Radical Notation:

$$\sqrt[n]{y}$$

- the expression above is read "the nth root of y", where y is the radicand, $\sqrt{}$ is the radical sign, and n is the index or root
- if no index is denoted, it is understood to be an index of 2 (a square root)
  - $\sqrt{y} = \sqrt[2]{y}$
- the definition of $\sqrt[n]{y}$ is the value that must be taken to the power of n to produce y
  - the expression $\sqrt{25}$ represents the value that is taken to the power of 2 to produce 25
    - since 5 taken to the power of 2 is 25, the square root of 25 is 5
    - $\sqrt{25} = 5$ because $5^2 = 25$

  - the expression $\sqrt[3]{-64}$ represents the value that is taken to the power of 3 to produce -64
    - since -4 taken to the power of 3 is -64, the cubed root of -64 is -4
    - $\sqrt[3]{-64} = -4$ because $(-4)^3 = -64$

Example 1: Evaluate each expression.

a. $\sqrt{16}$

Why?

b. $\sqrt[4]{16}$

Why?

c. $\sqrt{64}$

Why?

d. $\sqrt[3]{64}$

4

Why?

Because $4^3 = 64$

e. $\sqrt[3]{27}$

3

Why?

Because $3^3 = 27$

f. $\sqrt{81}$

9

Why?

Because $9^2 = 81$

Even Roots:

- a radical with an index of $2, 4, 6, \ldots$
- the radicand of a radical with an even root must be positive or zero (no negative values)
  - the even root of a positive number is a positive number
    - $\sqrt[4]{16} = 2$ because $2^4 = 16$
  - the even root of zero is zero
    - $\sqrt{0} = 0$ because $0^2 = 0$
  - the even root of a negative number does not exist with real numbers because no real number (negative, positive, or zero) can be taken to an even power and produce a negative value
    - $(-4)^2 = (-4) \cdot (-4)$
      $= 16$
    - $(-3)^4 = (-3) \cdot (-3) \cdot (-3) \cdot (-3)$
      $= 81$
    - $(-2)^6 = (-2) \cdot (-2) \cdot (-2) \cdot (-2) \cdot (-2) \cdot (-2)$
      $= 64$
    - a negative base taken to an **<u>even</u>** exponent will **<u>ALWAYS</u>** result in a positive value, so that is why the **<u>even</u>** root of a negative number does not exist with real numbers

Odd Roots:

- a radical with an index of $3, 5, 7, \ldots$
- the radicand of a radical with an odd root can be any real number (negative, positive, or zero)
  - the odd root of a positive number is a positive number
    - $\sqrt[5]{32} = 2$ because $2^5 = 32$
  - the odd root of zero is zero
    - $\sqrt[3]{0} = 0$ because $0^3 = 0$
  - the odd root of a negative number is a negative number
    - $\sqrt[3]{-64} = -4$ because $(-4)^3 = -64$
  - a negative base taken to an **<u>odd</u>** exponent will **<u>ALWAYS</u>** result in a negative value, so that is why the **<u>odd</u>** root of a negative number does exist with real numbers, and is negative

Example 2: Evaluate each expression; if a solution does not exist in real numbers, write DNE.

a. $\sqrt{36}$

Why?

b. $\sqrt{-81}$

Why?

c. $-\sqrt{25}$

Why?

d. $\sqrt[3]{8}$

Why?

e. $\sqrt[3]{-8}$

Why?

f. $-\sqrt[3]{125}$

Why?

g. $\sqrt[4]{81}$

$\mathbf{3}$

Why?

Because $3^4 = 81$

h. $\sqrt[6]{-\pi}$

DNE

Why?

Because no real number to the power of 6 will be negative

i. $-\sqrt[8]{1}$

$\mathbf{-1}$

Why?

Because $1^8 = 1$ and then it's negated

j. $\sqrt[5]{0}$

$\mathbf{0}$

Why?

Because $0^5 = 0$

k. $\sqrt[7]{-1}$

$\mathbf{-1}$

Why?

Because $(-1)^7 = -1$

l. $-\sqrt[9]{1}$

$\mathbf{-1}$

Why?

Because $1^9 = 1$ and then it's negated

Answers to Examples:

1a. 4 *; 1b.* 2 *; 1c.* 8 *; 1d.* 4 *; 1e.* 3 *; 1f.* 9 *; 2a.* 6 *;*
2b. DNE *; 2c.* -5 *; 2d.* 2 *; 2e.* -2 *;*
2f. -5 *; 2g.* 3 *; 2h.* DNE *; 2i.* -1 *; 2j.* 0 *; 2k.* -1 *; 2l.* -1 *;*