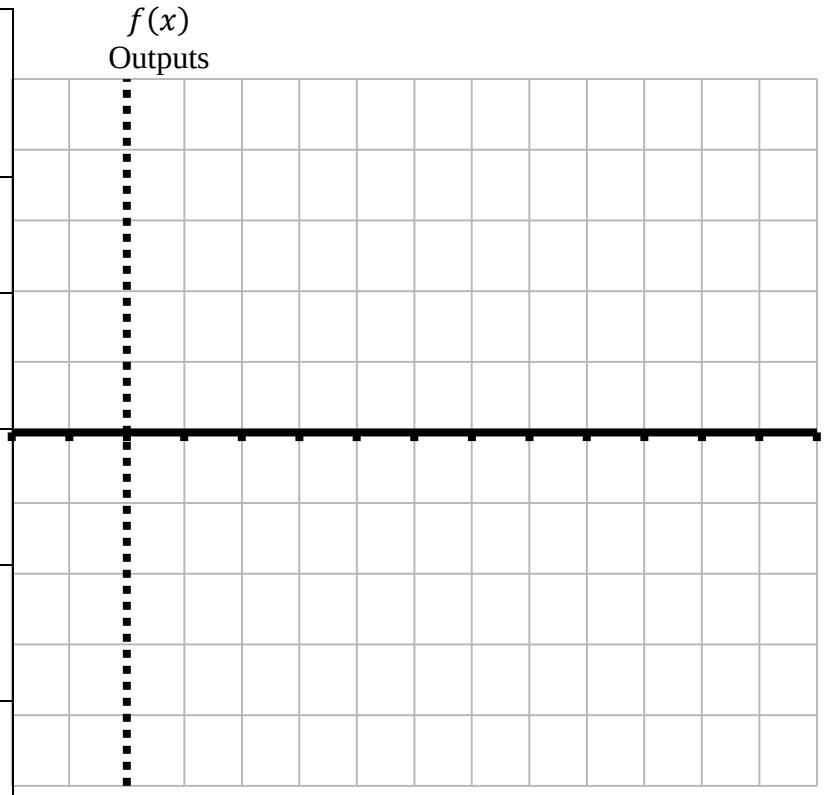
Example 1: Complete the input/output table for the function $f(x) = \log(x)$, and use the ordered pairs to sketch the graph of the function. After graphing, list the domain, range, zeros, positive/negative intervals, increasing/decreasing intervals, and the intercepts.

Inputs (these are powers of the base 10)	**Outputs** (these are exponents that produce the powers of the base 10)
x	$f(x) = \log(x)$
$x \to 0$	
$\frac{1}{1000}$	
$\frac{1}{100}$	
$\frac{1}{10}$	
1	
10	
100	
1000	
$x \to \infty$	



Domain: $(\mathbf{0}, \infty)$

Range: $(-\infty, \infty)$

Zeros:
$f(x) = 0$ when $x =$ **1**

Positive intervals:
$f(x) > 0$ when x is $(\mathbf{1}, \infty)$

Negative intervals:
$f(x) < 0$ when x is $(\mathbf{0}, \mathbf{1})$

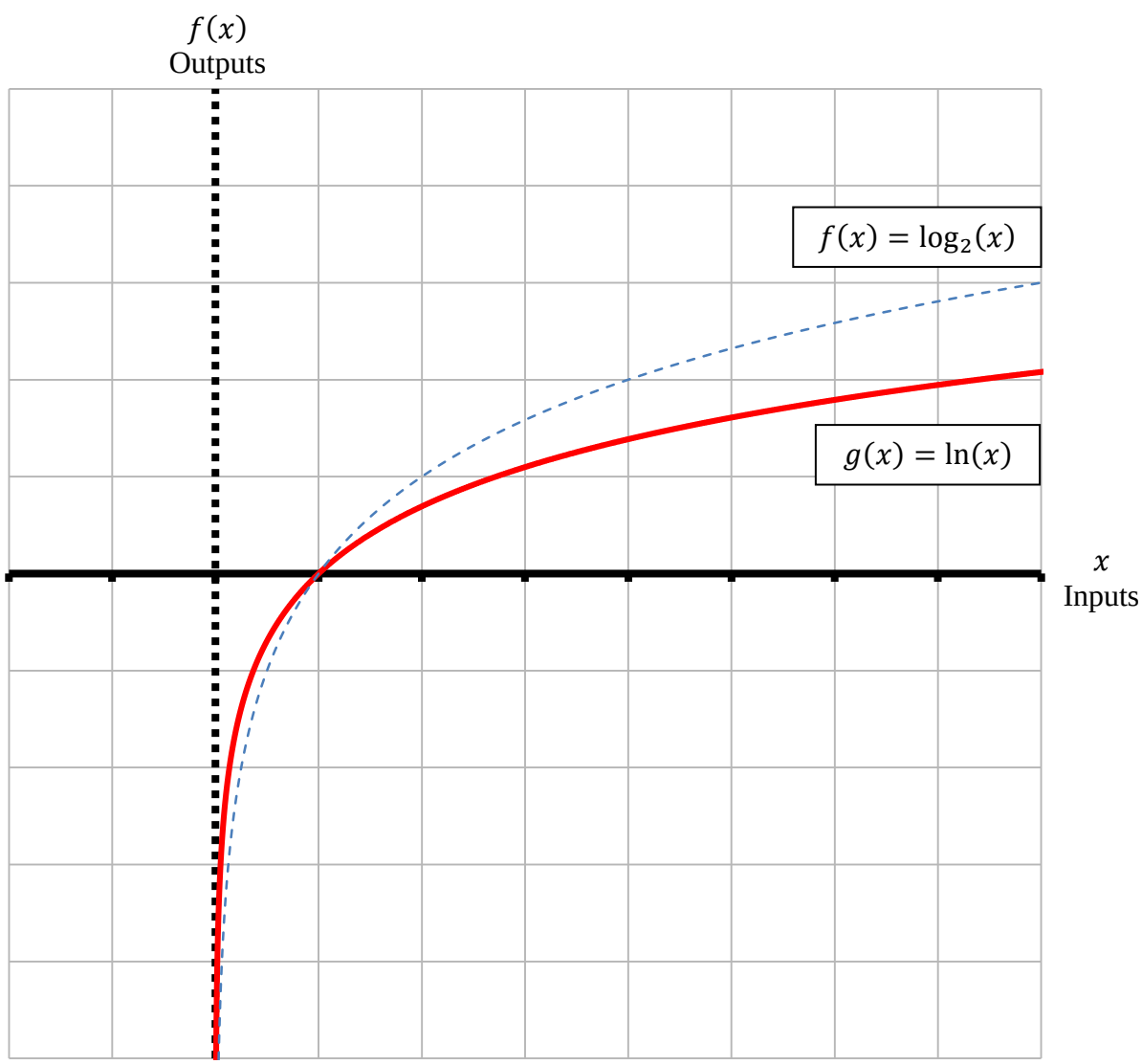
Increasing intervals:
$f(x)$ is rising when x is $(\mathbf{0}, \infty)$

Decreasing intervals:
$f(x)$ is falling when x is **NONE**

Intercepts:
$x-$intercept: $(\mathbf{1}, \mathbf{0})$
$y-$intercept: **NONE**

All of this information (domain, range, zeros, etc.) is the same for any logarithmic function $f(x) = \log_a(x)$ that has **NOT** been transformed. As shown before, transformations will change some or all of this information based on what type of transformation is performed.

Since the domain, range, zeros, positive/negative intervals, increasing/decreasing intervals, intercepts, and asymptote will be the same for all logarithmic functions of the form $f(x) = \log_a(x)$, we do not need to repeat this exercise for the natural logarithmic function $g(x) = \ln(x)$. Remember that the natural logarithmic function $\ln(x)$ is $\log_e(x)$, and e is the natural number 2.71828 ....; therefore it will have the same domain, range, zeros, etc. as the $f(x) = \log(x)$. I will however include its graph, along with the graph of $f(x) = \log_2(x)$ for comparison.

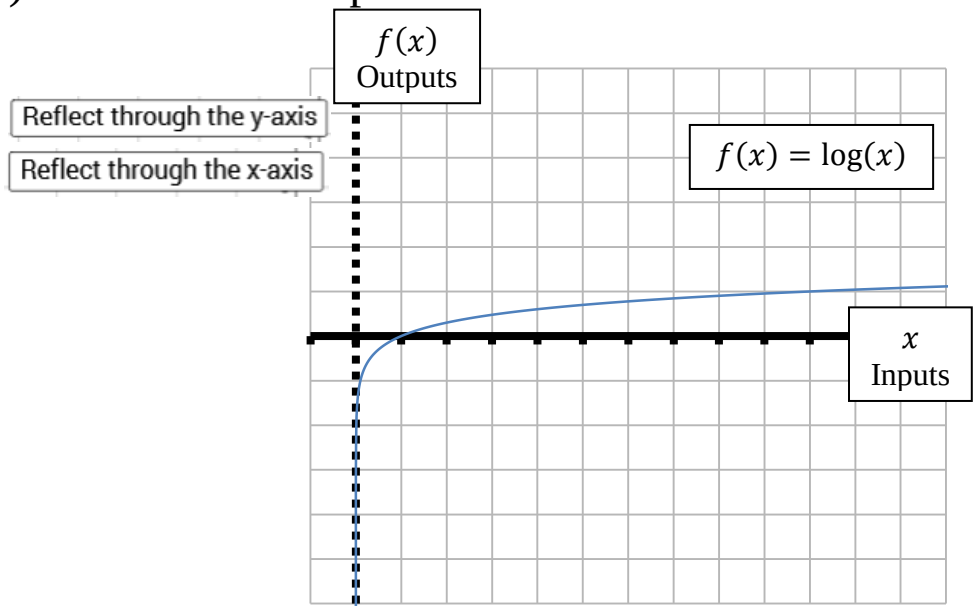


<u>Example 2:</u> Re-write the function $j(x) = -\log(x)$ in terms of $f(x) = \log(x)$. Then find the x-intercept of j and find its graph by transforming the graph of the original function f. Enter exact answers only (no approximations) for the x-intercept.

$\boldsymbol{j(x) =}$

$\boldsymbol{x - \text{intercept}}$:

$(\quad ,0)$



When solving problems like this in LON-CAPA, you can identify the transformation and find the x-intercept first, and then use both in order to transform the graph of the original function. Or you can simply transform the original graph first, and then identify the x-intercept from the graph, like I did above. If you want to identify the x-intercept first, before getting the graph of the new function, you'll have to find the x-intercept algebraically:

$$\underline{\boldsymbol{x - \text{intercept of } j(x) = -\log(x)}}:$$

$0 = -\log_{10}(x)$

$0 = \log_{10}(x)$ ← I multiplied both sides of the equation by -1 to change the sign of the logarithm.

$10^0 = x$ ← Then I converted from log form to exponential form in order to isolate x.

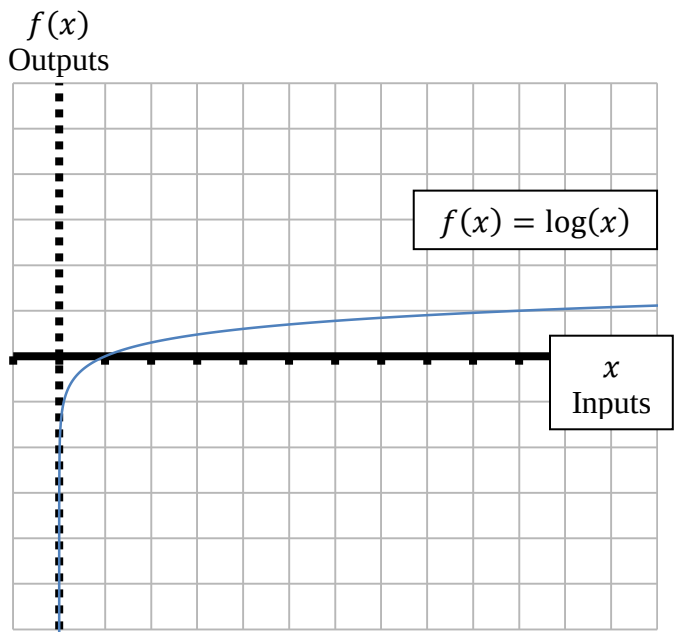
$1 = x$

$(\mathbf{1}, \mathbf{0})$

Finding the x-intercept algebraically was optional on this problem because were able to find it graphically as well. That will not always be the case, as we'll see on the next example.

<u>Example 3:</u> Re-write the function $k(x) = \log(x) + 2$ in terms of $f(x) = \log(x)$. Then find the x-intercept of k and find its graph by transforming the graph of the original function f. Enter exact answers only (no approximations) for the x-intercept.

$\boldsymbol{k(x) = f(x) + 2}$



To transform the graph of the function f, we need to shift it up 2 units. Doing so will mean the old x-intercept of $(1, 0)$ will become $(1, 2)$. The new x-intercept of the function k might not be identifiable from its graph, so it may be necessary to find the x-intercept algebraically.

<u>$x-$intercept of $k(x) = \log(x) + 2$:</u>

$0 = \log_{10}(x) + 2$

$-2 = \log_{10}(x)$ I subtracted 2 from both sides of the equation to isolate the logarithm.

$10^{-2} = x$ Then I converted from log form to exponential form in order to isolate x.

$\frac{1}{100} = x$

$\boldsymbol{(0.01, 0)}$

This is an example of a problem where finding the x-intercept graphically may not be an option (at least on paper). Be aware that something like this may come up in LON-CAPA on the homework and/or on an exam.

<u>Example 4:</u> Re-write each of the following functions in terms of $f(x) = \log(x)$. Then find the x-intercept of each new function and find its graph by transforming the graph of the original function f. Enter exact answers only (no approximations) for the x-intercept.

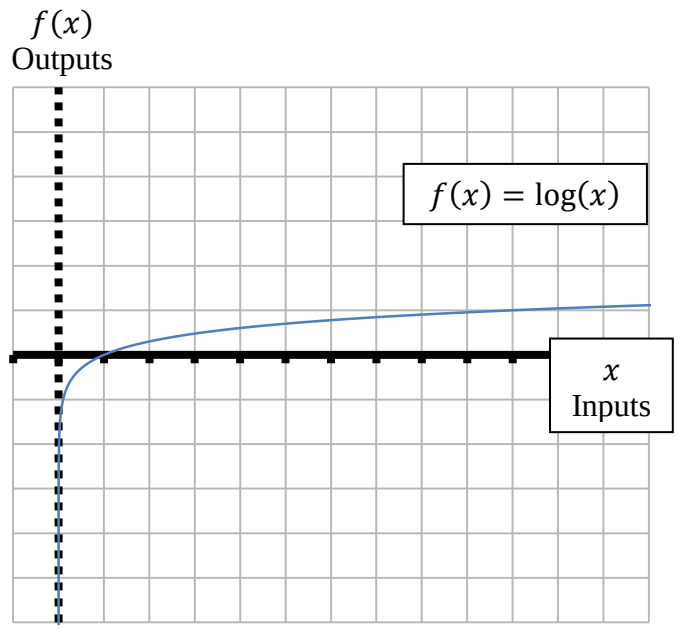
a. $m(x) = \log(2x)$

$\boldsymbol{m(x) =}$

$\boldsymbol{x - \text{intercept}}$:

$0 = \log_{10}(2x) \quad \rightarrow$

$(\quad ,0)$



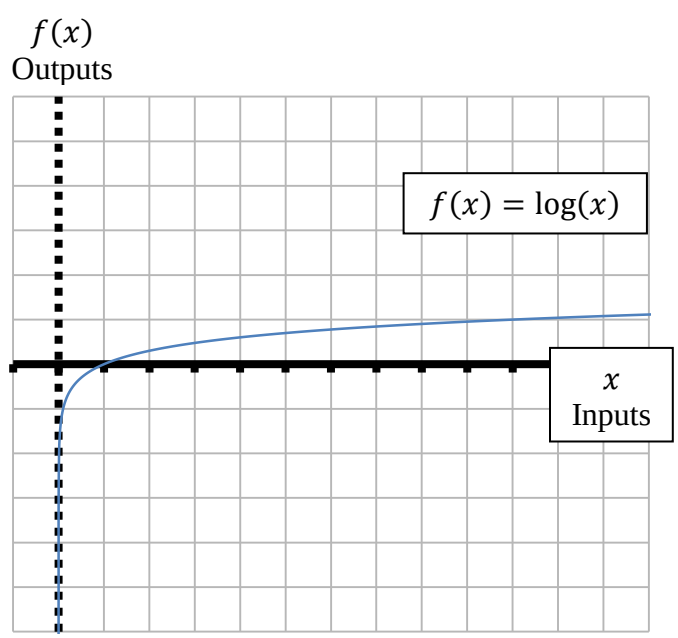
b. $r(x) = \log(x - 4)$

$\boldsymbol{r(x) =}$

$\boldsymbol{x - \text{intercept}}$:

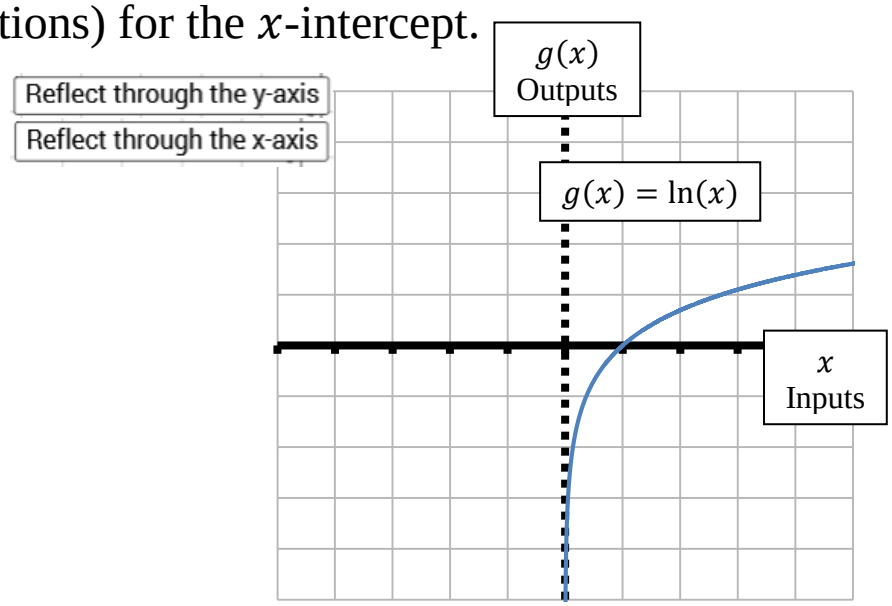
$0 = \log_{10}(x - 4) \quad \rightarrow$

$(\quad ,0)$



<u>Example 5:</u> Re-write each of the following functions in terms of $g(x) = \ln(x)$. Then find the x-intercept of each new function and find its graph by transforming the graph of the original function g. Enter exact answers only (no approximations) for the x-intercept.

a. $n(x) = \ln(-x)$



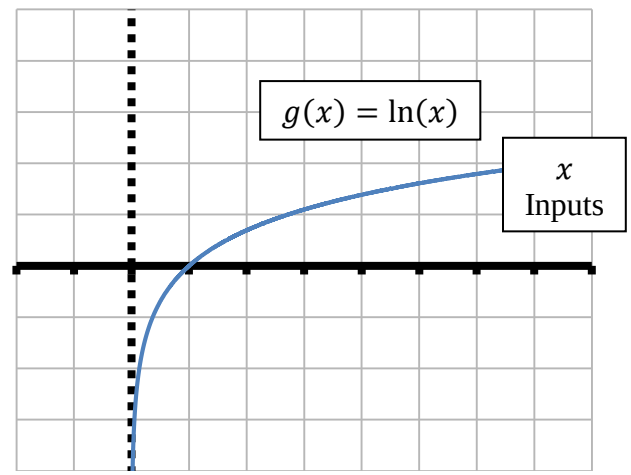
$\boldsymbol{n(x) =}$

$\boldsymbol{x - \text{intercept}}$:

$0 = \log_e(-x) \quad \rightarrow$

$(\quad ,0)$

b. $p(x) = \ln(x - 2)$



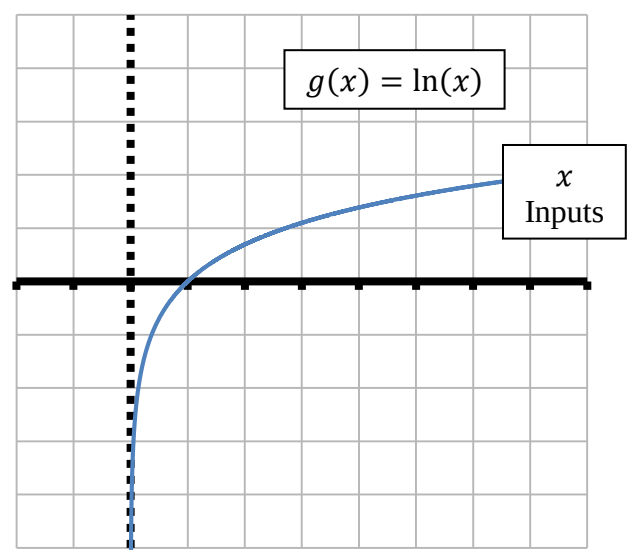
$\boldsymbol{p(x) =}$

$\boldsymbol{x - \text{intercept}}$:

$0 = \log_e(x - 2) \quad \rightarrow$

$(\quad ,0)$

c. $q(x) = 2\ln(x)$



$\boldsymbol{q(x) =}$

$\boldsymbol{x - \text{intercept}}$:

$0 = 2\log_e(x) \quad \rightarrow$

$(\quad ,0)$

Once again, when solving problems like these on the homework assignment, you can identify the transformation and find the x-intercept first, and then use both in order to transform the graph of the original function. Or you can simply transform the original graph first, and then identify the x-intercept from the graph. However keep in mind there could be instances where the x-intercept cannot be found graphically, so you should be prepared to find the x-intercept algebraically in those cases.

Answers to Examples:

2. $j(x) = -f(x), x - \text{intercept: } (1, 0)$;

3. $k(x) = f(x) + 2, x - \text{intercept: } \left(\frac{1}{100}, 0\right)$;

4a. $m(x) = f(2x), x - \text{intercept: } \left(\frac{1}{2}, 0\right)$;

4b. $r(x) = f(x - 4), x - \text{intercept: } (5, 0)$;

5a. $n(x) = g(-x), x - \text{intercept: } (-1, 0)$;

5b. $p(x) = g(x - 2), x - \text{intercept: } (3, 0)$;

5c. $q(x) = 2g(x), x - \text{intercept: } (1, 0)$;