

In the previous lesson, we worked on finding the standard form of a quadratic function ( $f(x) = a(x - h)^2 + k$ ) by converting from polynomial form ( $f(x) = ax^2 + bx + c$ ) to standard form. To do so, we first found the vertex using the formula  $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$ . Then we simply plugged the vertex into standard form for  $h$  and  $k$ , and we plugged in the leading coefficient  $a$ , which we already knew because it is the same leading coefficient  $a$  from polynomial form.

Today we will continue finding quadratic functions in standard form, only this time we will either be given the vertex directly, or we will be given the graph of a quadratic function so we can find the vertex from the graph. However we will not be given the quadratic function in any other forms, so we will not know the value of the leading coefficient  $a$ . To find it, we will first need to plug the vertex into standard form for  $h$  and  $k$ , then we will need to plug in any other point that the graph of the quadratic function passes through in order to find the value of  $a$ .

**Example 1:** Find the standard form  $f(x) = a(x - h)^2 + k$  of the quadratic function with a vertex  $(1, 2)$  that passes through the point  $(3, 4)$ .

$$f(x) = a(x - h)^2 + k$$

$$f(x) = a(x - 1)^2 + 2$$

$$f(x) = a(x - 1)^2 + 2$$

$$4 = a(3 - 1)^2 + 2$$

$$2 = a(4)$$

$$\frac{1}{2} = a$$

$$f(x) = \frac{1}{2}(x - 1)^2 + 2$$

To find the standard form of a quadratic function, always start by plugging in the vertex. In this case we'll replace  $h$  with 1 and  $k$  with 2.

Next we use the other point that we're given to find the value of the leading coefficient  $a$ . In this case we'll replace  $x$  with 3 and  $f(x)$  with 4.

Finally, we replace  $a$  with  $\frac{1}{2}$  in  $f(x) = a(x - 1)^2 + 2$  and we have the standard form of the quadratic function  $f$ .

The standard form of a quadratic function  $f(x) = a(x - h)^2 + k$  is convenient for identifying the vertex of the function, and it is also convenient for graphing the function by using transformations (more on this in a later set of notes). To determine the standard form of a quadratic function based its graph, or based on some information about the function, we simply plug in the vertex  $(h, k)$  and then use any other point that the parabola is passing through to determine the leading coefficient  $a$ .

Keep in mind that when expressing quadratic functions in standard form, it may be necessary to plug in values for  $x$  and  $f(x)$  in order to identify the leading coefficient  $a$ , but ALWAYS leave  $x$  and  $f(x)$  as  $x$  and  $f(x)$  when expressing your final answer. In other words, final answers will **ALWAYS** be expressed as  $f(x) = \#(x - \#)^2 + \#$ , where  $f(x)$  and  $x$  remain as is, but  $a$ ,  $h$ , and  $k$  are replaced with numbers.

**Example 2:** Find the standard form  $f(x) = a(x - h)^2 + k$  of the quadratic function that has a vertex at  $(-2, 2)$  and that passes through the point  $(4, -1)$ .

**Example 3:** Find the standard form  $f(x) = a(x - h)^2 + k$  of the quadratic function that has a vertex at  $(0, -2)$  and that passes through the point  $(3, 25)$ .

$$f(x) = a(x - h)^2 + k$$

$$f(x) = a(x - 0)^2 + (-2)$$

$$f(x) = ax^2 - 2$$

$$25 = a(3)^2 - 2$$

$$27 = a(9)$$

$$3 = a$$

$$f(x) = 3x^2 - 2$$

**Example 4:** Find the standard form  $f(x) = a(x - h)^2 + k$  of the quadratic function that has a vertex at  $\left(-\frac{1}{2}, -\frac{1}{4}\right)$  and has a y-intercept of  $-\frac{9}{4}$ .

**Example 5:** Find the standard form  $f(x) = a(x - h)^2 + k$  of the quadratic function that has a vertex at  $(3, 5)$  and has an  $x$ -intercept of  $0$ .

$$f(x) = a(x - h)^2 + k$$

$$f(x) = a(x - 3)^2 + 5$$

$$(0, 0)$$

$$0 = a(0 - 3)^2 + 5$$

$$-5 = a(9)$$

$$-\frac{5}{9} = a$$

$$f(x) = -\frac{5}{9}(x - 3)^2 + 5$$

To find the standard form of a quadratic function, always start by plugging in the vertex. In this case we'll replace  $h$  with  $3$  and  $k$  with  $5$ .

Next we use the other point that we're given to find the value of the leading coefficient  $a$ . In this case we're told that the  $x$ -intercept is  $0$ , so as an ordered pair that will be the point  $(0, 0)$ . So we'll replace  $x$  with  $0$  and  $f(x)$  with  $0$ .

Finally, we replace  $a$  with  $\frac{5}{9}$  in  $f(x) = a(x - 3)^2 + 5$  and we have the standard form of the quadratic function  $f$ .

**Example 6:** Find the standard form  $f(x) = a(x - h)^2 + k$  of the quadratic function that has a  $y$ -intercept of  $-3$  and  $x$ -intercepts of  $-3$  and  $3$ , if the vertex and the  $y$ -intercept are the same point.

**Example 7:** Find the standard form  $f(x) = a(x - h)^2 + k$  of the quadratic function that has a  $y$ -intercept of  $\frac{1}{4}$  and an  $x$ -intercept of  $\frac{7}{2}$ , if the vertex is the point  $(\frac{7}{2}, 0)$ .

Since we're told the vertex is  $(\frac{7}{2}, 0)$ , we can plug  $\frac{7}{2}$  in for  $h$  and  $0$  in for  $k$ .

$$f(x) = a(x - h)^2 + k$$

$$f(x) = a\left(x - \frac{7}{2}\right)^2 + 0$$

Now we need to use any other point that the quadratic function passes through in order to determine the leading coefficient  $a$ . We're told that the quadratic function has a  $y$ -intercept of  $\frac{1}{4}$ , which is the ordered pair  $(0, \frac{1}{4})$ . We're also told that the quadratic function has an  $x$ -intercept of  $\frac{7}{2}$ , which is the ordered pair  $(\frac{7}{2}, 0)$ . However since that is also the vertex, we can't re-use that point. So I'll use the  $y$ -intercept, plugging  $0$  in for  $x$  and  $\frac{1}{4}$  in for  $f(x)$  in order to find the value of  $a$ .

$$f(x) = a\left(x - \frac{7}{2}\right)^2$$

$$\frac{1}{4} = a\left(0 - \frac{7}{2}\right)^2$$

$$\frac{1}{4} = a\left(\frac{49}{4}\right)$$

$$\frac{1}{49} = a$$

Now I can simply plug the value of  $a$  into the quadratic function:

$$f(x) = \frac{1}{49}\left(x - \frac{7}{2}\right)^2$$

*Answers to Examples:*

$$1. f(x) = \frac{1}{2}(x - 1)^2 + 2; \quad 2. f(x) = -\frac{1}{12}(x + 2)^2 + 2;$$

$$3. f(x) = 3x^2 - 2; \quad 4. f(x) = -8\left(x + \frac{1}{2}\right)^2 - \frac{1}{4};$$

$$5. f(x) = -\frac{5}{9}(x - 3)^2 + 5; \quad 6. f(x) = \frac{1}{3}x^2 - 3;$$

$$7. f(x) = \frac{1}{49}\left(x - \frac{7}{2}\right)^2;$$