When a function is transformed, its domain and/or range will change. If only the inputs are transformed, then only the domain will change. If only the outputs are transformed, then only the range will change. If both the inputs and outputs are transformed, then both the domain and range will change.

Remember that the domain represents the set of inputs for a function, and the range represents the set of outputs.

Example 1: Let $y = f(x)$ be a function with domain $D = [-6, 5]$ and range $R = [0, 14]$. Find the domain D and range R for each of the following functions. **Keep in mind order of operation and the order of your intervals.**

a. $y = -3f(x)$

Changes **IN**side the parentheses change the **IN**puts and we do the **IN**verse; remember that the Domain is the set of inputs

$\text{inputs}, -3(\text{outputs})$

Since the inputs of this function are not being changed with the transformation $y = -3f(x)$, that means the domain is also not being changed. So the domain will still be $D = [-6, 5]$.

Range: $[-3(0), -3(14)]$

Range: $[0, -42]$

Domain: $[-6, 5]$
Range: $[-42, 0]$

b. $y = f\left(\frac{1}{2}x\right)$

Changes **OUT**side the parentheses change the **OUT**puts and we do exactly what we see; remember that the Range is the set of outputs

$\frac{\text{inputs}}{\frac{1}{2}}, \text{outputs}$

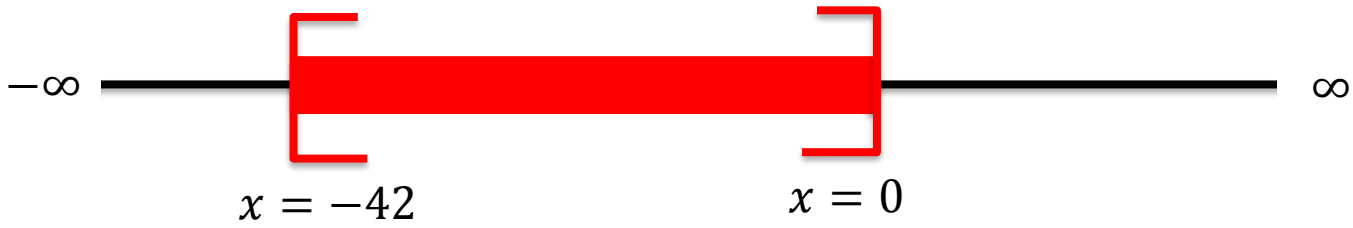
Since the outputs of this function are not being changed with the transformation $y = f\left(\frac{1}{2}x\right)$, that means the range is also not being changed. So the range will remain $D = [0, 14]$.

Domain: $\left[-\frac{6}{\frac{1}{2}}, \frac{5}{\frac{1}{2}}\right]$

Domain: $[-12, 10]$

Domain: $[-12, 10]$
Range: $[0, 14]$

<u>Be sure to keep in mind that intervals (such as a domain or range), just like number lines, always go in order from smallest to largest as you go from left to right. On Example 1a, the range is listed as</u> $[-42, 0]$ **<u>because -42 is smaller than 0.</u>**



<u>Be sure to re-arrange intervals as needed so they are in the correct order.</u>

<u>Example 2:</u> Let $y = f(x)$ be a function with domain $D = [-6, 5]$ and range $R = [0, 14]$. Find the domain D and range R for each of the following functions. **<u>Keep in mind order of operation and the order of your intervals.</u>**

a. $y = f(x + 3) - 2$

b. $y = f(x - 4) + 1$

<u>Example 3:</u> Let $y = f(x)$ be a function with domain $D = [-6, 5]$ and range $R = [0, 14]$. Find the domain D and range R for each of the following functions. **<u>Keep in mind order of operation and the order of your intervals.</u>**

a. $y = \frac{1}{2} f(-x)$

b. $y = -f(3x)$

Example 4: Let $y = f(x)$ be a function with domain $D = [-6, 5]$ and range $R = [0, 14]$. Find the domain D and range R for each of the following functions. **Keep in mind order of operation and the order of your intervals.**

a. $y = \frac{2}{3}f(x) - 1$

b. $y = -f\left(-\frac{3}{2}x\right)$

Example 5: Let $y = f(x)$ be a function with domain $D = [0, \infty)$ and range $R = (-\infty, 0]$. Find the domain D and range R for each of the following functions. **Keep in mind order of operation and the order of your intervals.**

a. $y = \frac{1}{2}f(-x) + 3$

$\frac{\text{inputs}}{-1}, \frac{1}{2}(\text{outputs}) + 3$

Domain: $\left[\frac{0}{-1}, \frac{\infty}{-1}\right)$

Range: $\left(\frac{1}{2}(-\infty) + 3, \frac{1}{2}(0) + 3\right]$

Domain: $[0, -\infty)$

Range: $(-\infty, 3]$

Domain: $(-\infty, \mathbf{0}]$

Range: $(-\infty, \mathbf{3}]$

b. $y = -f(2x) - 2$

$\frac{\text{inputs}}{2}, -1(\text{outputs}) - 2$

Domain: $\left[\frac{0}{2}, \frac{\infty}{2}\right)$

Range: $(-1(-\infty) - 2, -1(0) - 2]$

Domain: $[0, \infty)$

Range: $(\infty, -2]$

Domain: $[\mathbf{0}, \infty)$

Range: $[-\mathbf{2}, \infty)$

Example 6: Let $y = f(x)$ be a function with domain $D = [0, \infty)$ and range $R = (-\infty, 0]$. Find the domain D and range R for each of the following functions. **Keep in mind order of operation and the order of your intervals.**

a. $y = \frac{2}{3}f(x-4) - 1$

b. $y = -3f\left(-\frac{1}{3}x\right)$

Example 7: Let $y = f(x)$ be a function with domain $D = [-9, 0]$ and range $R = (-\infty, \infty)$. Find the domain D and range R for each of the following functions. **Keep in mind order of operation and the order of your intervals.**

a. $y = f(x-4) + 1$

$\text{inputs} + 4, \text{outputs} + 1$

Domain: $[-9+4, 0+4]$
Range: $(-\infty + 1, \infty + 1)$

Domain: $[-5, 4]$
Range: $(-\infty, \infty)$

Domain: $[-5, 4]$
Range$(-\infty, \infty)$

b. $y = -f(3x)$

$\frac{\text{inputs}}{3}, -1(\text{outputs})$

Domain: $\left[-\frac{9}{3}, \frac{0}{3}\right]$
Range: $\left(-1(-\infty), -1(\infty)\right)$

Domain: $[-3, 0]$
Range: $(\infty, -\infty)$

Domain: $[-3, 0]$
Range: $(-\infty, \infty)$

Example 8: Let $y = f(x)$ be a function with domain $D = (-\infty, \infty)$ and range $R = [-5, 4]$. Find the domain D and range R for each of the following functions. **Keep in mind order of operation and the order of your intervals.**

a. $y = \frac{1}{2}f(-x) + 3$

b. $y = -5f(2x) - 2$

Once again, keep in mind that the domain of a function is the set of inputs, while the range of a function is the set of outputs. So any changes to the inputs of a function are made to the domain, and any changes to the outputs of a function are made to the range.

Answers to Examples:

1a. $D: [-6, 5], R: [-42, 0]$; *1b.* $D: [-12, 10], R: [0, 14]$;

2a. $D: [-9, 2], R: [-2, 12]$; *2b.* $: [-2, 9], R: [1, 15]$;

3a. $: [-5, 6], R: [0, 7]$; *3b.* $D: \left[-2, \frac{5}{3}\right], R: [-14, 0]$;

4a. $: [-6, 5], R: \left[-1, \frac{25}{3}\right]$; *4b.* $D: \left[-\frac{10}{3}, 4\right], R: [-14, 0]$;

5a. $D: (-\infty, 0], R: (-\infty, 3]$; *5b.* $D: [0, \infty), R: [-2, \infty)$;

6a. $D: [4, \infty), R: (-\infty, -1]$; *6b.* $D: (-\infty, 0], R: [0, \infty)$;

7a. $D: [-5, 4], R: (-\infty, \infty)$; *7b.* $D: [-3, 0], R: (-\infty, \infty)$;

8a. $D: (-\infty, \infty), R: \left[\frac{1}{2}, 5\right]$; *8b.* $D: (-\infty, \infty), R: [-22, 23]$;