As shown in Lesson 29, one application of exponential functions is compound interest, which is when interest is calculated on the total value of a sum and not just on the principal like with simple interest. We saw in Lesson 29 that one way interest can be compounded is n times per year, where n represents some number of compounding periods (quarterly, monthly, weekly, daily, etc.). The other way interest can be compounded is continuously, where interest is compounded essentially every second of every day for the entire term. This means n is essentially infinite, and so we will use a different formula which contains the natural number e to calculate the value of an investment. The formula for interest compounded continuously is $A = Pe^{rt}$.

<u>Formula for Interest Compounded Continuously:</u>

- when interest is compounded continuously, we use the formula $A = Pe^{rt}$
  - when interest is compounded continuously, there are essentially an infinite number of compounding periods ($n \to \infty$), so that is why we use the natural number e
  - A is the accumulated value of the investment
  - P is the principal (the original amount invested)
  - r is the annual interest rate
  - t is the number of **<u>years</u>** the principal is invested (the term)

<u>Example 1:</u> If \$17,000 is invested at a rate of 6.25% per year for 39 years, find value of the investment to the nearest penny if the interest is compounded continuously. Use either $A = P\left(1 + \frac{r}{n}\right)^{nt}$ or $A = Pe^{rt}$.

$$A = Pe^{rt}$$

$$A = 17000e^{(0.0625)(39)}$$

$$A = 17000e^{2.4375}$$

$$A = 17000(11.44439396\ldots)$$

$$\boldsymbol{A = \$194,554.70}$$

When working with compound interest formulas, remember to keep in mind order of operation (PEMA):

1. simplify parentheses
2. simplify exponents
3. simplify multiplication/division, working from left to right
4. simplify addition/subtraction, working from left to right

Example 2: If \$20,000 is invested at a rate of 6.5% per year compounded continuously, find value of the investment at each given time and round to the nearest cent. Use either $A = P\left(1+\frac{r}{n}\right)^{nt}$ or $A = Pe^{rt}$.

a. 8 months b. 18 months c. 21 years d. 100 years

For each of these problems you will use the formula $A = Pe^{rt}$ since interest is compounded continuously. The principal will be 20000 for each problem part ($P = 20000$) and the interest rate will be 6.5% ($r = 0.065$). However the term will vary from part to part:

$t = \frac{8}{12} = \frac{2}{3}$ $t = \frac{18}{12} = 1.5$ $t = 21$ $t = 100$

$A = 20000e^{0.065\cdot\frac{2}{3}}$

Once again do your best to leave all calculated values in your calculator. For instance when calculating $A = 20000e^{0.065\cdot\frac{2}{3}}$ from Example 2 part a, do not calculate $e^{0.065\cdot\frac{2}{3}}$ and then try to write that down on paper to 5 or 6 decimal places. Leave calculated values in your calcul ator to avoid approximating.

$A = 20000e^{0.065\cdot\frac{2}{3}}$

$\boldsymbol{A = \$20,885.72}$

Once again for help with entering expressions such as $20000e^{0.065\cdot\frac{2}{3}}$ in your calculator, take a look at the Calculator Tips document in Brightspace or stop by my office hours. Also, be sure to use the same calculator on homework (handheld or computer calculator) as you will on the exam.

Example 3: A recent college graduate decides to open a credit card in order to pay for their upcoming trip across Europe. In order to get a card with a large enough credit limit to pay for their trip ($5,500), the student agrees to an interest rate of 38.99% compounded continuously. If no payments are made for an entire year, what will be the balance on the card rounded to the nearest penny? Use either $A = P\left(1 + \frac{r}{n}\right)^{nt}$ or $A = Pe^{rt}$.

Example 4: Parents of a newborn baby are given a gift of $20,000 and will choose between two options to invest for their child's college fund. Option 1 is to invest the gift in a fund that pays an average annual interest rate of 8% compounded semiannually; option 2 is to invest the gift in a fund that pays an average annual interest rate of 7.75% compounded continuously. Assuming each investment has a term of 18 years, calculate the value of each investment and round your answer to the nearest penny. Use either $A = P\left(1 + \frac{r}{n}\right)^{nt}$ or $A = Pe^{rt}$.

Option 1 =

Option 2 =

If the rates are the same, which is the better option for the parents?

Answers to Examples:

1. $194,554.70 *; 2a.* $20,999.16 *; 2b.* $22,048.23 *; 2c.* $78,314.46 *; 2d.* $13,302,832.66 *; 3.* $8,122.58 *;*

4. Option 1 = $82,078.65 ; Option 2 = $80,699.49 ;

if the rates are equal, Option 2 is the better option *;*