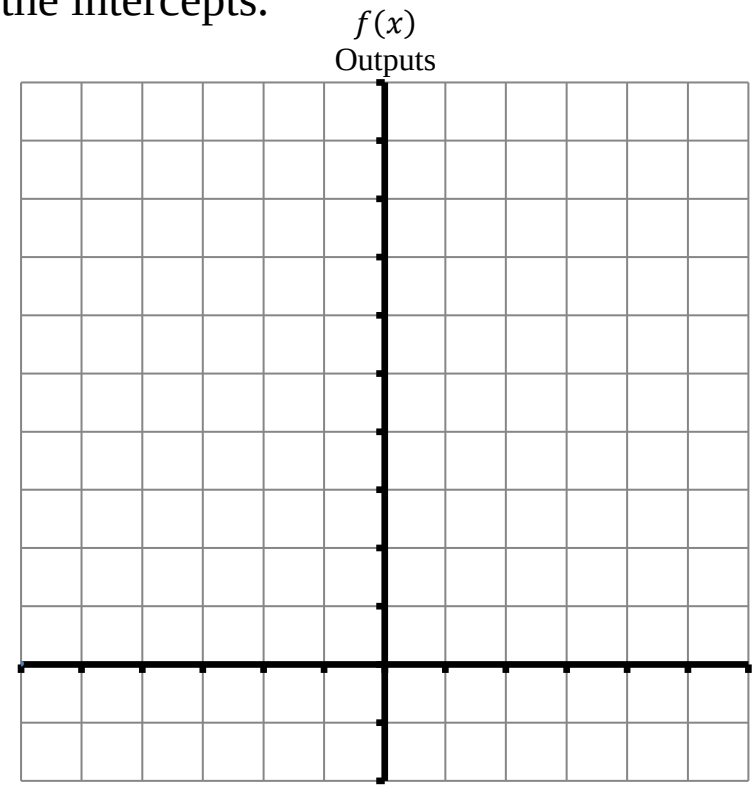
Example 1: Complete the input/output table for the function $f(x) = 2^x$, and use the ordered pairs to sketch the graph of the function. After graphing, list the domain, range, zeros, positive/negative intervals, increasing/decreasing intervals, and the intercepts.

Inputs	**Outputs**
x	$f(x) = 2^x$
$x \to -\infty$	
-5	$f(-5) = 2^{-5} = \frac{1}{32} = 0.03125$
-4	$f(-4) = 2^{-4} = \frac{1}{16} = 0.0625$
-3	$f(-3) = 2^{-3} = \frac{1}{8} = 0.125$
-2	$f(-2) = 2^{-2} = \frac{1}{4} = 0.25$
-1	$f(-1) = 2^{-1} = \frac{1}{2} = 0.5$
0	$f(0) = 2^0 = 1$
1	$f(1) = 2^1 = 2$
2	$f(2) = 2^2 = 4$
3	$f(3) = 2^3 = 8$
4	$f(4) = 2^4 = 16$
5	$f(5) = 2^5 = 32$
$x \to \infty$	



Domain: $(-\infty, \infty)$

Range: $(\mathbf{0}, \infty)$

Zeros: **NONE**
$f(x) \neq 0$

Positive intervals:
$f(x) > 0$ when x is $(-\infty, \infty)$

Negative intervals:
$f(x) < 0$ when x is **NONE**

Increasing intervals:
$\boldsymbol{f(x)}$ **is rising when** $\boldsymbol{x}$ **is** $(-\infty, \infty)$

Decreasing intervals:
$f(x)$ is falling when x is **NONE**

Intercepts:
$x-$intercept: **NONE**
$y-$intercept: $(\mathbf{0}, \mathbf{1})$

Horizontal Asymptote:

- a horizontal line ($y = \#$) that the graph of a function approaches when the inputs are large positive numbers ($x \to \infty$) or large negative numbers ($x \to -\infty$)
  - in the case of $f(x) = 2^x$, as the inputs get smaller and smaller ($x \to -\infty$), the outputs get closer and closer to zero ($f(x) \to 0$), so the graph has a horizontal asymptote at $y = 0$
  - the graph of every exponential function will have a horizontal asymptote
- in the next example (Example 2) I denoted the horizontal asymptote with a dotted line to make it easier to identify

Example 2: Re-write the function $g(x) = -2^x$ in terms of $f(x) = 2^x$. Then find the y-intercept of g and find its graph by transforming the graph of the original function f. Enter exact answers only (no approximations) for the y-intercept.

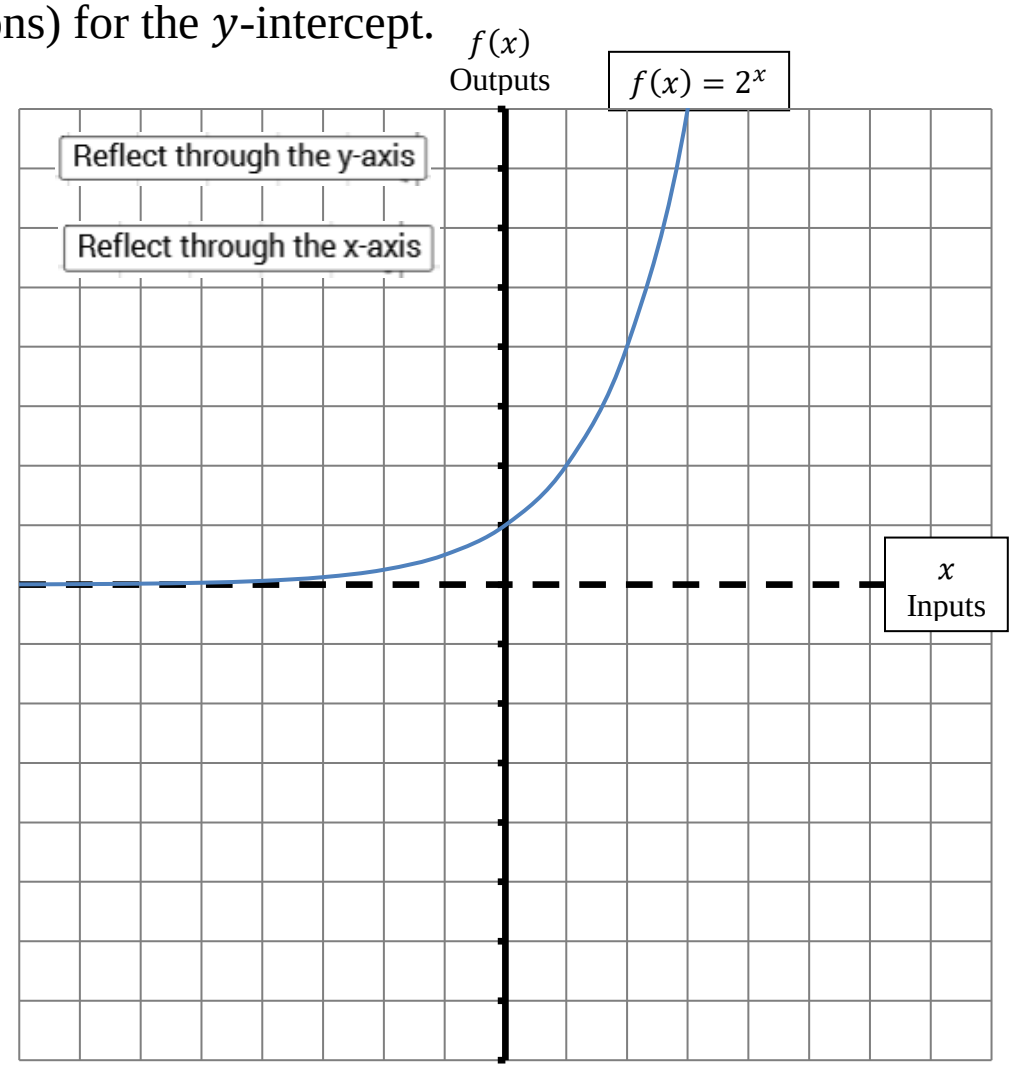
Re – write $g(x)$ in terms of $f(x)$:

$\boldsymbol{g(x) =}$

$\boldsymbol{y - \text{intercept}}$:

$g(0) =$

$(\mathbf{0}, \quad)$

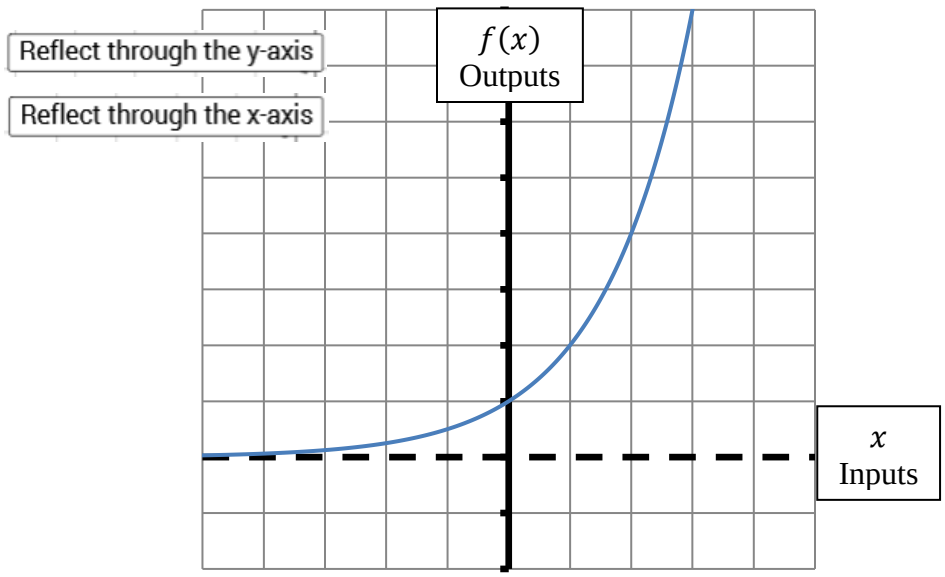


<u>Example 3:</u> Re-write each of the following functions in terms of $f(x) = 2^x$, then transformation the graph of f. Also, find the y-intercept for each function and enter exact answers only, no approximations.

a. $h(x) = 2^{-x}$

$\boldsymbol{h(x) =}$

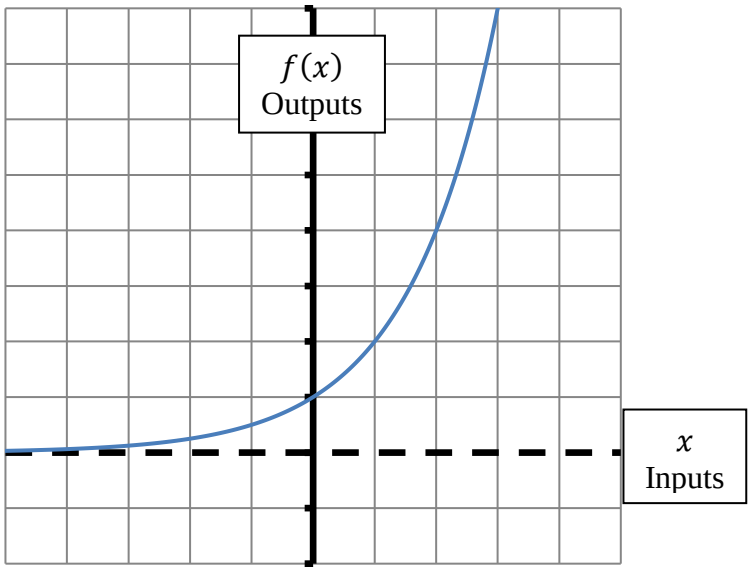
$\boldsymbol{y - \text{intercept}}$:



b. $j(x) = 2^x - 1$

$\boldsymbol{j(x) =}$

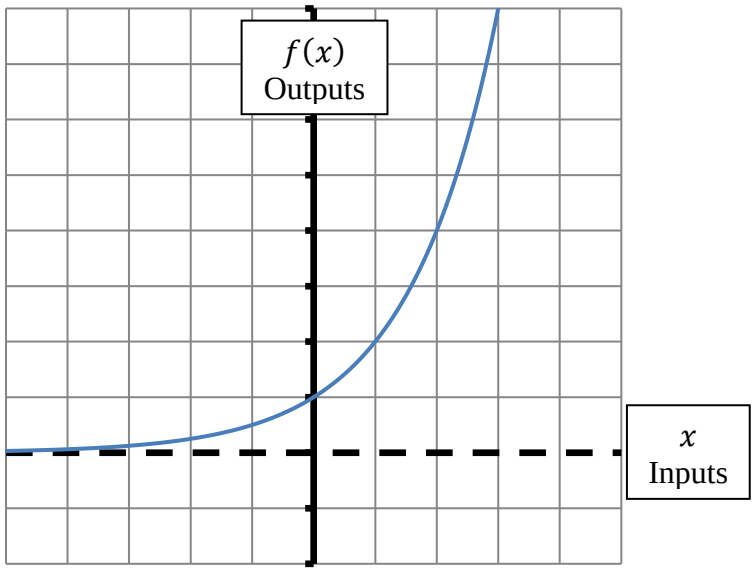
$\boldsymbol{y - \text{intercept}}$:



c. $k(x) = 3(2^x)$

$\boldsymbol{k(x) =}$

$\boldsymbol{y - \text{intercept}}$:



LON-CAPA Problem :

Given the function $f(x) = 2^x$, along with it's graph below, complete the following:

a. Express the new function $g(x) =$ in terms of the original function $f(x) = 2^x$.

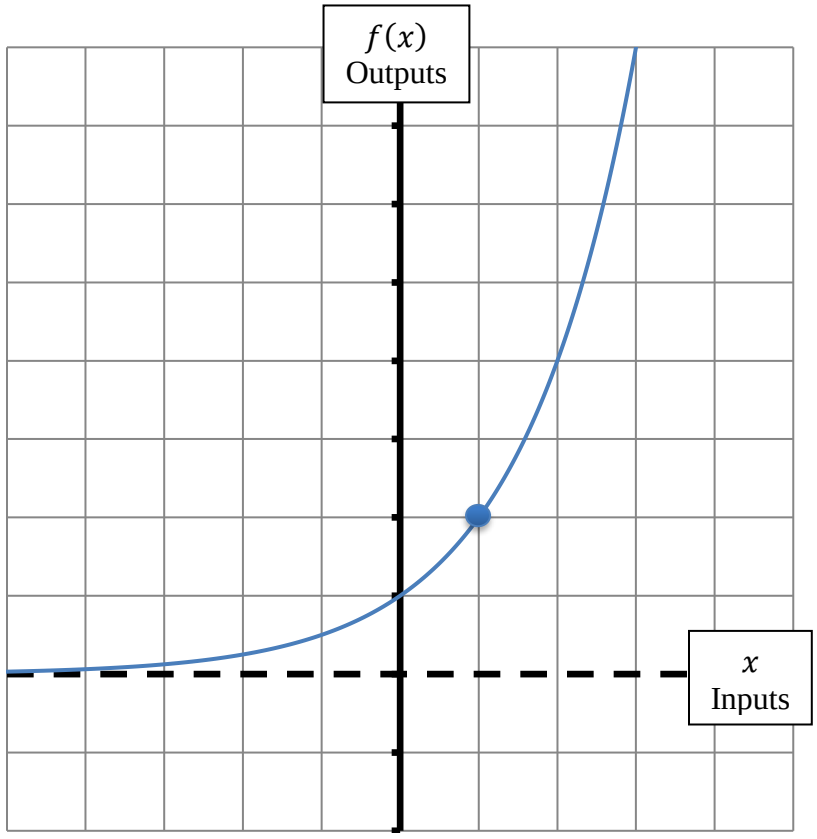
$\boldsymbol{g(x) =}$

b. Find the y-intercept of the function $g(x) =$ and enter your answer as an ordered pair (x, y). Enter exact answers only, no approximations.

$\boldsymbol{y - \text{intercept } (x, y) =}$

c. Transform the graph of $f(x) = 2^x$ to get the graph of $g(x) =$. Use the y-intercept of g to verify that your transformation is correct.

Keep in mind that even though the point you're given on some graphs in LON-CAPA maybe the y-intercept, after transforming the point it may no longer be an intercept. Use the answer from part a. to determine how to transform the point that you are given on the graph. After transforming the graph, use your answer from part b. to verify that the y-intercept on the new graph is correct.

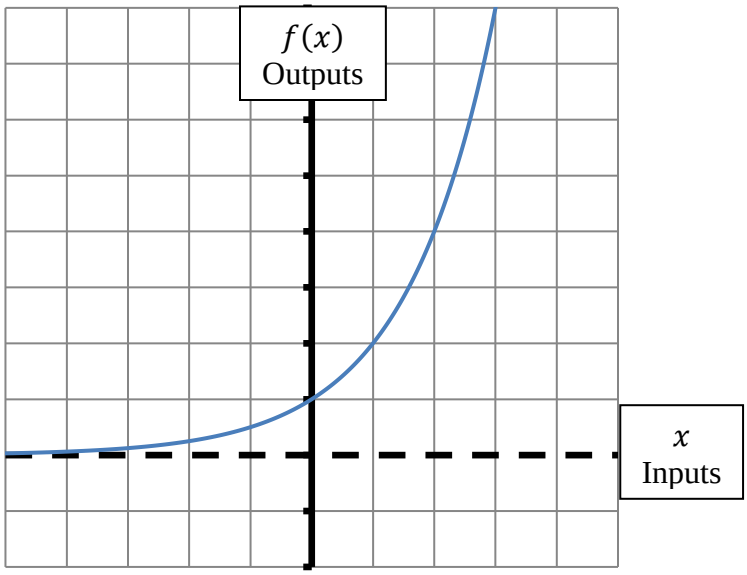


<u>Example 4:</u> Re-write each of the following functions in terms of $f(x) = 2^x$, then transformation the graph of f. Also, find the y-intercept for each function and enter exact answers only, no approximations.

a. $m(x) = 2^{x-3}$

$\boldsymbol{m(x) =}$

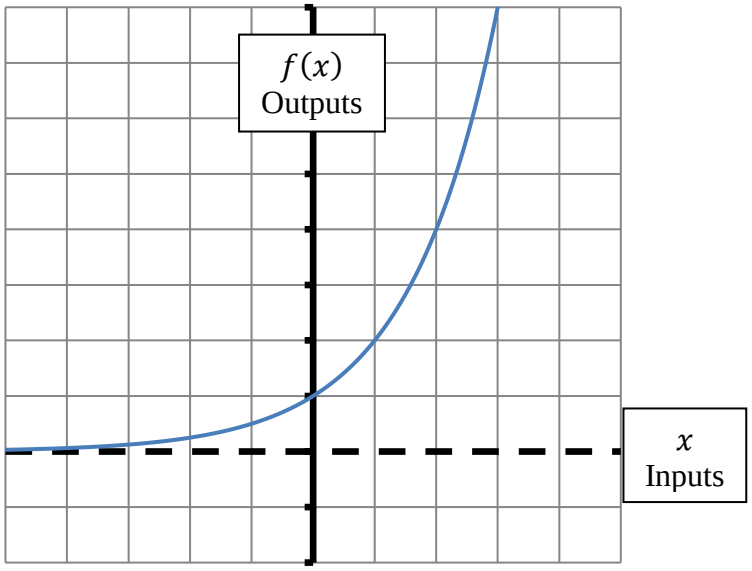
$\boldsymbol{y-}$ **intercept**:



b. $n(x) = -2^x + 1$

$\boldsymbol{n(x) =}$

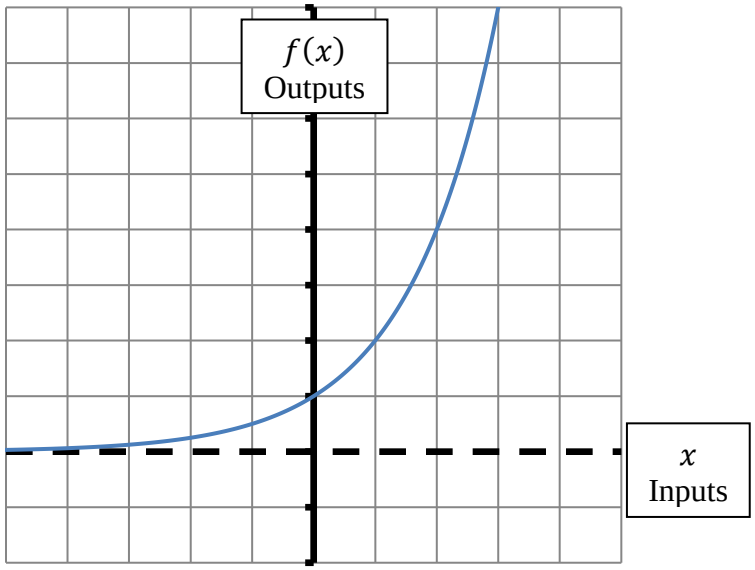
$\boldsymbol{y-}$ **intercept**:



c. $p(x) = 2^{\frac{x}{2}}$

$\boldsymbol{p(x) =}$

$\boldsymbol{y-}$ **intercept**:



Answers to Examples:

2. $g(x) = -f(x), y-\text{intercept: } (0,-1)$;

3a. $h(x) = f(-x), y-\text{intercept: } (0,1)$;

3b. $j(x) = f(x)-1, y-\text{intercept: } (0,-2)$;

3c. $k(x) = 3f(x), y-\text{intercept: } (0,3)$;

4a. $m(x) = f(x-3), y-\text{intercept: } \left(0,\frac{1}{8}\right)$;

4b. $n(x) = -f(x)+1, y-\text{intercept: } (0,0)$;

4c. $k(x) = f\left(\frac{x}{2}\right), y-\text{intercept: } (0,1)$;