In this set of notes we'll be covering finding exponential functions algebraically by using ordered pairs that the graph of the function passes through. We'll use a similar process to how we found quadratic functions from ordered pairs in Lesson 24. In Lesson 24 we were always given the vertex of the quadratic function, and we'd simply plug that in for h and k. Then we'd use the other ordered pair we were given to find the value of a. With exponential functions, $f(x) = b \cdot a^x$, we will always be given the y-intercept of the function and we'll simply plug that in for b. Then we'll use the other ordered pair we're given to find the value of the base a.

<u>Example 1:</u> Find an exponential function of the form $f(x) = b \cdot a^x$, given that the graph of the function passes through the points $P(0,3)$ and $Q(1,2)$. Simplify your answer completely and enter exact answers only

Always start by plugging in the y-intercept in order to find the value of b.

$$f(x) = b \cdot a^x$$
$$f(0) = b \cdot a^0$$
$$3 = b \cdot 1$$

The ordered pair $(0,3)$ says that when 0 is the input of the function f, 3 is the output. That means that $f(0) = 3$. And since those two values are equivalent, they are also interchangeable.

Now that we know that value of b is 3, we can replace b with 3 in our exponential function.

$$f(x) = 3 \cdot a^x$$

Now use the other ordered pair $(1,2)$ to find the value of a.

$$f(1) = 3 \cdot a^1$$
$$2 = 3 \cdot a$$
$$\frac{2}{3} = a$$

The ordered pair $(1,2)$ says that when 1 is the input of the function f, 2 is the output. That means that $f(1) = 2$. And since those two values are equivalent, they are also interchangeable.

Now that we know that value of a is $\frac{2}{3}$, we can replace a with $\frac{2}{3}$ in our exponential function.

$$\boldsymbol{f(x) = 3 \cdot \left(\frac{2}{3}\right)^x}$$

<u>Always start by plugging in the y-intercept to find the value of b, then use the other ordered pair to find the value of a.</u>

<u>Example 2:</u> Find an exponential function of the form $f(x) = b \cdot a^x$, given that the graph of the function passes through the points $P(0, 2)$ and $Q(-2, 50)$. Simplify your answer completely and enter exact answers only (no approximations).

Remember from the notes on Exponential Functions in Lesson 29 that for any exponential function $f(x) = a^x$, the base a must be a positive number other then one $(a > 0, a \neq 1)$. So when you encounter an a value such as $\pm\frac{1}{5}$, be sure to disregard the negative value.

<u>Example 3:</u> Find an exponential function of the form $f(x) = b \cdot a^x$, given that the y-intercept of the function is 1, and the graph of the function passes through the point$Q\left(-1, \frac{4}{5}\right)$. Simplify your answer completely and enter exact answers only (no approximations).

<u>Example 4:</u> Find an exponential function of the form $f(x) = b \cdot a^x$, given that the y-intercept of the function is $-\frac{1}{3}$, and the graph of the function passes through the point $Q\left(3, -\frac{343}{3}\right)$. Simplify your answer completely and enter exact answers only (no approximations).

$f(x) = b \cdot a^x$

$f(0) = b \cdot a^0$

$-\frac{1}{3} = b \cdot 1$

$b = -\frac{1}{3}$

$f(x) = -\frac{1}{3} \cdot a^x$

$f(x) = -\frac{1}{3} \cdot a^x$

$f(3) = -\frac{1}{3} a^3$

$-\frac{343}{3} = -\frac{1}{3} a^3$

$343 = a^3$

$7 = a$

$\boldsymbol{f(x) = -\frac{1}{3} \cdot 7^x}$

<u>Example 5:</u> Find an exponential function of the form $f(x) = b \cdot a^x$, given that the graph of the function passes through the points $P\left(0, \frac{3}{2}\right)$ and $Q\left(-1, \frac{3}{e}\right)$. Simplify your answer completely and enter exact answers only (no approximations).

<u>Example 6:</u> Find an exponential function of the form $f(x) = b \cdot a^x$, given that the graph of the function passes through the points $P(0, -1)$ and $Q(2, -e^2)$. Simplify your answer completely and enter exact answers only (no approximations).

<u>Example 7:</u> Find an exponential function of the form $f(x) = b \cdot a^x$, given that the y-intercept of the function is -2, and the graph of the function passes through the point $(-2, -2e^2)$. Simplify your answer completely and enter exact answers only (no approximations).

Always start by plugging in the y-intercept in order to find the value of b. In this case we need to express the y-intecept of -2 as the ordered pair $(0, -2)$.

$$f(x) = b \cdot a^x$$
$$f(0) = b \cdot a^0$$
$$-2 = b \cdot 1$$

Now that we know the value of b, we can replace b with -2.

$$f(x) = -2 \cdot a^x$$

Now use the other ordered pair we were given to find the value of a.

$$f(-2) = -2 \cdot a^{-2}$$
$$-2e^2 = -2 \cdot a^{-2}$$
$$e^2 = \frac{1}{a^2}$$
$$e^2 \cdot a^2 = 1$$
$$a^2 = \frac{1}{e^2}$$
$$a = \pm\sqrt{\frac{1}{e^2}}$$
$$a = \pm\frac{1}{e}$$

Since the base of an exponential function must be a positive number other than 1, we drop the negative root and keep only the positive root

$$\boldsymbol{f(x) = -2 \cdot \left(\frac{1}{e}\right)^x}$$

Answers to Examples:

1. $f(x) = 3\left(\frac{2}{3}\right)^x$ *; 2.* $f(x) = 2\left(\frac{1}{5}\right)^x$ *; 3.* $f(x) = \left(\frac{5}{4}\right)^x$ *;*

4. $f(x) = -\frac{1}{3}(7)^x$ *; 5.* $f(x) = \frac{3}{2}\left(\frac{e}{2}\right)^x$ *; 6.* $f(x) = -e^x$*;*

7. $f(x) = -2\left(\frac{1}{e}\right)^x$*;*