In this set of notes we will set-up a systems of equations based on information from story problems, and then we will solve them. Just like the previous set of notes, I will use the Substitution Method to solve each system. However, you are welcome to use any method you'd like.

Method of Substitution:

1. solve one equation for one variable (it doesn't matter which equation you choose or which variable you choose).
2. substitute the solution from step 1 into the other equation.
3. solve the new equation from step 2.
4. back substitute to solve the equation from step 1.

Steps for solving applications:

1. assign variables to represent the unknown quantities
2. set-up equations using the variables from step 1
3. solve using substitution or elimination; it makes no difference which method you use

All of these application problems should result in a system of equations with two equations and two variables:

$$\begin{cases} ax + by = c \\ dx + ey = f \end{cases}$$

The equations will usually be linear, but not always.

<u>Example 1:</u> Set-up a system of equations and solve using any method.

When a popular band played at Elliott Hall, the box office receipts totaled $120,000. Student tickets cost $10 and non-student tickets cost $25. If 6,000 people attended the concert, how many were students and how many were non-students?

How many students attended the concert?

How many non-students attended the concert?

Write an equation to represent the number of people who attended the concert.

How much revenue was generated from students?

How much revenue was generated from non-students?

Write an equation to represent the amount of money generated by the concert.

$$\begin{cases} x + y = 6{,}000 \\ 10x + 25y = 120{,}000 \end{cases}$$

To solve this system of equations using substitution, I will take the first equation $(x + y = 6{,}000)$ and solve for x:

$$x + y = 6{,}000$$

$$x = 6{,}000 - y$$

Now I will substitute my answer for x into the original second equation in order to solve for y:

$$10x + 25y = 120{,}000$$

$$10(6{,}000 - y) + 25y = 120{,}000$$

$$60{,}000 - 10y + 25y = 120{,}000$$

$$15y = 60{,}000$$

$$y = 4{,}000$$

Now that I know the value of y, I will back substitute my answer for y into the equation $x = 6{,}000 - y$ in order to find the numeric value of x:

$$x = 6{,}000 - y$$

$$x = 6{,}000 - 4{,}000$$

$$x = 2{,}000$$

Since x represents the number of students at the concert and y represents the number of non − students, that means there were 2,000 students and 4,000 non − students at the concert.

Example 2: Set-up a system of equations and solve using any method.

A large solar heating panel requires 260 gallons of a fluid that is 45% antifreeze. The fluid comes in either a 75% solution or a 10% solution. How many gallons of each should be used to prepare the 260-gallon solution?

How much of the 75% solution should be used (in gallons)?

How much of the 10% solution should be used (in gallons)?

Write an equation to represent the number of gallons used.

How much antifreeze (in gallons) comes from the first portion?

How much antifreeze (in gallons) comes from the second portion?

Write an equation to represent the amount of antifreeze in the final mixture.

$$\begin{cases} x + y = 260 \\ 0.75x + 0.1y = 117 \end{cases}$$

To solve this system of equations using substitution, I will take the first equation $(x + y = 260)$ and solve for x:

$$x + y = 260$$

$$x = 260 - y$$

Now I will substitute my answer for x into the original second equation in order to solve for y:

$$0.75x + 0.1y = 117$$

$$0.75(260 - y) + 0.1y = 117$$

$$195 - 0.75y + 0.1y = 117$$

$$-0.65y = -78$$

$$y = 120$$

Now that I know the value of y, I will back substitute my answer for y into the equation $x = 260 - y$ in order to find the numeric value of x:

$$x = 260 - y$$

$$x = 260 - 120$$

$$x = 140$$

Since x represents the gallons of 75% antifreeze solution and y represents the gallons of 10% solution, that means we need 140 gallons of 75% solution and 120 gallons of 10% solution to fill the solar heating panel.

<u>Example 3:</u> Set-up a system of equations and solve using any method.

A salesperson purchased an automobile that was advertised as averaging $\frac{23 \text{ miles}}{\text{gallon}}$ of gasoline in the city and $\frac{41 \text{ miles}}{\text{gallon}}$ on the highway. A recent sales trip that covered 1977 miles required 57 gallons of gasoline. Assuming that the advertised mileage estimates were correct, how many gallons of gas were used in the city and how many gallons of gas were used on the highway?

How many gallons of gasoline were used in the city?

How many gallons of gasoline were used on the highway?

Write an equation to represent the total number of gallons of gasoline used on the trip.

How many miles were driven in the city, assuming the advertised mileage of $\frac{23 \text{ miles}}{\text{gallon}}$ *is correct?*

$$23\frac{\text{miles}}{\text{gallon}} \cdot x \text{ gallons} =$$

How many miles were driven on the highway, assuming the advertised mileage of $\frac{41 \text{ miles}}{\text{gallon}}$ *is correct?*

$$41\frac{\text{miles}}{\text{gallon}} \cdot y \text{ gallons} =$$

Write an equation to represent the total number of miles driven on the trip.

$$\begin{cases} x + y = 57 \\ 23x + 41y = 1977 \end{cases}$$

To solve this system of equations using substitution, I will take the first equation $(x + y = 57)$ and solve for x:

$$x + y = 57$$

$$x = 57 - y$$

Now I will substitute my answer for x into the original second equation in order to solve for y:

$$23x + 41y = 1977$$

$$23(57 - y) + 41y = 1977$$

$$1311 - 23y + 41y = 1977$$

$$18y = 666$$

$$y = 37$$

Now that I know the value of y, I will back substitute my answer for y into the equation $x = 57 - y$ in order to find the numeric value of x:

$$x = 57 - y$$

$$x = 57 - 37$$

$$x = 20$$

Since x represents the gallons of gasoline used in the city and y represents the gallons used on the highway, that means there were 20 gallons of gasoline used in the city and 37 gallons used driving on the highway.