Once again the idea of using inverses to solve equations continues when solving exponential equations. The inverse of an exponential function is a logarithmic function, so we will convert exponential equations to logarithmic form to solve them.

<u>Example 1:</u> Solve each of the following equations by converting to exponential form, and simplify your answers completely.

a. $2^x = 5$ b. $10^{2x} = 2$ c. $e^{x-1} = 15$

To solve each exponential equation, simply convert to logarithmic form. Remember that a logarithm is an exponent, so set each exponent equal to the logarithm you set-up.

$2^x = 5$	$10^{2x} = 2$	$e^{x-1} = 15$
converts to	converts to	converts to
$x = \log_2(5)$	$2x = \log_{10}(2)$	$x - 1 = \log_e(15)$
$\boldsymbol{x = \log_2(5)}$	$\boldsymbol{x = \frac{\log(2)}{2}}$	$\boldsymbol{x = \ln(15) + 1}$

Keep in mind that in the expression $\frac{\log(2)}{2}$ we can **<u>NOT</u>** cancel the 2 in parentheses with the 2 in the denominator. The 2 in parentheses is part of a function (the argument of a logarithm), so adding, subtracting, multiplying, or dividing the logarithm will not affect the argument unless we use one of the Properties of Logarithms, such as the Power Rule:

$$\frac{\log(2)}{2}$$

$$\frac{1}{2}\log(2)$$

$$\log\left(2^{\frac{1}{2}}\right)$$

$$\log(\sqrt{2})$$

All of these expressions are correct ways to express the answer of $10^{2x} = 2$. Even though they may look different, they are all equivalent.

Once again, keep in mind that exponential functions and logarithmic functions are inverses, which means each one undoes the other. Converting exponential equations to logarithmic equations gives us the primary way that we will use to solve exponential equations. There are other ways to solve exponential equations, such as using common logarithms and natural logarithms, but I will stick with simply converting to logarithmic form.

Also, keep in mind that when converting from exponential form to logarithmic form, **THE BASE DOES NOT CHANGE**. Base a in one form is base a in the other form; we simply **switch the inputs and outputs because logarithms and exponentials are inverses.**

Example 2: Solve the exponential equation $2^{-x} = 6$ by converting to logarithmic form and then isolating the variable x. **LEAVE ANSWERS IN EXACT FORM, DO NOT APPROXIMATE**.

$$2^{-x} = 6$$

$$-x = \log_2(6)$$

$$x = -\log_2(6)$$

The logarithm $\log_2(6)$ cannot be simplified any further, so I will leave my answer as $x = -\log_2(6)$. Had this been $-\log_2(4)$ or $-\log_2(8)$, I would have been able to simplify as -2 or -3.

$$\boldsymbol{x = -\log_2(6)}$$

Keep in mind that anytime a logarithm can be simplified, such as $\log(10)$ or $\ln(1)$, you will be expected to do so in order to simplify your answer completely. Simplifying will also result in an easier answer to input in LON-CAPA, as we'll see on the next example.

Example 3: Solve the exponential equation $2^{x-3} = 16$ by converting to logarithmic form and then isolating the variable x. **LEAVE ANSWERS IN EXACT FORM, DO NOT APPROXIMATE**.

$$2^{x-3} = 16$$

$$x - 3 = \log_2(16)$$

Remember that a logarithm represents an exponent, so to simplify $\log_2(16)$, we need to think about what exponent makes 2 become 16. In this case the answer is 4, because $2^4 = 16$, so $\log_2(16) = 4$.

$$x - 3 = \log_2(16)$$

$$x - 3 = 4$$

$$\boldsymbol{x = 7}$$

One benefit of solving exponential equations in this lesson as opposed to logarithmic equations like we solved in the previous lesson is that we are not required to check our answers. This is because exponential functions have unrestricted domains, so x can represent any real number. Logarithmic functions have restricted domains, since the argument of a logarithm must be positive. Therefore when solving logarithmic equations, we must verify that our answers result in positive arguments. However when solving exponential equations, x can be any real number, so checking our answers is not mandatory.

In the case of Example 3, plugging 7 back into the original equation for x results in the following:

$$2^{7-3} = 16$$

$$2^4 = 16$$

Again, checking your answer on exponential equations is optional.

Example 4: Solve each exponential equation by converting to logarithmic form and then isolating the variable x. **LEAVE ANSWERS IN EXACT FORM, DO NOT APPROXIMATE**.

a. $3^{4-x} = 5$

b. $3^{x^2} = 12$

c. $3^{5-x} = 27$

d. $3^{\frac{1}{x}} = 7$

e. $2^{5x+3} = 3$

$5x + 3 = \log_2(3)$

$5x = \log_2(3) - 3$

$x = \frac{\log_2(3)-3}{5}$

$\boldsymbol{x = \frac{\log_2(3)-3}{5}}$

f. $\frac{1}{81} = 3^{2x+1}$

$\log_3\left(\frac{1}{81}\right) = 2x + 1$

$-4 = 2x + 1$

$-5 = 2x$

$\boldsymbol{-\frac{5}{2} = x}$

Once again, anytime a logarithm can be simplified, such as $\log_3\left(\frac{1}{81}\right)$ above, it should be. Notice that because $\log_3\left(\frac{1}{81}\right)$ was able to be simplified, we ended up with an easier answer to input in LON-CAPA.

Answers to Examples:

2. $x = -\log_2(6)$ *; 3.* $x = \log_2(16) + 3 = \mathbf{7}$ *; 4a.* $x = 4 - \log_3(5)$ *; 4b.* $x = \sqrt{\log_3(12)}, -\sqrt{\log_3(12)}$ *; 4c.* $x = 5 - \log_3(27) = \mathbf{2}$ *;*

4d. $x = \frac{1}{\log_3(7)}$ *; 4e.* $x = \frac{\log_2(3)-3}{5}$ *; 4f.* $x = \frac{\log_3\left(\frac{1}{81}\right)-1}{2} = \boldsymbol{-\frac{5}{2}}$ *;*