<u>Rational expression:</u>
- a fraction with polynomials in the numerator and denominator (also called a quotient or ratio of polynomials)
  - $\frac{x+2}{x^2-5x-6}$ is an example of a rational expression; the numerator is a binomial and the denominator is a trinomial

Since a rational expression is simply a fraction, it can be simplified just like any other fraction by factoring the numerator and denominator and then canceling any common factors that they may have.

<u>Example 1:</u> Simplify the following fractional expressions completely.

a. $\frac{6}{8}$

b. $\frac{14x^4y}{18x^3y^2}$

$$\color{red}\frac{2\cdot7\cdot x\cdot x\cdot x\cdot x\cdot y}{2\cdot9\cdot x\cdot x\cdot x\cdot y\cdot y}$$

$$\color{red}\frac{2\cdot7\cdot \cancel{x}\cdot \cancel{x}\cdot \cancel{x}\cdot x\cdot \cancel{y}}{2\cdot9\cdot \cancel{x}\cdot \cancel{x}\cdot \cancel{x}\cdot y\cdot \cancel{y}}$$

$$\color{red}\frac{7\cdot x}{9\cdot y}$$

$$\color{red}\mathbf{\frac{7x}{9y}}$$

<u>Steps for Simplifying Rational Expressions:</u>
1. remove parentheses and combine like terms (if necessary)
2. factor all the polynomials
3. cancel common factors

A rational expression is simplified if its numerator and denominator have no common **<u style="color:red">factors</u>** other than 1, just like the rational expression $\frac{x-2}{x+2}$.

The reason that you don't cancel the x's or the 2's in the expression $\frac{x-2}{x+2}$ is because they are terms and not factors.

<u>It is imperative that you understand how to factor polynomials prior to simplify rational expressions. Keep in mind that rational expressions are simply fractions, and just like any other type of fraction, they should be simplified completely by canceling common factors.</u>

Example 2: Simplify the rational expressions completely.

a. $\dfrac{x^2-5x+6}{x^2+10x-24}$

$\dfrac{x^2-2x-3x+6}{x^2-2x+12x-24}$

$\dfrac{x(x-2)-3(x-2)}{x(x-2)+12(x-2)}$

$\dfrac{(x-2)(x-3)}{(x-2)(x+12)}$

$\dfrac{\cancel{(x-2)}(x-3)}{\cancel{(x-2)}(x+12)}$

$\dfrac{\boldsymbol{x-3}}{\boldsymbol{x+12}}$

b. $\dfrac{x^2-25}{x^3-125}$

c. $\dfrac{8+x^3}{x^4-16}$

$\dfrac{2\cdot2\cdot2+x\cdot x\cdot x}{x^2\cdot x^2-4\cdot4}$

$\dfrac{(2+x)\left(2^2-(2)(x)+x^2\right)}{(x^2-4)(x^2+4)}$

$\dfrac{(2+x)\left(4-2x+x^2\right)}{(x\cdot x-2\cdot2)(x^2+4)}$

$\dfrac{(2+x)\left(4-2x+x^2\right)}{(x-2)(x+2)(x^2+4)}$

$\dfrac{\cancel{(2+x)}\left(4-2x+x^2\right)}{(x-2)\cancel{(x+2)}(x^2+4)}$

$\dfrac{\boldsymbol{x^2-2x+4}}{\boldsymbol{(x-2)(x^2+4)}}$

d. $\dfrac{12x^4-17x^2+5}{8x^9-8x^8}$

Answers to Examples:

1a. $\dfrac{3}{4}$; *1b.* $\dfrac{9x^2}{16y}$; *2a.* $\dfrac{x-3}{x+12}$; *2b.* $\dfrac{x+5}{x^2+5x+25}$; *2c.* $\dfrac{x^2-2x+4}{(x-2)(x^2+4)}$;

2d. $\dfrac{(12x^2-5)(x+1)}{8x^8}$;