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On physical and mathematical causality in quantum mechanics

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ABSTRACT

This paper critically examines the view of quantum phenomena that has persisted since the introduction of quantum mechanics and that still remains prevalent in the foundational literature on quantum theory. According to this view, the independent behavior of quantum systems is causal, while the experimentally manifest lack of causality in observable quantum phenomena and, as a result, the probabilistic nature of our predictions concerning these phenomena are due to the disruption of this causal behavior by interfering with it through the measuring process. It appears that this view originates with P. A. M. Dirac and his work on the transformation theory (introduced by him and P. Jordan), which brought together W. Heisenberg's and E. Schrödinger's versions of quantum mechanics within a single scheme. Other founding figures of quantum theory, specifically N. Bohr, W. Heisenberg, and J. von Neumann, also advanced this view and helped to establish its prominence. The paper discusses these arguments and contends them to be insufficient to support the view that the independent behavior quantum systems is *physically* causal. It suggests that one can meaningfully speak of *mathematical* causality in quantum theory, and advocates an alternative, physically noncausal, interpretation of quantum mechanics.

1. Introduction

This paper offers a critical examination of the view of quantum theory that emerged shortly after the introduction of quantum mechanics and that has been persistent and prominent ever since. Dirac, who appears to have been first to advance this view around 1927 [1, p. 641], expressed it as follows in his famous book *The Principle of Quantum Mechanics*, first published in 1930: "We must [in view of the nature of quantum phenomena and quantum mechanics] revise our ideas of causality. Causality applies to a [quantum] system which is left undisturbed. If a system is [quantum-level] small, we cannot observe it without producing a serious disturbance and hence we cannot expect to find any causal connexion between the results of our observations" [2, p. 4]. I am not claiming that this view is necessarily wrong, but only that Dirac, or Bohr, Heisenberg, von Neumann, and others who subscribe to this view do not appear to offer adequate arguments supporting it. While it would be impossible to make a definitive claim in this regard given an enormous amount of literature on the foundations of quantum mechanics currently available, the classic major treatments to be discussed here appear to be representative of this view. One might speak of 'mathematical causality' in quantum theory, whereby the equations of quantum mechanics *determine* the relevant *mathematical* object, say, a wave function in the case of Schrödinger's equation, at any time once it is known at a given time. Unlike in classical physics, however, where the same mathematical causality holds as well, it does not translate into a physical causality even when the system in question is undisturbed by measurement, but only into probabilistic estimates concerning the outcomes of certain possible measurements on the basis of other, already performed measurements. Not everyone subscribes to the view under criticism here. Thus, although both N. Bohr and W. Heisenberg subscribed to this view, respectively, at the time and following Bohr's introduction of complementarity in 1927, both rejected the idea quantum-level physical causality earlier, as did other founders of matrix mechanics, M. Born, in particular. Bohr was not only quick to abandon this view in favor of his earlier position against causality but also, at least *de facto*, challenged it in his work following his initial exchanges on quantum mechanics with Einstein in 1927. More recently, this view was challenged, at least, again, *de facto*, by those who pursue the Bayesian approach to quantum information theory (e.g. [3]).

Given their prominence and impact, Dirac's *The Principles of Quantum Mechanics*, published in 1930, Heisenberg's Chicago lecture, delivered in 1929 and published as *The Physical Principles of the Quantum Theory* in 1930 [4], and J. von Neumann's *Mathematical Foundations of Quantum Mechanics*, published in 1932 [5], might have been most responsible for the prevalence of this view. Dirac, as I said, expressed a similar view earlier, via the

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transformation theory (developed by him and, independently, Jordan), in his 1927 paper "The Physical Interpretation of the Quantum Dynamics" ([1]), and was arguably the first to do so. The paper had a major impact on Heisenberg's thinking and his paper introducing the uncertainty relations ([6]), where, however, Heisenberg still maintained a strong position against the quantum-level physical causality. Both papers influenced Bohr's thinking at the time, as did Schrödinger's wave mechanics. While, however, Schrödinger's program also aimed at a causal theory of quantum processes, and while it had influenced all of the arguments concerning quantum causality just mentioned, his view of the situation was different. The concept of (noncausal) disturbance of quantum processes by observation played no role in his argumentation, and he hoped that the causal behavior of quantum systems would, at least in principle, be accessible. Bohr appears to have been the first to formulate expressly the idea of the juxtaposition between "causality" and "observation" (seen as "disturbance") in his so-called Como lecture of 1927, "The Quantum Postulate and the Recent Development of Atomic Theory," famous for his introduction of the concept of complementarity, which this juxtaposition exemplified ([7], v. 1, pp. 52–91). The juxtaposition was only implied, albeit difficult to miss, in Dirac's paper just mentioned.

As I said, Bohr abandoned this view under the impact of his exchanges with Einstein in 1927. He returned to his earlier view, stemming from Heisenberg's initial work on quantum mechanics that he (or Heisenberg) entertained before Schrödinger's wave mechanics and the Dirac–Jordan transformation theory. As Bohr said in 1925, "In contrast to ordinary mechanics, the new mechanics [of Heisenberg] does not deal with a space-time description of the motion of atomic particles," and, hence, with the question of causality of their independent behavior, which Heisenberg argued to be unobservable ([7], v. 1, p. 48). Heisenberg's mechanics dealt only with the probabilities of transitions, manifest in atomic spectra, between stationary states. Eventually, Bohr developed this view into a full-fledged noncausal interpretation of quantum mechanics, known as complementarity, which this paper by and large adopts as an alternative to the view that the independent behavior of quantum systems is (physically) causal.

Before proceeding to my argument, I would like to establish the key terms of my discussion. I use 'causality' as an *ontological* category relating to the physical behavior defined by the fact that the state of a given system is exactly determined at all points by its state at a particular point, indeed at any given point. At least, such is the case at the level of idealized models, which qualification will be presupposed from now on, since modern physics, classical or quantum, only deals with such models. It appears that the authors to be considered here mean 'causality' essentially in this sense. I use 'determinism' as an *epistemological* category having to do with our ability to predict the state of a system at any and all points once we *know* its state at a given point. Thus, classical mechanics deals with causal systems deterministically, while classical statistical physics deals with causal systems only probabilistically or statistically, rather than deterministically. Classical mechanics is also, correlatively, 'realist,' insofar as a mapping of the behavior of individual systems is assumed to take place. Classical statistical physics is not realist in this sense, since its equations do not describe the behavior of the individual objects comprising the systems considered. It is, however, based on the realist assumption that this behavior conforms to the causal laws of classical mechanics.

On experimental grounds, as currently established, quantum mechanics is neither a deterministic nor a causal theory. A given quantum event or phenomenon, observed in the measuring instruments involved, can be connected to other quantum events or phenomena only probabilistically. Indeed, since, as is, again, well-established experimentally, identically prepared quantum experiments in general lead to different outcomes, any theory properly accounting for these phenomena can only be probabilistic. The situation is more complex as concerns the behavior of quantum objects, because our observations, as manifest in measuring instruments, appear to relate only indirectly to quantum objects and their behavior. In view of the uncertainty relations, it is difficult to sustain an argument for a classical-like causal quantum level behavior in the way it is done in classical physics. Such a *behavior* is a feature of certain alternative interpretations of quantum mechanics, such as the many-worlds interpretation, or alternatives to quantum mechanics itself, for example, in Bohmian theories, where realism and causality carries at the cost of nonlocality (in the sense of the possibility of instantaneous physical connections between spatially separated events, forbidden by relativity). Even in Bohmian theories, however, no undistorted *description* of this behavior is possible, and the uncertainty relations are still valid.

By 'randomness' or 'chance' I refer to a manifestation of the unpredictable. A random event is an unpredictable event. Physically, such events may or may not manifest some ultimate underlying causal dynamics unavailable to us. In classical statistical physics, randomness and probability might be seen as resulting from insufficient information concerning systems that are at bottom causal but whose mechanical complexity prevents us from accessing their causal behavior and making deterministic predictions concerning this behavior. The situation is, again, more complex in quantum mechanics, given the difficulties of sustaining arguments for the causality of the independent behavior of quantum systems.

Probability considerations concern numerical estimates of occurrences of individual or collective events, in accordance with mathematical probability theories, which theorize such estimates. In particular, Bayesian approaches deal with the probabilities of the *individual* events, rather than ensembles of events, as in the frequentist (ensemble) approaches, where it is more appropriate to speak of 'statistics.' In either type of interpretation, however, in the case of quantum phenomena and quantum mechanics one deals with a peculiar combination of randomness and probability. *On the one hand*, given that identically prepared quantum experiments lead to different outcomes, there is randomness inherent in each individual quantum event. *On the other hand*, our expectations concerning such outcomes can be helped by rigorously defined rules, encoded in the mathematical formalism of quantum mechanics.

The next section examines the arguments concerning quantum causality offered by Bohr, Dirac, Heisenberg, and von Neumann. Section 3 discusses some among the historical and conceptual reasons for the emergence of the arguments of this type, in particular the role of the transition from Heisenberg's initial approach to quantum mechanics in 1925 to those developed in (and following) Schrödinger's wave mechanics and the transformation theory. I close with a brief conclusion on how the problematic of quantum causality may bear on the search for possible alternatives to the standard quantum mechanics or, conversely, on one's acceptance of the latter as a sufficiently adequate theory of quantum phenomena.

2. Quantum mechanics and causality in Bohr, Dirac, Heisenberg, and von Neumann

Bohr advanced the idea that the independent evolution of a given quantum system is causal, while the lack of causality in the observed quantum phenomena is due to an act of measurement (which "disturbs" this evolution), in 1927 in the so-called Como lecture. While possibly following Dirac's initial insight in Ref. [1], Bohr gave this idea a new twist through his concept of complementarity, introduced in the lecture. Indeed complementarity was exemplified and even defined there by the mutual exclusivity of the quantum-level causality and observation (the space-time co-ordination), which disturbs the causal evolution of a given quantum system. According to Bohr:

> On the one hand, the definition of the state of a physical system, as ordinarily understood [i.e. in classical physics], claims the elimination of all external disturbances. But in that case, according to the quantum postulate [of Planck], any observation will be impossible, and, above all, the concepts of space and time lose their immediate sense. On the other hand, if in order to make observation possible we permit certain interactions with suitable agencies of measurement, not belonging to the system, an unambiguous definition of the state of the system is naturally no longer possible, and there could be no question of causality in the ordinary sense of the word. The very nature of the quantum theory thus forces us to regard *the space-time co-ordination and the claim of causality, the union of which characterizes the classical theories, as complementary but exclusive features of the description*, symbolizing the idealization of observation and definition, respectively ([7], v. 1, pp. 54–55: emphasis added).

The idealization of both observation and definition is primarily mathematical, and, in the case of definition, it refers to the mathematical formalism of quantum mechanics, such as Schrödinger's equation, which reflects the causality in question. Bohr's statement "*the space-time co-ordination and the claim of causality, the union of which characterizes the classical theories, as complementary but exclusive features of the description*" introduces his concept of complementarity. Bohr, however, does not rigorously define what he means by "complementary" in the lecture. Indeed, it is not clear what "complementary" means beyond "exclusive." Eventually, Bohr defined the concept more rigorously as follows ([7], v. 2, p. 40). It designates (a) a mutual exclusivity of certain phenomena, entities, or conceptions; and yet (b) the possibility of applying each one of them separately at any given point and (c) the necessity of using all of them at different moments for a comprehensive account of the totality of phenomena that we must consider. The term also came to designate Bohr's overall interpretation of quantum mechanics, based on this concept. Parts (b) and (c) of this definition are just as important as part (a), and disregarding them often leads to misunderstandings of Bohr's concept. These aspects of the concept are not stated in the Como lecture, although (c) appears to be implied and to be the reason why Bohr sees the space-time co-ordination and the claim of causality not only as "mutually exclusive" but also as "complementary." On the other hand, as will be seen presently, both (b) and (c) pose difficulties in the case of this particular complementarity. The definition itself just given is, however, more general and allows for other complementary configurations, such as those of the (exact) position and momentum measurements, correlative to the uncertainty relations, in which case all three features (a), (b), and (c) rigorously apply. This complementarity became central for Bohr after he abandoned the idea that the claim of causality can apply to either the description of observed quantum phenomena or to the behavior of quantum objects.

If, however, by being mutually exclusive to observation, the independent behavior of quantum objects is, by definition, unobservable and ultimately undefinable, at least on the classical model, in view of the uncertainty relations, in what sense could one speak of "the claim of causality" here, at least in physical terms? One could easily speak of mathematical causality or determination in quantum mechanics, via, say, Schrödinger's equation. Bohr, however, appears to imply a certain physical causality here. Thus, he says that, while the space-time co-ordination and the claim of causality are mutually exclusive in quantum physics, the classical theory is characterized by the unity of both. In classical physics, however, the claim of causality is physical, and the equations of classical physics map the physically causal processes considered. Hence, some physical causality, now as mutually exclusive with the space-time co-ordination, is at least implied by Bohr in quantum theory as well.

What is correct is that, unlike in classical physics, in quantum physics the independent behavior (causal or not) of the quantum system and the observation *are mutually exclusive*, in view of the irreducible "disturbance" in question affecting the behavior of quantum systems in any act of observation or measurement, on which our predictions concerning this behavior are based. Even the slightest possible observational interference (which may be a better term than disturbance), say, by a single photon, would be sufficient to "disturb" this behavior, as Heisenberg explained in his paper on the uncertainty relations ([6], p. 65). It follows that this independent behavior of quantum systems is mutually exclusive with observation and, hence, is unobservable. In classical physics, the independent behavior of a given system is not mutually exclusive with observation because the interference in question can, at least in principle, be neglected. Hence, this behavior can be considered independently and happens to be causal, and the formalism of classical mechanics maps this behavior, at least, again, in the case of idealized models. Bohr is, thus, right to say that "*the union … of the space-time co-ordination and the claim of causality… characterizes the classical theories.*" No such models appear to be possible in quantum theory, since the independent behavior of quantum objects is mutually exclusive with observation and hence, again, is unobservable. In what sense, then, could one speak of the independent quantum behavior as causal? Or, to begin with, in what sense could one meaningfully claim that the formalism of quantum mechanics describes this independent behavior? Schrödinger, whose earlier thinking was shaped by the idea of a causal evolution of an electron (on a wave, rather than particle, model) made the point later, in his cat-paradox paper (1935), by way of a very different assessment of quantum mechanics, which he saw as "a doctrine born of distress." By that time his program for wave mechanics was abandoned by him. He said: "if a classical [physical] state does not exist at any moment, it can hardly change causally" ([8], p. 154).

Heisenberg, Dirac, and von Neumann follow Bohr's argument in, respectively, *The Physical Principles of the Quantum Theory* ([4]), *The Principles of Quantum Mechanics* ([2]) and *Mathematical Foundations of Quantum Mechanics* ([5]), all published around the same time (1930–1932). These were the most important early books on quantum mechanics, which quickly became classics and have shaped our thinking concerning quantum mechanics from then on and still continue to do so. As such they might well have been most responsible for the propagation of the argument concerning quantum causality that I question here.

I would like to consider Dirac's arguments first because Dirac held the type of view in question already in his paper, "The Physical Interpretation of the Quantum Dynamics," completed in 1926, while in Copenhagen, and published in 1927. Thus, Dirac's argument for this view precedes both Bohr's Como argument for it and even Heisenberg's work on the uncertainty relations, influenced by this paper, without, as I said, yet subscribing to this view itself, which Heisenberg only adopted in the wake of Bohr's Como argument. It is difficult to say whether Dirac's argument there influenced Bohr or was, conversely, influenced by

Bohr's thinking as shaped, as it was by then, by Schrödinger's wave mechanics, and the definitive answer is not that important at the moment. What is important is that, stimulated by his transformation theory (which rigorously connect both Heisenberg's and Schrödinger's versions of quantum mechanics in mathematical and predictive terms), Dirac sees his argument for the causality of the behavior of the "undisturbed" quantum system as fully compatible with Heisenberg's matrix mechanics. This is also true for Bohr's Como argument, also influenced by the transformation theory, both directly and via Heisenberg's paper on the uncertainty relations. According to Dirac: "One can suppose that the initial state of a system determines definitively the state of the system at any subsequent time. … The notion of probability does not enter into the ultimate description of mechanical processes; only when one is given some information that involves a probability … can one deduce results that involve probabilities" ([1], p. 641). The last point is correct. The question is, again, whether the first part of this statement is true and in what sense, which in part hinges on the concept of 'quantum state.' Dirac's statement and those of others to the same effect cause no problem if one sees a quantum state as a mathematical object (say, a vector in a complex Hilbert space) that, via Born's or equivalent rule, enables one to predict the probabilities in question, rather than a physical concept, especially if conceived on the model of classical mechanics. The mathematical formalism of quantum mechanics allows for an unequivocal determination of a state vector as a mathematical object under a given mathematical transformation, say, under an action of an energy operator. Just as Bohr does in his Como argument, however, Dirac clearly implies more by invoking "the ultimate [quantum-level] description of mechanical processes," clearly seen as causal here, or by the title-phrase of his paper, "the *physical interpretation* of the quantum dynamics" (emphasis added). Accordingly, Dirac's similar argument concerning the case in *The Principles of Quantum Mechanics* should not be surprising. The influence of Bohr's Como argument is now palpable, however. Dirac says:

> [We] must revise our ideas of causality. Causality applies to a system which is left undisturbed. If a system is [quantum-level] small, we cannot observe it without producing a serious disturbance and hence we cannot expect to find any causal connexion between the results of our observations. Causality will still apply to undisturbed [quantum] systems and the equations [those of quantum mechanics] which will be set up to describe an undisturbed system will be differential equations expressing a causal connection between conditions at one time and conditions at a latter time. These equations will be in close correspondence with the equations of classical mechanics, but they will be connected only indirectly with the results of observations. There is an unavoidable indeterminacy in the calculation of observational results, the theory enabling us to calculate in general only the probability of our obtaining a particular result when we make an observation. ([2], p. 4).

It follows that the results of observation in space-time and the determination, including that of time itself, and causality of the connections between conditions at one time and conditions at a latter time are mutually exclusive or complementary. Thus, the complementarity of space-time co-ordination and the claim of causality is *de facto* used by Dirac, even if the term itself is not invoked and the concept is avoided in the book. By the same token, the statement carries the same difficulties as Bohr's Como statement discussed above. The "conditions at one time and conditions at a latter time" that are claimed to be causally connected by differential equations of quantum mechanics cannot be ever physically determined and are, in principle, unobservable, since any observation, by definition, destroy the presumed causal connection in question. On what basis, then, can one claim the causality of this connection and in what sense, beyond, again, the mathematical determination of variables involved by the differential equations in question? Just as Bohr, however, Dirac, too, appears to imply the physical causality of independent quantum behavior by saying that "causality will still apply to undisturbed [quantum] systems," rather than only to the equations of quantum mechanics. These equations, Dirac says, are "connected only *indirectly* to the results of observation," one presumes that "indirectly" means in terms of probabilistic predictions, given Dirac's final sentence here, which is crucial. The equations of quantum mechanics properly respond to the situation as concerns our predictions, in general probabilistic, of the outcomes of quantum observations or measurements. But what are these equations connected *directly* to in physical terms? Dirac himself inadvertently suggests that his contention could be questioned along the lines here suggested in the next page, and perhaps the rigor of his thinking compels him to do so. He says: "A question about what will happen to a particular photon under certain conditions is not really very precise. To make it precise one must imagine some experiment performed having a bearing on the question and inquire what will be the result of the experiment. Only questions about the results of experiments have real significance and it is only such questions that theoretical physics has to consider" ([2], p. 5). In contrast to the statement cited above, this sounds closer to the post-Como Bohr, who no longer thinks of the independent ("undisturbed") behavior of quantum systems, than to the Como Bohr. Once, however, one makes this statement, it becomes difficult to sustain the claim that "causality will still apply to undisturbed [quantum] systems."

Heisenberg's *The Physical Principles of the Quantum Theory* (1930), shaped as much by Bohr's Como argument as by his own work on the uncertainty relations, gives Bohr's scheme an elegant diagrammatical form:

> CLASSICAL THEORY:
> Causal relationships described in terms of space and time.
> QUANTUM THEORY:
> *Either*
> Phenomena described in terms of space and time.
> *But*
> Uncertainty Principle
> *Or*
> Causal relations expressed by mathematical laws.
> *But*
> Physical description of phenomena in space-time is impossible.
> These two alternatives are related statistically ([4], p. 65).

The meaning of the last sentence appears to be that, although our predictions concerning quantum experiments are enabled by certain mathematical laws, defined "causally" by the formalism of quantum mechanics, these predictions are nevertheless only probabilistic or statistical. Heisenberg adds: "It is only after attempting to fit this fundamental complementarity of space-time description and causality into one's conceptual scheme that one is in a position to judge the degree of consistency of the methods of quantum theory (particularly of the transformation theory)' (p. 65). The statement itself is opened to questioning along the lines of the preceding discussion. Heisenberg does, however, pinpoint a crucial aspect of the situation and the main difficulty of Bohr's Como scheme by invoking "causal relationships expressed by mathematical laws," rather than implying physical causality, since a "physical description of phenomena in space-time is impossible"—*any physical description*, let alone a causal one. This contention is not surprising, since Heisenberg strongly argued against introducing any physical causality into the quantum-mechanical situation in his uncertainty-relations paper as well. "There exists a body of exact mathematical laws," he also says here, "but these cannot be interpreted as expressing simple relationships between objects existing in space and time" (p. 64). This qualification, again, compels us to ask what is the meaning of causality under these conditions. Which relationships and between which elements are causal and are, as such, expressed by mathematical laws, defined by the formalism of quantum theory? Heisenberg does not answer this question either, any more than do Bohr and Dirac, or von Neumann. *Posing* the question in this form, however, allows us to think through the situation differently and reach a deeper understanding of it.

I have considered von Neumann's argument for the causal character of the independent behavior of quantum systems in detail in [9] and shall only state the key relevant points here. According to von Neumann:

On the one hand, a state ϕ is transformed into the state ϕ' under the action of an energy operator H in the time interval $0 \le \tau \le t$:

$$\partial\phi_\tau/\partial t = -(2\pi i/h)H\phi_\tau \; (0 \le \tau \le t), \tag{1}$$

so if we write $\phi_0 = \phi$, $\phi_t = \phi'$, then

$$\phi' = e^{-(2\pi i/h)tH}\phi \tag{2}$$

which is *purely causal*. A mixture U [U is a statistical operator] is correspondingly transformed into

$$U' = e^{-(2\pi i/h)tH}Ue^{(2\pi i/h)tH} \tag{3}$$

Therefore, *as a consequence of the causal change of ϕ into ϕ'*, the state $U = P_{[\phi]}$ go over into the states $U' = P_{[\phi']}$

$$U \to U_t = e^{-(2\pi i/h)tH}Ue^{(2\pi i/h)tH}$$

On the other hand, the state ϕ – which may measure a quantity with a pure discrete spectrum, distinct eigenvalues and eigenfunctions $\phi_1, \phi_2, \ldots$ – undergoes in a measurement *a non-causal change* in which each of the states $\phi_1, \phi_2, \ldots$ can result, and in fact does result with the respective probabilities $|(\phi,\phi_1)|^2, |(\phi,\phi_2)|^2, \ldots$ That is, the mixture

$$U' = \sum_{n=1}^{\infty} |\phi, \phi_n|^2 P_{[\phi_n]} \tag{4}$$

obtains. More generally, the mixture U goes over into

$$U' = \sum_{n=1}^{\infty} |U\phi_n, \phi_n|^2 P_{[\phi_n]} \tag{5}$$

Since the states go over into mixtures, the process is not causal.

The difference between these two processes $U \to U'$ is a very fundamental one: aside from the different behaviors in regard to the principle of causality, they are also different in that the former is (thermodynamically) reversible, while the latter is not. ([4], pp. 417–418; emphasis added)

While having a more rigorously mathematical form, this statement and von Neumann's argument supporting it largely follow those of Bohr, Heisenberg, and Dirac, as just discussed. Throughout the book, von Neumann's elaborations on the subject are marked by similar ambivalences concerning the possibly physical nature of the causality invoked here vis-à-vis the relationships between mathematical entities (vectors and operators in Hilbert spaces), which express the laws linked to quantum objects or phenomena. These ambivalences, again, reflect the ambiguity of the term 'state,' which designates both a mathematical concept (a vector in a complex Hilbert space) and a certain physical concept, conceived on the model of classical physics. In quantum mechanics, these relationships are indirect and probabilistically predictive, as against the direct, descriptive ones, found in classical physics. Thus, on the one hand, von Neumann's formulation here and related formulations found in the book appear to suggest that he has in mind primarily a certain mathematical "causality" or determination (e.g. p. 357). So do other arguments in the book. On the other hand, von Neumann does not expressly say here or elsewhere in the book that this causality is only mathematical, and, since the noncausal measurement process corresponding to Eq. (4) is physical, it is difficult to avoid an implication that the causal process corresponding to Eq. (2) is physical as well. His claim that the process $U \to U_t = e^{-(2\pi i/h)tH}\, U\, e^{(2\pi i/h)tH}$ (the state $U = P_{[\phi]}$ go over into the states $U' = P_{[\phi']}$, where U is a statistical operator), which is "*a consequence of the causal change of ϕ into ϕ'*," is "thermodynamically reversible" also suggests, in view of the generally physical nature of the latter concept, that this process and the causality itself in question may be physical in nature, as does his analysis of "measurement and reversibility" in Chapter 5. Leaving aside the independent evolution of mixtures, if the *independent evolution* of "a [quantum] state ... under the action of an energy operator," $\phi' = e^{-(2\pi i/h)tH}\,\phi$, is assumed to relate to any physical process, this process is not observable, and hence the *assumption* that the evolution of a quantum state is "purely causal" is only an assumption. Eqs. (1) and (2) are not given a physical content in von Neumann's analysis. As I argue, it may not be possible to do so, as opposed to the case of the noncausal process defined by measurement, represented by Eq. (4). Coupling the overall scheme to Schrödinger's equation, with a wave function ψ (which can also be connected to a certain ket-vector in a Hilbert space, $|\psi>$) enables one to make a prediction concerning the outcome of a future measurement, at time t, on the basis of a measurement performed at time t_0. We do need Schrödinger's equation to make proper predictions, but we need not assume that it relates to any causal evolution of the object in question, any more than do Eqs. (1) or (2).

3. From acausality to causality and from causality to acausality in quantum theory

Given the preceding argument, beginning with its starting point, recognized by the authors discussed here, that the independent ("undisturbed") physical behavior of quantum objects is unobservable, what, then, is *the basis* for the view that this behavior is *physically*, rather than only *mathematically*, causal? That is, given that, as I argue here, there does not appear to be a sufficient conceptual argument for it, how did this view *come about*? Both the history of modern physics, beginning with Galileo and Newton, and the history of quantum theory before and after quantum mechanics, or in the process of its development, have played their roles in the emergence of this view. The nature and effectiveness of classical physics led to powerful philosophical imperatives or, since there are other forces (philosophical and cultural) that have been at work shaping them before the rise of modern physics, reinforced and stimulated such imperatives. Arguably the most powerful of them is the idea and the ideal, "the classical ideal," as Schrödinger called it, of describing or at least approximating by way of idealized models the independent, and presumably causal, behavior of individual physical systems. This ideal was challenged by quantum theory, as Schrödinger noted on the same occasion ([8], p. 152). It is not possible to trace this history here. The invention of Schrödinger's wave quantum mechanics, however, and the key philosophical differences between the latter and Heisenberg's matrix quantum mechanics appear to be responsible most for the turn or return to causality in quantum theory. Heisenberg's theory was based on a suspension of the causal view of the independent behavior of quantum objects, as in principle unobservable. By contrast, Schrödinger's wave mechanics was based on this view, and, at least initially, the hope was that the intervention of measurement could, at least in principle, be neglected, just as it could be in classical mechanics.

In particular, in Heisenberg's initial formulation of quantum mechanics the (continuous) orbital motion of electrons (retained in the old quantum theory along with discontinuous and noncausal quantum jumps from one orbit to another) and the *orbital* frequencies of this motion were seen, as, if existent at all, in principle unobservable. Hence, these frequencies were excluded from his new mechanics and its mathematical formalism altogether, based, in Heisenberg's famous words, on the relationships between the "quantities which in principle are observable" ([10], p. 263). In Heisenberg's scheme or in the full-fledged matrix mechanics there were no mathematical variables corresponding to these frequencies, unlike in Schrödinger's scheme or Dirac's q-number formalism. Only the *transition* "frequencies," corresponding *physically*, to quantum jumps and *mathematically*, but *not* physically, to classical frequencies, were retained, along with quantum "amplitudes," corresponding, again, mathematically, to classical amplitudes in Fourier's representation of classical motion. This mathematical correspondence gave a new, more mathematical form to Bohr's correspondence principle. It meant that the classical equations of motion (in the full-fledged scheme of matrix mechanics, the Hamiltonian equations) were taken over directly from classical mechanics. However, Heisenberg's "new kinematical elements," as he called them, as the variables used in these equations, were mathematically different from those of classical theory. They were (unbounded) infinite matrices of complex quantities, which could be linked to the probabilistic predictions, always real numbers, by means of the rule similar to Born's rule for Schrödinger's wave function. As a result, their formal correspondence notwithstanding, the physical content of the equations of quantum mechanics did not correspond to the physical content of the equations of classical physics, which, or ultimately even the old quantum theory, did not yield correct predictions concerning quantum phenomena, especially, for low quantum numbers. So a departure from classical physics and the (more classically based) old quantum theory was essential to and defined Heisenberg's project. In the region of high quantum numbers the same predictions could be obtained by means of either classical or quantum-mechanical equations (the ultimate physical point of the correspondence principle). This does not mean that the two schemes, classical and quantum, would become the same or that the actual process could be seen as classical in this region, but only that classical theory provides a good approximation there, while it failed for low quantum numbers.

The key physical point was that, instead of relating to physical motion of quantum objects, Heisenberg's new kinematical elements referred to, and properly predicted, the probabilities of discontinuous and, given that we can only estimate their probabilities, noncausal transitions (Bohr's quantum jumps) from one quantum energy level to another. It is preferable to speak of "energy levels," since quantum mechanics offers no orbital or other representation of what physically happens at these stationary states. The concept of orbits or more generally trajectories, initially retained by the old quantum theory, eventually posed insurmountable problems for it and was abandoned by Heisenberg. Heisenberg's approach thus also contained the seeds of the idea of probability amplitudes, only which, rather than probabilities themselves (derived from amplitudes via Born's rule), enter the equations of quantum mechanics.

In his initial commentary on Heisenberg's paper in 1925, Bohr said:

> In [Heisenberg's] theory the attempt is made to transcribe every use of mechanical concepts in a way suited to the nature of the quantum theory, and such that in every stage of the computation only directly observable quantities enter. In contrast to ordinary mechanics, the new mechanics does not deal with a space-time description of the motion of atomic particles. It operates with manifolds of quantities which replace the harmonic oscillating components of the motion and symbolize the possibilities of transitions between stationary states in conformity with the correspondence principle. These quantities satisfy certain relations which take the place of the mechanical equations of motion and the quantization rules [of the old quantum theory]. ([7], v.1, p. 48)

In other words, in Heisenberg's scheme one does not deal with the evolution of a quantum objects, such as an electron in the atom, either physically or mathematically. There is not even a transformation of the state under the action of the energy operator, and hence no physical evolution, however related to this transformation. There are only stationary states of an electron at certain energy levels, each numerically determinable by a measurement, and the probabilities, calculated from the matrix formalism, of the discontinuous transitions from a given state to other possible states, the transition physically manifest in spectra. It is true that, given that it is mathematically equivalent to Schrödinger's scheme or Dirac's q-number scheme, matrix mechanics can be adjusted to incorporate, *mathematically*, the general transformation of the state under the action of the energy operator. It is also true, however, that one can use the transformation theory, along with Born's probabilistic interpretation of the wave function as part of the theory, to re-interpret Schrödinger's scheme along more Heisenbergian lines of dealing only with the probabilities of measurement outcomes, without referring to any independent behavior of quantum objects. Thus, Born, who never accepted the view that the independent behavior of quantum systems is causal, returned quantum mechanics to its Heisenbergian epistemological roots.

My main point is that this type of mathematical machinery was not part Heisenberg's initial thinking, since his mathematics only reflected, in terms of probabilistic predictions, physically discontinuous and a-causal transitions ("jumps") from one state to another, from one set of quantum numbers to another. In other words, Heisenberg's mathematics, including its matrix and hence noncommutative nature (his great discovery), came from physics, from what can be actually observed, and not what happens between observations. It did not reflect any conjectural thinking concerning what happens at the *unobservable* quantum level, how electrons actually, mechanically "behave," especially while in stationary states. Nor was Heisenberg trying or aiming to integrate, as Dirac was a bit later, stationary states into the overall scheme in quantum-mechanical mathematical terms, to find quantum-mechanical mathematical analogues for the corresponding classical quantities. Hence, the profound physical difference not only from Schrödinger's original approach, but also from the kind of thinking, influenced by Schrödinger, concerning quantum causality in question here.

Schrödinger's wave scheme, then, especially once the time-dependent Schrödinger's equation was introduced, and, around the same time, Dirac's q-number scheme allowed one also to integrate the orbital rather than only the transitions frequencies, or again, the quantum analogues of orbital frequencies, into their quantum scheme. On the one hand, this was a major theoretical advance, amplified by Born's probabilistic interpretation of the wave function, which generalized Heisenberg's rules for the probabilities of transitions of electrons between the energy levels. Dirac's and Jordan's transformation theory allowed one to combine both with a single scheme. On the other hand, this mathematical representation and the transformation theory were also conducive to the view that the independent physical behavior of quantum systems, as reflected mathematically in the transformation of a state vector, within the formalism, is causal.

This shift is particularly interesting in the case of Bohr and Heisenberg, both in his uncertainty-relations paper and following Bohr's Como argument (rather than Schrödinger's approach) in *The Physical Principle of the Quantum Theory*, given their sharply different previous positions, defined by the matrix version of quantum mechanics. Heisenberg's case is especially revealing in this respects, because Heisenberg's paper introducing the uncertainty relations was in part aimed as a critique of Schrödinger's approach. It also strongly attacked Schrödinger's critique of matrix quantum mechanics as concerns visualization or/as intuitiveness [*Anschaulichkeit*], including, one might add, a causal visualization. He said: "Schrödinger calls [matrix] quantum mechanics a formal theory of frightening, indeed repulsive, unvisualizability and abstractness. Certainty one cannot overestimate of the mathematical (and in that sense also physical) mastery of the quantum-mechanical law as achieved by Schrödinger. However, as regards question of physical principle, the popular visualizability of wave mechanics has, in my opinion, led away from the straight path which has been marked by the papers of Einstein and de Broglie, on the one hand and by the work of Bohr and [matrix] quantum mechanics, on the other ([6], p. 82, n.).

And yet, as part of his alternative form of visualization of the quantum situation via the uncertainty relation, Heisenberg expressly appealed to the idea of "disturbing" the *independent* (and possibly causal) behavior of quantum objects by an act of observation in his gamma-ray microscope thought experiment. While Bohr, famously, corrected some of Heisenberg's argumentation even before the paper was published, he, at this point, did not critique the ingredients under discussion here and indeed adopted them in his own discussion of the experiment in the Como lecture. Eventually Bohr rejected the use of the term "disturbance" as implying that there is some in principle describable or even conceivable, let alone, causal, independent quantum behavior, which is then disturbed by observation. He speaks instead, more cautiously, of our *interference* with quantum objects, which interference affects their subsequent behavior and, hence, our probabilistic predictions concerning future experiments (e.g. [7], v. 2. p. 56). In this view, nothing can any longer be said about the independent behavior of quantum objects themselves. As he said, "In quantum mechanics, we are not dealing with an arbitrary renunciation of a more detailed analysis of atomic phenomena [i.e. an analysis reaching to quantum objects themselves and their behavior], but with a recognition that such an analysis is *in principle* excluded" ([7], v. 2, p. 62). That one cannot under these conditions speak of any quantum-level causality, again, follows automatically. Bohr moves to this type of view (it took awhile to crystallize it) shortly after advancing his Como argument under the impact of his exchanged with Einstein. Einstein himself vastly preferred Schrödinger's approach to that of Heisenberg, even though he did not think that quantum mechanics, in whatever version, offered a satisfactory approach to the quantum "enigma," as he called it. Nor ultimately did Schrödinger. That did not deter Bohr from pursuing his approach, although he was much chagrined by Schrödinger and, especially, Einstein's rejection of his argument, as he noted on the same occasion ([7], v. 2, p. 62).

Not very many followed Bohr or thought it necessary to go, in his words, that "far in renouncing customary demands as regards the explanation of natural phenomena," ([7], v. 2, p. 63). Some have followed, however, or have arrived at this type of thinking on their own, specifically a number of quantum information theorists ([3]). Consider, for example, how the uncertainty relations look from this perspective, according to A. Peres, one of the founders of quantum information theory:

> An uncertainty relation such as [$\Delta q \Delta p \cong h$] is not a statement about the accuracy of our measuring instruments. On the contrary, its derivation assumes the existence of *perfect* instruments (the experimental errors due to common laboratory hardware are usually much larger than these quantum uncertainties). The only [available?] correct interpretation of [$\Delta q \Delta p \cong h$] is the following: If the *same* preparation procedure [defined by the classical control of measuring instruments] is repeated many times, and is followed either by a measurement of [q], or by a measurement of [p], the various results obtained for [q] and for [p] have standard deviations, [Δq] and [Δp], whose product cannot be less than [h]. There is never any question here that a measurement of [q] "disturbs" the value of [p] and vice-versa, as [is] sometimes claimed. These measurements are incompatible, but they are performed on *different* [*quantum objects*] (all of which are identically prepared) and therefore these measurements cannot disturb each other in any way. An uncertainty relation ... only reflects the intrinsic randomness of the outcomes of quantum tests. ([11], p. 93)

As Heisenberg and Bohr do, Peres sees the uncertainty relations as experimentally given, as a law of nature, along with, and correlatively to, the probabilistic nature of our quantum predictions, since, as I said, identically prepared experiments lead to different outcomes. Accordingly, any theory that would properly cover the situation is bound to be probabilistic. Quantum mechanics does this, and the uncertainty relations can be automatically derived from it, as Heisenberg showed as part of his discovery of them.

4. Conclusions

I would like to close by reiterating that I am not saying that the assumption that the independent behavior of quantum systems is causal in physical terms is necessarily wrong. I am saying, however, that, to my best knowledge, it has not been given an adequate explanation or justification thus far, and that, for the reasons explained above, I am not sure that such an explanation is possible in the case of the standard quantum mechanics. Accordingly, if one wants a causal or, to begin with, realist theory of quantum objects and processes, the search for alternative to quantum mechanics is understandable from this viewpoint as well (this search also has other reasons). On the other hand, for those of us for whom such a theory is not an imperative or at least not a categorical imperative, as Kant called it, quantum mechanics suffices. In other words, it suffices for those of us whose imperatives are different: it is difficult not to have any imperatives, although it is possible not to have categorical ones.

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