\frac{x^2}{x^2} = x^0$, and since we already know that $\frac{x^2}{x^2} = 1$, that means x^0 must equal 1. This leads us to the Zero-Exponent Rule.

<u>Zero-Exponent Rule:</u>

- any base taken to the power of zero is 1
  - the exception to this rule is a base of zero, because using the $\frac{x^2}{x^2}$ example, you cannot have a denominator of zero
- this is true for a factor like $x^0 = 1$, as well as a product like $(5x^7y^6)^0 = 1$ or a quotient like $\left(\frac{-3x^2y}{4y^9}\right)^0 = 1$

- $xy^0 =$
- $(xy)^0 =$
- $-4^0 =$
- $(-4)^0 =$

The final topic in this lesson is negative exponents. Our goal when working with negative exponents is to make them positive, since we have already covered exponent rules with positive integers. One way to understand how to change a negative exponent to a positive exponent is to think about canceling common factors within a fraction. For instance, $\frac{x^2}{x^3} = \frac{x \cdot x}{x \cdot x \cdot x}$, and since this fraction has common factors in the numerator and denominator, we can simply cancel two factors of x from both to get $\frac{1}{x}$. However, what happens if we simplify $\frac{x^2}{x^3}$ using the Quotient Rule?

$$\frac{x^2}{x^3} = x^{2-3} = x^{-1}$$

So what we see is that $\frac{x^2}{x^3}$ simplifies to both x^{-1} and $\frac{1}{x}$; and since x^{-1} and $\frac{1}{x}$ are both equal to $\frac{x^2}{x^3}$, they are also equal to each other. So this shows that to change the sign of an exponent, we can simply take the reciprocal of the factor that has a negative exponent.

Negative Exponent Rule:

- to change the sign of an **exponent**, take the reciprocal of the expression or factor with the negative exponent
  - $x^{-2} = \frac{1}{x^2}$
  - $xy^{-5} = x \cdot \frac{1}{y^5}$
    $= \frac{x}{y^5}$
    - notice we do not take the reciprocal of the exponent, but rather the factor that contains a negative exponent
- remember that when an exponent is a positive integer, exponential notation represents the product of repeated factors (something times itself times some number of times)
- the sign of the base does **NOT** change
  - $(-2)^{-4} =$
  - $-2^{-4} =$

- again, this is true for a factor or a product/quotient
  - Product to a Power
    - $(-2x^4)^{-3} =$

$$\frac{1}{(-2x^4)^3}$$

$$\frac{1}{(-2)^3(x^4)^3}$$

$$\boldsymbol{\frac{1}{-8x^{12}}}$$

  - Quotient to a Power
    - $\left(\frac{3x^3y^2}{-xy^3}\right)^{-4} =$

$$\left(\frac{-xy^3}{3x^3y^2}\right)^4$$

$$\frac{(-1)^4(x)^4(y^3)^4}{(3)^4(x^3)^4(y^2)^4}$$

$$\frac{1x^4y^{12}}{81x^{12}y^8}$$

$$\boldsymbol{\frac{y^4}{81x^8}}$$

Example 1: Simplify each expression **COMPLETELY**. Do **NOT** leave negative exponents in your answers.

a. $-8y^2(3y^3)^{-4}$

$-8y^2\frac{1}{(3y^3)^4}$

$\frac{-8y^2}{81y^{12}}$

$\boldsymbol{\frac{-8}{81y^{10}}}$

b. $(-8y)^2(3y^{-3})^{-4}$

$(-8)^2(y)^2(3)^{-4}(y^{-3})^{-4}$

$64y^2\frac{1}{3^4}y^{12}$

$\frac{64y^2}{1}\frac{1}{81}\frac{y^{12}}{1}$

$\boldsymbol{\frac{64y^{14}}{81}}$

c. $\left(\frac{1}{2}x^4y^{-3}\right)^5\left(-\frac{\sqrt{5}}{7}x^{-\pi}\right)^0$

d. $\left(\frac{2x^4}{y^{-7}}\right)^3\left(\frac{-x^{-5}}{2y^6}\right)^2$

$(2x^4y^7)^3\left(\frac{-1}{2x^5y^6}\right)^2$

$8x^{12}y^{21}\frac{1}{4x^{10}y^{12}}$

$\frac{8x^{12}y^{21}}{4x^{10}y^{12}}$

$\boldsymbol{2x^2y^9}$

e. $-3^{-1}(6x^{-4})^2\left(\frac{x^0y^{-1}}{4y^3}\right)^{-3}\left(\frac{1}{2}x^{-4}y^3\right)^5$

$-1\cdot\frac{1}{3}\left(\frac{6}{x^4}\right)^2\left(\frac{1}{4y^4}\right)^{-3}\left(\frac{y^3}{2x^4}\right)^5$

$-\frac{1}{3}\cdot\frac{36}{x^8}\cdot\left(\frac{4y^4}{1}\right)^3\cdot\frac{y^{15}}{32x^{20}}$

$-\frac{1}{3}\cdot\frac{36}{x^8}\cdot\frac{64y^{12}}{1}\cdot\frac{y^{15}}{32x^{20}}$

$-\frac{36\cdot64y^{27}}{3\cdot32x^{28}}$

$\boldsymbol{-\frac{24y^{27}}{x^{28}}}$

f. $\left(\frac{-2x^{-4}}{y^7}\right)^3\left(\frac{-2x^6}{y^5}\right)^{-2}$

g. $\left(\frac{1}{3}x^{-5}y^{2}\right)^{-1}(9x^{2}y^{3})^{-2}$

h. $\dfrac{\left(\frac{1}{2}x^{-3}y^{0}\right)^{-1}}{xy^{-2}}$

$$\frac{\left(\frac{1}{2}\right)^{-1}(x^{-3})^{-1}(y^{0})^{-1}}{xy^{-2}}$$

$$\frac{2x^{3}(1)}{xy^{-2}}$$

$$\frac{2x^{3}y^{2}}{x}$$

$$\mathbf{2x^{2}y^{2}}$$

i. $-3^{2}+7^{0}-2^{-1}$

j. $-\pi^{0}+4^{-2}+\left(\frac{16}{7}\right)^{-1}$

$$-1\cdot\pi^{0}+\frac{1}{4^{2}}+\left(\frac{7}{16}\right)^{1}$$

$$-1\cdot 1+\frac{1}{16}+\frac{7}{16}$$

$$-1+\frac{1}{16}+\frac{7}{16}$$

$$-\frac{16}{16}+\frac{1}{16}+\frac{7}{16}$$

$$\mathbf{-\frac{1}{2}}$$

Answers to Examples:

1a. $\frac{-8}{81y^{10}}$; *1b.* $\frac{64y^{14}}{81}$; *1c.* $\frac{x^{20}}{32y^{15}}$; *1d.* $2x^{2}y^{9}$; *1e.* $\frac{-24y^{27}}{x^{28}}$;
1f. $\frac{-2}{x^{24}y^{11}}$; *1g.* $\frac{x}{27y^{8}}$; *1h.* $2x^{2}y^{2}$; *1i.* $-\frac{17}{2}$; *1j.* $-\frac{1}{2}$