When adding or subtracting fractions, you must have a common denominator. There are multiple ways to get a common denominator, but the method I prefer is to simply determine which factors are missing from each denominator and multiply by those missing factors over themselves. In order to do this, I must first factor each denominator to determine which factors make up each denominator.

Example 1: Add or subtract the following fractions and simplify your answers completely.

a. $\frac{1}{4} + \frac{1}{7}$

$$\frac{1}{4} \cdot \frac{7}{7} + \frac{1}{7} \cdot \frac{4}{4}$$

$$\frac{7}{28} + \frac{4}{28}$$

I understand that at this point that both fractions can be reduced. However doing so will simply put us back to where we started, at $\frac{1}{4} + \frac{1}{7}$. Therefore you should wait until both fractions have been combined before you attempt to reduce.

$$\mathbf{\frac{11}{28}}$$

Since our answer is a fraction, we still need to cancel common factors if possible. However in this case, 11 and 28 have no common factors other than 1, so it's not possible to simplify this fraction any further.

b. $\frac{4x}{21} - \frac{1}{3xy}$

c. $\frac{1}{4} + \frac{1}{x} - \frac{1}{4x}$

$$\frac{1}{4} \cdot \frac{x}{x} + \frac{1}{x} \cdot \frac{4}{4} - \frac{1}{4x}$$

$$\frac{x}{4x} + \frac{4}{4x} - \frac{1}{4x}$$

$$\frac{x+4-1}{4x}$$

$$\boldsymbol{\frac{x+3}{4x}}$$

d. $\frac{2x}{5} + \frac{5}{2x} - 1$

$$\frac{2x}{5} \cdot \frac{2x}{2x} + \frac{5}{2x} \cdot \frac{5}{5} - \frac{1}{1} \cdot \frac{5}{5} \cdot \frac{2x}{2x}$$

$$\frac{4x^2}{10x} + \frac{25}{10x} - \frac{10x}{10x}$$

$$\boldsymbol{\frac{4x^2-10x+25}{10x}}$$

Once again keep in mind that only common factors are canceled when reducing a fraction, not terms. In the simplified fraction $\frac{4x^2-10x+25}{10x}$, the $10x$ in the numerator and the $10x$ in the denominator cannot be canceled because the $10x$ in the numerator is a term, not a factor.

Steps for Adding or Subtracting Rational Expressions:
(no common denominator)

1. factor each denominator completely
   - this is done to determine which factors each denominator has, and which factors each denominator is missing
2. multiply each rational expression by 1 $\left(\frac{\text{missing factor}}{\text{missing factor}}\right)$ to get a common denominator for each expression
3. once you have a common denominator, combine the numerators
4. factor the numerator and denominator completely, then cancel common factors (if possible)

If you are adding or subtracting rational expressions that already have common denominators, you can skip steps 1 & 2.

Example 2: Add or subtract the following rational expressions and simplify your answers completely.

a. $\frac{3}{x} - \frac{7}{x^2}$

$\frac{3}{x} \cdot \frac{x}{x} - \frac{7}{x^2}$

$\frac{3x}{x^2} - \frac{7}{x^2}$

$\mathbf{\frac{3x-7}{x^2}}$

b. $\frac{x}{x+1} + \frac{x}{x+2}$

c. $\frac{2}{(x-1)^2} - \frac{x}{x-1}$

d. $\frac{1}{x+h} - \frac{1}{x}$

$$\frac{x}{x} \cdot \frac{1}{x+h} - \frac{1}{x} \cdot \frac{x+h}{x+h}$$

$$\frac{x}{x(x+h)} - \frac{x+h}{x(x+h)}$$

$$\frac{x-(x+h)}{x(x+h)}$$

$$\frac{x-x-h}{x(x+h)}$$

$$\boldsymbol{\frac{-h}{x(x+h)}}$$

e. $\frac{7}{x+2} - \frac{1}{x} + 3$

f. $\frac{4}{x+2} - \frac{3}{x-2} + \frac{12}{x^2-4}$

$$\frac{7}{x+2} \cdot \frac{x}{x} - \frac{1}{x} \cdot \frac{x+2}{x+2} + \frac{3}{1} \cdot \frac{x}{x} \cdot \frac{x+2}{x+2}$$

$$\frac{7x}{x(x+2)} - \frac{x+2}{x(x+2)} + \frac{3x(x+2)}{x(x+2)}$$

$$\frac{7x-(x+2)+3x(x+2)}{x(x+2)}$$

$$\frac{7x-x-2+3x^2+6x}{x(x+2)}$$

$$\frac{3x^2+12x-2}{x(x+2)}$$

Since the trinomial $3x^2 + 12x - 2$ in the numerator cannot be factored, the rational expression $\frac{3x^2+12x-2}{x(x+2)}$ cannot be simplified any further.

$$\boldsymbol{\frac{3x^2+12x-2}{x(x+2)}}$$

Negative Exponent Rule:

- to change the sign of an **exponent**, take the reciprocal of the factor or expression that has the negative exponent
  - $x^{-3} = \frac{1}{x^3}$
  - $5x^{-3} = \frac{5}{x^3}$
  - $(5x)^{-2} = \frac{1}{25x^2}$

Example 3: Add or subtract the following rational expressions and simplify your answers completely. Do not include negative exponents in your answers.

a. $6x^{-2} - (3x)^2$

b. $2x - 2x^{-1} + (2x)^{-1}$

Remember that to change the sign of a negative exponent we take the reciprocal of the factor or the expression that has the negative exponent. Notice on the middle term that only the factor of x has a negative exponent, so we only take the reciprocal of x, not 2.

$$2x - \frac{2}{x} + \frac{1}{2x}$$

$$\frac{2x}{1} \cdot \frac{2x}{2x} - \frac{2}{x} \cdot \frac{2}{2} + \frac{1}{2x}$$

$$\frac{4x^2}{2x} - \frac{4}{2x} + \frac{1}{2x}$$

$$\frac{4x^2 - 4 + 1}{2x}$$

$$\boldsymbol{\frac{4x^2 - 3}{2x}}$$

Example 4: Combine the following rational expressions using the given operations, and simplify your answers completely. Keep in mind order of operation (PEMDAS).

a. $\left(\frac{2x+3}{x+1} \cdot \frac{x^2+4x-5}{2x^2+x-3}\right) + \frac{2}{x+2}$

b. $\left(\frac{x-1}{x+1} - \frac{x+3}{x-3}\right) \div \frac{x}{x-3}$

$$\left(\frac{x-3}{x-3} \cdot \frac{x-1}{x+1} - \frac{x+3}{x-3} \cdot \frac{x+1}{x+1}\right) \div \frac{x}{x-3}$$

$$\left(\frac{(x-3)(x-1)}{(x-3)(x+1)} - \frac{(x+3)(x+1)}{(x-3)(x+1)}\right) \div \frac{x}{x-3}$$

Once again, both fractions could be reduced at this point in the problem.

However once again, doing so would simply put us back at the start. That is why I waited until I had one factored fraction before I canceled common factors.

$$\left(\frac{x^2-4x+3-(x^2+4x+3)}{(x-3)(x+1)}\right) \div \frac{x}{x-3}$$

$$\frac{x^2-4x+3-x^2-4x-3}{(x-3)(x+1)} \div \frac{x}{x-3}$$

$$\frac{-8x}{(x-3)(x+1)} \div \frac{x}{x-3}$$

$$\frac{-8x}{(x-3)(x+1)} \cdot \frac{x-3}{x}$$

$$\frac{(-8)(\cancel{x})(\cancel{x-3})}{(\cancel{x-3})(x+1)(\cancel{x})}$$

$$\boldsymbol{\frac{-8}{x+1}}$$

Answers to Examples:

1a $\frac{11}{28}$ *; 1b.* $\frac{4x^2y-7}{21xy}$ *; 1c* $\frac{x+3}{4x}$ *; 1d.* $\frac{4x^2-10x+25}{10x}$ *; 2a* $\frac{3x-7}{x^2}$ *;*
2b. $\frac{x(2x+3)}{(x+1)(x+2)}$ *; 2c.* $\frac{-(x-2)(x+1)}{(x-1)^2}$ *; 2d* $\frac{-h}{x(x+h)}$ *; 2e.* $\frac{3x^2+12x-2}{x(x+2)}$ *; 2f.* $\frac{1}{x+2}$
; 3a $\frac{3(2-3x^4)}{x^2}$ *; 3b.* $\frac{4x^2-3}{2x}$ *; 4a* $\frac{x^2+9x+12}{(x+2)(x+1)}$ *; 4b.* $-\frac{8}{x+1}$ *;*