

On Lacan and Mathematics

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Imaginary roots are a subtle and wonderful resort of the divine spirit, a kind of hermaphrodite between existence and non-existence (inter Ens and non Ens Amphibio).

Leibniz

As I was, in October of 1997, contemplating the idea of this essay, an e-mail arrived from a physicist friend of mine. The message concerned Lacan and reflected the recent events sometimes referred to as the “Science Wars,” in the wake of Paul Gross and Norman Levitt’s book *Higher Superstition: The Academic Left and its Quarrel with Science* and physicist Alan Sokal’s hoax article published in the journal *Social Text*. Sokal and his co-author, a Belgian physicist Jean Bricmont, then just published their book, *Impostures intellectuelles*, devoted to the misuse or even abuse (alleged by the authors) of mathematics and science by some among leading French intellectuals. Lacan’s work appears to be seen by the authors as arguably the most notorious case of this alleged abuse, and some of Lacan’s statements they cite were bound to attract a special attention, which prompted my friend’s e-mail. It said:

Does Lacan really talks about the penis and the square root of minus 1 with a straight face, as reported in Saturday’s NY Times article on the Sokal and Bricmont book? And if so is there any way to view this as anything but a complete nonsense? I am testing the limits of my open-mindedness. These seems to go beyond them.

I shall, by way of replying to these questions here, sketch an argument applicable to Lacan’s usage of mathematical ideas other than imaginary numbers (such as the square root of -1), for example, those borrowed from topology and mathematical logic, two other prominent areas of mathematics ventured into by Lacan. I shall deal directly, however, only with imaginary numbers and Lacan’s argument, leading to the statement in question, in “The Subversion of the Subject and the Dialectic of Desire in the Freudian Unconscious” (*Écrits*). It is worth noting at the outset that, as a psychoanalytically informed reader would be aware (my physicist friend wasn’t), the erectile organ of Lacan’s statement is not the same as the penis.

It may not even quite be seen as the phallus, defined by Lacan in the same essay as “the image of the penis,” but instead as in turn the *image* of the phallus – the image of the *image* of the penis.¹

I shall more or less bypass the “Science Wars” debates here.² Regardless of potential problems with – the work of Lacan and other authors under criticism, arguments against them by Gross and Levitt, Sokal and Bricmont, and other recent critics in the scientific community can hardly be seen as ethically, scholarly, and intellectually appropriate, or indeed as in accord with the spirit of scientific inquiry itself. Let me hasten to add that I here refer specifically to the “Science War” criticism, such as that by the authors just mentioned, and not to the views or opinions concerning these subjects of the scientific community in general. Indeed, it is my view that such critics as Gross and Levitt or Sokal and Bricmont do not represent, and should not be seen as representing, science and scientists. The criticism of these particular authors is disabled by: a) their lack of necessary familiarity with specific subject matter, arguments, idiom, and context of many works they criticize; b) their inattentiveness to the historical circumstances of using mathematical and scientific ideas in these works; c) their lack of the general philosophical acumen, which is necessary for understanding most of the works in question; and d) their insufficient expertise in the history and philosophy of mathematics and science. These factors, which are, as will be seen, manifest in Sokal and Bricmont’s “treatment” of Lacan, make any constructive criticism virtually impossible. Lacan’s statement in question is a part of a complex psychoanalytical and philosophical conceptual assemblage, and of an equally complex textual network. It makes little, if any, sense without taking both and their context into account, or without translating Lacan’s ideas into a more accessible idiom. Even such translations are bound to retain considerable complexity for the general audience. The psychoanalytical or even philosophical substance of Lacan’s argument requires no mathematics as such, which one can “decouple” from this argument by “translating” Lacan’s statements containing mathematical references into statements free from them. The reverse, however, cannot be done: one cannot decouple “Lacan” from the “mathematics” he uses. One cannot meaningfully read Lacan mathematical or quasi-mathematical statements by extracting them from their psychoanalytical and philosophical content and context.

¹ Indeed, as will be seen, if considered as the square root of -1 of Lacan’s “algebra” (which, I shall argue here, is not mathematics), “the erectile organ” in Lacan is a formalization of the *image* of the *image* of the penis.

² I have addressed the subject elsewhere, in “But It Is Above All Not True”: Derrida, Relativity and the ‘Science Wars,’” and “On Derrida and Relativity: A Reply to Richard Crew.”

Admittedly, the task of reading Lacan is not easy, in view of their idiosyncracies, convolutions, fragmented or even spasmodic textual economy, and other complications, in part resulting from the fact that one usually deals with transcripts of oral presentations. Luckily, I need not deal with these problems here, since I need not fully spell out Lacan’s psychoanalytical argument for my purposes. My argument and claims are of a different nature. They concern, first, the *way* mathematics is used in Lacan, not the mathematical accuracy of his mathematical references (although, as will be seen, Lacan is far from being as bad on this score as some of his critics claim), and, second, *philosophical*, rather than psychoanalytical, dimensions of Lacan’s work. More generally, I am interested in the interconnections between, on the one hand, philosophical and, on the other, mathematical ideas. I am also interested in the structure of *philosophical concepts* as such, and Lacan’s concepts will be here considered as philosophical concepts. Such concepts often entail an engagement of different disciplines and fields of inquiry. The term “concept” itself is used here in the sense Deleuze and Guattari give it in *What Is Philosophy?*, rather than in any common sense of it, such as an entity established by a generalization from particulars, or indeed “any general or abstract idea,” as Deleuze and Guattari argue, via Hegel (*What Is Philosophy?* pp. 11–12, 24). A philosophical concept is an irreducibly complex, multilayered structure – a multi-component conglomerate of concepts (in their conventional sense), figures, metaphors, particular (ungeneralized) elements, and so forth. This complexity is, I argue, manifest in Lacan’s concepts. Psychoanalytical dimensions of Lacan’s conceptual economy is a separate matter, which I will not be able properly to consider here, although they are of course crucial to Lacan’s work.³ This essay, thus, concerns primarily the philosophical component of Lacan’s discourse, and the role of mathematics there will be considered accordingly. I shall return to the question of the relationships between mathematics and philosophy in the end of this essay. It may be recalled here, by way of justifying this approach, that Lacan’s essay in question was a contribution to a philosophical conference entitled “La Dialectique.” Its first reference is Hegel and *The Phenomenology of the Spirit*. Hegel is one of the key, even if mostly implicit, subjects of the essay, indelibly inscribed in the phrase “the dialectic of desire” of its title. The structure of philosophical concepts is, in my view,

³ The secondary literature on the subject is, of course, immense. See, in particular, Guy Le Gaufey’s contribution to this volume and his work in general, which offers a cogent and rigorous treatment of the subject, including as concerns the connections between the Lacanian psychoanalysis and the philosophical problematics of modern mathematics and science.

where Lacan's usage of mathematics most fundamentally belongs and the best perspective from which this usage can be meaningfully considered.

From this perspective, there is a way, at least one way, to argue that the statement in question and the *connections* (rather than an identification or even a metaphor) between the erectile organ and the square root of -1 as making sense. Ironically, in order to pursue this argument one has indeed to know something not only about Lacan but also about imaginary and complex numbers, and their history. On that score, Sokal and Bricmont appear to be rather less informed than they could have been and, even more ironically, in some respects perhaps less informed than Lacan was. They appear to be taking complex numbers for granted as a self-evident mathematical object. The situation, however, is more complicated, both mathematically and, especially, philosophically.

Accordingly, it may be useful to review basic facts concerning imaginary and complex numbers, and numbers in general.⁴ Given their crucial role in defining first irrational and then complex numbers, square roots will be my primary focus. Let us recall, first, that the square root is the mathematical operation reversing the square of a number. The square of 2 is 4, the square root of 4 is 2, or of course -2 , which is of some significance here. I hope I will be forgiven for being so elementary, but I want even those who know nothing, or forgot everything, about mathematics – unlike Plato we do admit them into the Academy these days to understand my argument. Besides, things get more complicated rather quickly. Thus, the square root of 2 is already a far more complex matter, both mathematically and philosophically, although it is of a rather straightforward mathematical genealogy. One needs it if one wants to know the length of the diagonal of the square. This is how the Greeks discovered it. If the length of the side is 1 the length of the diagonal is the square root of 2. I would not be able to say – nobody would – what its exact numerical value is. It does not have an *exact* numerical value: it cannot be represented (only approximated) by a finite, or an infinite periodical, decimal fraction, and accordingly, by a regular fraction – by a ratio of two whole numbers. It is what is called an irrational number, and

⁴ The discussion to follow is indebted to a number of technical and semi-technical accounts, in particular, Elie Cartan, "Nombres complexes. Exposé, d'après l'article allemand de E. Study (Bonn)" (which, along with Study's article itself appears to shape most accounts of the subject by mathematicians), Reinhold Remmert's chapter, "Complex Numbers" in Heinz-Dieter Ebbinghaus, et al, *Numbers* (which, too, follows Cartan rather closely) and David Reed, *The Figures of Thought*. I am grateful to David Reed for especially helpful discussions of several key mathematical and philosophical questions to be addressed here. I also grateful to Barry Mazur, Michael Harris, and David Mermin for productive exchanges on these subjects.

it was the first or one of the first such a number – or (they would not see it as a number) mathematical object – discovered by the Greeks, specifically by the Pythagoreans. The discovery is sometimes attributed to Plato's friend and pupil Theaetetus, although earlier figures are mentioned. It was an extraordinary and, at the time, shocking discovery – both a great glory and a great problem, almost a scandal, of Greek mathematics. The diagonal and the side of a square were *mathematically proven* to be incommensurable, their "ratio" irrational. The very term "irrational" – both *alogon* (outside logos) and *arreton* (incomprehensible) were used – was at the time of its discovery also used in its direct sense. The discovery, made by the Pythagoreans against themselves, may be seen as the "Gödel theorem" of antiquity.⁵ It undermined the Pythagorean belief that, as everything rational, the harmony of the cosmos was expressible in terms of (whole) numbers and their commensurable ratios (proportions). This discovery was also in part responsible for a crucial shift from arithmetical to geometrical thinking in mathematics and philosophy. For, while the diagonal of the square was well within the limits of geometrical representation, it was outside those of arithmetical representation, as the Greeks conceived of it.

We now call fractions and whole numbers *rational* numbers. Rational numbers together with irrational numbers (such as roots of all powers and still other irrational numbers, such as *pi*, which cannot be represented as roots or even as solutions of polynomial equations) are called *real* numbers. Real numbers can be either positive or negative, or zero (the latter, incidentally, unknown to the Greeks).⁶ The main reason for using this term is that real numbers are suitable for measurements, in particular of the length of line-segments, straight or curved, in the material world around us, the world of things that are, or appear to be, real. We can also represent and visualize them as points on the continuum of the straight line. We can do all standard arithmetic with real numbers and generate new real numbers in the process – add them, subtract them, multiple them, divide them, and so forth. (The same is true for rational numbers, but, because of division, not for whole numbers.)

Now "there's the rub" – the square root. If a number is positive, there is no problem. We can always mathematically define its square root and calculate it to any degree of approximation. However (this is the rub), in the domain of real number the square root can be defined, can be given

⁵ It is worth noting here that the method used in the proof – an argument based on the so-called excluded middle (still the most common and effective form of mathematical proof) – was originally used by Parmenides and Zeno, and formed one of the foundations of dialectic, the invention of which is credited to them.

⁶ It is worth observing here that the mathematical legitimacy of negative numbers, too, was a matter of a long debate, extending to the eighteenth century.

mathematical sense, only for positive numbers. This is so for a very simple reason (recall that the square root is the reversal of the square): whether you square a positive or a negative number – that is, multiply any number by itself – the result is always positive. Thus, 2 by 2 is 4, and -2 by -2 is also 4, and the same is for 1 and -1 – the square of both is 1. In a sense, square roots of negative numbers, such as -4 or -1, do not exist, at least in the way real numbers exist, or appear to exist. This is why, when introduced, they were called imaginary, and sometimes even impossible, numbers.

Why bother, then? First, from early on it appeared (correctly) that one could operate with square roots of negative numbers as with any other numbers – add them, subtract them, multiply them, divide them, and so forth. Moreover, the impossible square root of -1 appears most naturally in the simplest algebraic equations – such as $X_2 + 1 = 0$. This is how the square root of -1 and other “imaginary quantities,” as they were called, made their first appearance during the Renaissance. Indeed roots of negative numbers naturally emerge throughout mathematics. In short, on the one hand, mathematics at a certain point appeared to need to be able to deal with square roots of negative numbers, beginning with -1. On the other hand, it was clear that such “numbers” could not be any numbers already available.

It took the mathematical community a while (nearly two centuries) to accept the mathematical legitimacy, let alone reality, of these new numbers, and rigorously to define them. Their status as mathematical objects has remained in question for much longer, especially in philosophical terms of their mathematical reality, or as concerns their possible role in describing material reality, as in physics (which remains a complex question to this day). The resolution required a very great and protracted effort and the best mathematical minds available. It was achieved by a seemingly simple, especially from our vantage point, but in truth, at least at the time, highly nontrivial stratagem – by formally adjoining the square root of -1 to real numbers. This “simple” resolution amounted to the introduction of new numbers and of a new kind of numbers, which could be manipulated in the manner of all other numbers. This is why they were first called first imaginary (and sometimes impossible) numbers, and then complex numbers, which are entities a little more complicated than square roots, although they have been known for just as long. The square root of -1, also called i , is the simplest such number. Other complex numbers are written in the form $A + Bi$, where A and B are real numbers (in the case of imaginary numbers $A = 0$). The square root of -1, may be seen as the fundamental element, by adjoining which to the old domain of real numbers the new domain is generated. Just as real (or rational) numbers do, complex numbers form what is in mathematics called a “field” – a multiplicity with whose elements one can

perform standard arithmetical operations with the outcome being again an element of the same multiplicity.

With the introduction of *complex* numbers it also became possible to represent the whole system on the regular real (in mathematical sense) two-dimensional plane, with the line representing real numbers serving, symbolically, as the horizontal axis and the line representing imaginary numbers (strictly square roots of negative numbers) as the vertical axis in the Cartesian-like mapping of the plane. The square root of minus 1 or i would be plotted at the length equal to 1 above zero on the vertical axis. In this representation the domain of complex numbers is two-dimensional, in contrast to the one-dimensional domain of real numbers as represented by the line. This representation is sometimes called the Argand plane, although it was a great, one of the greatest ever, mathematician Karl Friedrich Gauss, who legitimized it as part of giving legitimacy and perhaps reality to complex numbers.⁷ “You made possible the impossible” was a phrase (which also refers to complex numbers) used in a congratulatory address on the 50-year jubilee of his doctorate. In 1977 the German Post Office issued a stamp illustrating the Gauss-Argand plane to celebrate the bicentenary of his birth in 1777 (obviously a very lucky sequence of numbers).

The picture, however, is not without complications, although I can only indicate some among the complexities involved, not offer a full argument here. In particular, the real two-dimensional plane is – this is a mathematical fact – mathematically *not* the same object as complex numbers. Complex numbers, such as the square root of -1, and their operations, may be “represented” and “visualized” geometrically, via the two-dimensional real plane, only as a kind of *diagram* (the Gauss-Argand “plane”) – a schematic illustration, comprehensive (point by point) as it is – but *not* in themselves, not as mathematical objects with their actual (individual and collective) mathematical properties.⁸ The main reason for this is that a real (in mathematical sense) point on the two dimensional plane, for example, with Cartesian (now indeed Cartesian rather than Cartesian-like) coordinates, is not a “number.” In contrast to real numbers and their geometrical representation as the (real) line, there is no “natural” way to conceive of all necessary arithmetical operations, in particular multiplication or division

⁷ Gauss was also one of the discoverers of the non-Euclidean geometry, which discovery he, however, suppressed for twenty years for his fear of being laughed at by philistines, or perhaps for the reason the Pythagoreans thought it wise to conceal the existence the irrationals. Gauss did not think, as it happened rightly, that the world was quite ready for this.

⁸ In the language of mathematics, these two objects are not isomorphic, insofar as one can assign (as will be seen, one can) to the real plane an algebraic structure at all.

(addition and subtraction are not a problem) – that is, in the way the real line is “naturally” converted into arithmetics in the case of real number. As a result, complex numbers as such cannot in all rigor be seen as *represented* as points on the two-dimensional real plane and indeed are epistemologically unavailable as a visualizable or, more generally, geometrical, object. Their properties can of course be spelled out and rigorously comprehended algebraically. In the sense of algebraic representation, there is no epistemological difference between real and complex numbers (although there are fundamental differences in algebraic properties of two domains). Ultimately, complex numbers may remain not only imaginary, but, at least geometrically, strictly unimaginable. They (in the ultimate structure of their properties and attributes) are certainly nonvisualizable as such, at least not in the way real numbers are. Epistemologically, at least in terms of its geometrical representability, the square root of -1 or, more accurately, the signifier “the square root of -1 ” signals – “represents” – the ultimate lack of geometrical representation. It is something that in itself is geometrically unvisualizable or unrepresentable, or, one might say, geometrically un-epistemologizable. This radical epistemological complexity of mathematical complex numbers explains the ambivalent attitude toward them on the part of the key figures involved in their discovery or creation.⁹

⁹ Thus, Augustin-Louis Cauchy (1789–1857), a contemporary of Gauss and a great mathematician in his own right, had reservations concerning the geometrical representation of complex numbers throughout his life. He considered them as purely symbolic (algebraic) entities and at one point even attempted a general mathematical definition of “symbolic expression” in explaining his attitude – to some discontent among his colleagues. (See Remmert’s commentary in Ebbinghaus, et al., *Numbers*, pp. 62–63). This situation and a more general problem of geometrical representation of post-18th century mathematics that it reflects has far reaching implications for modern mathematics, such as topology, of some of which Lacan appears to be aware. It is also worth noting here that certain, indeed quite radical, epistemological complexities are involved in the case of real numbers as well, or indeed of all numbers. I cannot consider these subjects here. Lacan aside, however, the question may well be: Do we know what is the number like the square root of -1 , or indeed -1 (which, as I said, gave mathematicians some pause even as late as XVIIth century), or for that matter $1?$ It may be recalled that Frege once said that it is scandalous that we do not know what numbers really are. Lacan was aware of some of these complexities, as is clear from his comments on foundations of mathematics, including some of those cited by Sokal and Bricmont in *Impostures intellectuelles* (pp. 32–38). These passages cause Sokal and Bricmont much aggravation. In truth, however, they are at worst harmless, and often there is nothing especially wrong with them – if, again, one tries to understand Lacan’s actually argument where these passages are used. In general, Lacan and other radical thinkers in question in recent debates appear to be more aware of and attentive both to the philosophical dimension

It could appear “wondrous strange” indeed, and to some outright bizarre, that the theory of complex numbers has anything to do with the erectile organ. Given, however, the preceding discussion and some knowledge of Lacan, it is not so difficult to see that Lacan’s “formula” is in fact not so strange. The epistemological point just made concerning complex numbers – their ultimately unavailability to visualization and perhaps any geometrical conceptualization, while they seem to be represented as points on the two-dimensional real plane – gives one a hint here. The erectile organ may be seen as theorized by Lacan as a symbolic object (also in Lacan’s sense of the symbolic), specifically a *signifier* (in Lacan’s sense), that is epistemologically *analogous* to the signifiers one encounters in the case of complex numbers, and specifically the square root of -1 . Within the Lacanian psychoanalytic configuration, any image, in particular visual image, of the erectile organ, including that of an “erectile organ,” can only be an *image of the signifier* – the *signifier*, not the *signified*. (I shall further comment on this point presently.) This signifier itself is fundamentally, irreducibly non-visualizable. At the limit, this signifier – that is, its ultimate structure of, once again, the signifier designated as the erectile organ – may be inconceivable by any means, which epistemology or de-epistemization, and specifically de-visualization, are crucial to most of Lacan’s key concepts. Indeed, this signifier is in fact or in effect unnameable, for example, again, as the erectile organ, or the phallus, which, as I said, may not be the same as the erectile organ within the Lacanian economy of subjectivity and desire. That is, we can formally, “algebraically” manipulate its image or images, or names, or further formal symbols associated with it, just as we can formally manipulate complex numbers within their mathematical system, which Lacan’s “algebra” in part “mimics” but to which it is not identical. At the same time, however, we do not really know and perhaps cannot in principle conceive, at least from within the Lacanian psychoanalytical situation (defined by this economy of inaccessible signifiers), what the erectile organ really is as a *signifier* and what its properties are, if can speak in terms of properties here. The *image* of this signifier, and in particular its visual image, would, then, be analogous to the geometrical, hence visualizable, representation of complex numbers, and in particular of the square root of -1 , of which the erectile organ becomes an *analogon* within the Lacanian psychoanalytic “system,” rather than being a *mathematical* imaginary

of the mathematical concept in question and their history themselves and to the philosophical thought (often in turn quite radical) of the key mathematical and scientific figures involved than their recent critics in the scientific community. This difference, too, is far from irrelevant to the nature of the debates in question.

number.¹⁰ The situation may even be more subtle, insofar one may need to deal with further levels of formalization – that is, within still other “formal” symbols and structures associated with signifiers, such as the erectile organ – at which the analogy in question actually emerges. Let me stress that (whether one is within Saussure’s or Lacan’s scheme of signification) in question here is the irreducible inconceivability of the erectile organ as the *signifier*, not the *signified*. Its signified (such as, in Saussure, the concept behind it) and its referent, whatever they may be, may be in a certain sense even more “remote” and “inaccessible” or inconceivable. One would still need, however, to think in terms of the ultimate inaccessibility (which is not to say identity in terms of their functioning) of all three – the signifier, the signified, and the referent, which should be considered in the register of the Lacanian Real.¹¹

My main argument here may be summarized as follows. Both the signifier of the erectile organ in the Lacanian psychoanalytic field and the square root of -1 , i , in mathematics may be seen as fundamental formal, *symbolic*, entities that enable an introduction of, and may be seen as structurally generating, two new symbolic systems – that of the Lacanian psychoanalysis (his (re)interpretation of Freud’s Oedipal economy) and the field of complex numbers in mathematics. In each case, the introduction of these new symbols allows one to deal with problems that arise within previously established situations but that cannot be solved by their means: a pre-analytic situation, or a more naively (for example, by way of misreading Freud, conceivably, to a degree, even by Freud himself) constructed analytic situation in psychoanalysis (where one needed, and in the previous regime could not, approach certain particular forms of anxiety), and the system of real numbers in mathematics (where one needed but could not rigorously define complex numbers in order, for example, to solve certain polynomial equations). In both cases, the philosophical-epistemological status of these new symbolic systems is complex. In particular, in question are: a) the extent to which such systems represent or otherwise relate to, respectively, psychological/psychoanalytic and mathematical reality (with the question of material reality in the background in both cases – the question of the Real in Lacan’s case); and b) the extent to which the properties of such symbolic

¹⁰ Here and below the term “analogon” may also be understood in its Greek sense, as connoting a parallel or “proportionate” relation, rather than identity, of one *logos* (here as “discourse”) to another.

¹¹ See Note 20 below. It can be argued (although I cannot pursue this argument here) that the *epistemologically* analogous triple inconceivability is also encountered in modern, and perhaps all, mathematics, in particular in the mathematics of complex numbers.

systems and of their elements, such as what is designated as the square root on -1 , i , in mathematics, or the erectile organ in the Lacanian analysis, can themselves be accessed and specifically visualized by means of images, such as the geometrical representation of complex numbers or the image we form perceptibly or configure theoretically (and these are subtly linked in turn) of the erectile organ in the Lacanian psychoanalytic situation.¹²

In mathematics, these complexities, historically reflected in the term “imaginary numbers,” are, I would argue, not altogether resolved even now, although since and following Gauss most mathematicians stopped worried philosophically. Leibniz may well have given the problem its most glamorous expression: “Imaginary roots are a subtle and wonderful resort of the divine spirit, a kind of hermaphrodite between existence and non-existence (inter Ens and non Ens Amphibio)” (*Math. Schriften* 5: 357).¹³ Perhaps Descartes, who was one of the first to give serious consideration to imaginary roots and their nature, and indeed was first to use the very term “imaginary” (Cartan, “Nombres complexes,” 330 n.3), and who was the inventor of analytic geometry (which fundamentally relates geometrical and algebraic mathematical objects), should be given the last word here: “One is quite enable,” he said, “to visualize imaginary quantities.”¹⁴ Unless, the last word is Lacan’s, who in “Desire and the Interpretation of Desire in *Hamlet*,” says: “the square root of -1 does not correspond to anything that is subject to our intuition, anything real – in the mathematical sense of the term – and yet it must be conserved, along with its full functioning” (29). This may need to be more precisely stated, but is in essence right; and this statement grounds and guides my analysis here. It may be seen as an updated rendition of Leibniz’s early assessment: “From the irrationals are born the impossible or

¹² One must keep in mind here the difference between complex numbers, or indeed any mathematical object, and the Lacanian system in question as concerns their respective relationships with materiality (whether one sees the latter in terms of material reality in the classical sense or not). In the case of the Lacanian system, the relationships between the symbolic and the material are more immediately germane, somewhat similarly (although not identically) to the way mathematical models function in physics. In the case of mathematics, its symbolic systems may be seen as more or less independent of material objects – that is, such as those considered in physics, since other forms of materiality are irreducible in mathematics as well, and of course there is still the question of nonmaterial mathematical (or for that matter Lacanian) reality. I have considered the question of mathematical reality and its relations to physics in “Complementarity, Idealization, and Limits of the Classical Conceptions of Reality” (pp. 161–67).

¹³ Cited by Remmert (Ebbinghaus, et al., *Numbers*, p. 58).

¹⁴ The statement occurs in “La géométrie,” published in 1637; it is cited by Remmert (Ebbinghaus, et al., *Numbers*, p. 58).

imaginary quantities whose nature is very strange but whose usefulness is not to be despised," although numerous subsequent statements by leading mathematicians can be cited.¹⁵

In the same passage Lacan also speaks of imaginary numbers as "irrational." The passage is cited both in Sokal's hoax article and *Impostures intellectuelles* as an example of Lacan's confusion of irrational and imaginary numbers. Lacan's usage, however, does not appear to me due to his lack of understanding of the difference between real irrational numbers and imaginary numbers, imputed to him by Sokal and Bricmont. Instead it may be seen as a reflection of his sense of imaginary numbers as an extension of the idea of irrational numbers – both in the general conceptual sense, extending to its ancient mathematical and philosophical origins, as considered earlier, and in the sense of modern algebra – which is correct, and displays, conceptually, a better sense of the situation on Lacan's part than that of Sokal and Bricmont. Their description of irrational and imaginary numbers in their book is hardly edifying as concerns the substance and the beauty of the subject. It is also imprecise and misleading insofar as it suggests that there is no connection between irrational and imaginary numbers. Indeed, the claim even more strongly that they have nothing to do with each other (*Impostures intellectuelles*, p. 31). This is simply wrong. The profound connections between them define modern algebra. Certainly, complex numbers, beginning with i , are *irrational* numbers as the latter are defined by Sokal and Bricmont (as unrepresentable by a ratio of two whole numbers): no real fraction can be found to represent them, since no real number of any kind can represent them. The latter is a minor and trivial point, and one can hardly think that Sokal and Bricmont could be unaware of it. In general, I am not here holding Sokal and Bricmont responsible for their treatment of numbers as such, inadequate and imprecise as it is. They are physicists, not mathematicians or historians or philosophers of mathematics, and it is, in general, not their responsibility to know (or be precise about) mathematics and the philosophy and history of mathematics. It is, however, their responsibility to know those aspects of all three that they consider in Lacan, and, assuming that they do, it is their responsibility to carefully consider and appropriately explain these issues, if they want to criticize Lacan.

The erectile organ of the Lacanian or, Lacan argues (in part "against" Freud himself), already Freudian system is, then, analogous to the mathematical square root of -1 – *analogous*, but *not identical*. Indeed, the proper way of conceiving of the situation is to see the erectile organ, or, again, a certain formalization of it, *as* (as defined by and as defining) "the square root of -1 " of the Lacanian system itself – that is, as an *analogon* of the math-

ematical concept of the mathematical square root of -1 within this system – rather than anything identical, directly linked, or even metaphorized via the mathematical square root of -1 .¹⁶ In a word, the erectile organ *is* the square root of -1 of Lacan's system, the mathematical square root of -1 is *not* the erectile organ. There is no mathematics in the disciplinary sense in Lacan's analysis, only certain structural and epistemological analogies or homologies with the mathematics of complex numbers, most particularly the following. *First* – the structural analogy – the erectile organ, as a signifier, or indeed *the* signifier (in Lacan's sense), belongs to and gives rise to a psychoanalytical system different from the standard one or ones (based on misreadings of Freud, conceivably to a degree by Freud himself), and to a different formalization – "algebra" – of psychoanalysis, a formalization that is more effective both conceptually and in terms of the ensuing practice. *Second* – the epistemological analogy – the erectile organ, as a signifier, or again, *the* (Lacanian) signifier of this system, while and in a sense because it governs the economy of the system, can only be approached by means of tentative, oblique and ultimately inadequate metaphors. It is ultimately inaccessible, along with its signified and its referent, at the limits inaccessible even as that which absolutely inaccessible but definable in terms of independent properties and attributes.

In order to explain the reasons for my argument, I shall sketch here some of Lacan's logic and "algebra," *mimicking* complex numbers, without fully spelling out the structure of Lacan's key concepts – such as the subject, the signifier, desire, or indeed the erectile organ – which would require a perusal of a much larger textual field. Lacan defines a signifier in general, as "that which represents the subject for another signifier," rather than for another subject (*Écrits*, p. 316).¹⁷ The signifier "S" is introduced – first as the "signifier of a lack in the Other [Autre], inherent in its very function as the treasure of the signifier" (316). Ultimately, however, "S" is "the signifier for which all the other signifiers represent the subject: that is to say, in the

¹⁶ To the extent that one can speak of the metaphorical parallel, it operates at the level of two systems themselves. This, let me note in passing, is a classical Lacanian move, and it is often found elsewhere as well. For example, Poe's *The Purloined Letter* (in *Écrits*; the French edition) is read by Lacan as textualizing the scene and indeed the field of psychoanalysis, and is reread by Derrida as the scene of writing in Derrida's sense in "Le facteur de la vérité" (*The Postcard*), as part of his deconstruction of Lacan. In the sense just explained, however, one can also speak of a certain "repetition" of Lacan on Derrida's part, albeit a repetition in the sense of Derrida's *différance* as the interplay of differences and similarities, distances and proximities, and so forth.

¹⁷ See also the discussion in "Of Structure as an Inmixing" (*The Languages of Criticism*, pp. 193–94).

¹⁵ Cited by Remmert (Ebbinghaus, et al., *Numbers*, p. 55).

absence of this signifier, all the other signifiers represent nothing, since nothing is represented only *for something else*" (p. 316). This signifier is argued to "be symbolized by the inherence of (-1) in the whole set of signifier" (p. 316) This signifier is (symbolized as) the -1 of the psychoanalytical system in question in its "algebraic" representation ("algebra" is, again, that of Lacan). "As such, S is inexpressible, but its operation is not inexpressible, for it is that which is produced whenever a proper noun is spoken" (p. 316). In other words, "S" is operationally formalizable and this formalization is expressible.

The radical epistemology delineated earlier emerges already at this point, insofar as "S" is inexpressible as such. However, the *formal* object corresponding to "S" in Lacan's "algebra" – "-1" – is analogous to the epistemology of the mathematical -1, rather than of the square root on -1, which is radically inaccessible (at least to a geometrical representation) even as a formal object.¹⁸ Then, however, another signifier, "s" – the symbolic square root of -1 – is derived, and is defined as that which is radically inaccessible, *unthinkable*, for the subject – both as such, similarly to "S," and, I would argue, more radically, at the level of the corresponding element of the formal "algebra" built by Lacan. More generally, there are two interactive but distinct levels of the economy and epistemology of the signifier in Lacan – more general conceptual level (subject, the phallus, lack, and so forth), which is not quasi-mathematical, and the level of a certain "algebra," which is quasi-mathematical and at which the analogy between this "algebra" and mathematics must be considered. The signifier "s" is not yet equated with the erectile organ at this point. However, a certain radically inaccessible signifier is argued to be inherent in the dialectic in the subject, indeed as *the* signifier ultimately generating this system or, again, yet another formalization of it, as the square root of -1 of this formalization, rather than the 1 or -1 ("S") of the system. According to Lacan, "This [i.e. that which is designated or, again, formalized as the square root of -1] is what the subject lacks in order to think himself exhausted by his *cogito*, namely that which is unthinkable for him [although, we might add, appears as representable to him]" (p. 317). "But," Lacan asks next, "where does this being, who appears in some way defective in the sea of proper names originates?" (p. 317)

In order to answer this question, Lacan maps the passage from the imaginary to the symbolic order, especially, as regards the phallic imagery. First, thanks to Freud's "audacious step," the phallus is argued to acquire the privileged role in the overall economy of signification in question, via

¹⁸ See, however, Note 10 above. The question, crucial here, of "negativity," in psychoanalytical or (via Hegel) philosophical terms, in Lacan would require a separate discussion.

the castration complex. Here one must keep in mind the difference between Freud and Lacan insofar as the Lacanian economy of the signifier (replacing Freud's "signified") is concerned, in particular as it relates to the phallus and the difference between the phallus (as a Freudian signified) and what Lacan here designates as the erectile organ as a signifier. The latter, moreover, may need to be seen as formalized yet further as "the square root of -1," thus adding yet another "more distant" level of signification.¹⁹ Then, moving beyond, if not against, Freud, Lacan argues as follows:

The jouissance [associated with the infinitude involved in the castration complex in Freud]... brings with it the mark of prohibition, and, in order to constitute that mark, involves a sacrifice: that which is made in one and the same act with the choice of its symbol, the phallus.

This choice is allowed because the phallus, that is, the image of the penis, is negativity in its place in the specular image. This is what predetermines the phallus to embody the *jouissance* in the dialectic of desire.

We must distinguish therefore between the principle of sacrifice, which is symbolic, and the imaginary function that is devoted to that principle of sacrifice, but which, at the same time, masks the fact that it gives it its instrument. (*Écrits*, p. 319)²⁰

It follows, according to Lacan, that it is the erectile organ – the image or better the signifier as an un-image of the phallus, and thus un-image of the image of the penis (in the Lacanian symbolic order) – that is subject to the equation of signification at issue.²¹ It is, then, as such and only as such that the erectile organ is the square root of -1 – that is, as "the square root"

¹⁹ One can speak of "distancing" here only with considerable caution. For, although a certain efficacious materiality of the Lacanian Real can be seen as, in a certain sense, more "remote," it cannot be postulated as existing by itself and in itself, as absolutely anterior, prior to or otherwise independent of signification. Hence, it cannot be seen as something from which the distance of signifiers can be unequivocally "measured." One can speak in these terms only provisionally. Nor can the overall efficacy of the Lacanian signification can be contained by this materiality: this efficacy, along with its effects, such as signifiers, is fundamentally reciprocal in nature, including as concerns the relationships between materiality and phenomenality. In this sense, the expression "the image of the image of the image of the penis," used earlier, must be seen as designating the "cite" of the multidirectional and ultimately interminable reciprocal network of the "material" and the "phenomenal," although, indeed by the same token, other terms will be necessary in order to follow and to consider this network.

²⁰ Here one would need, of course, to consider the question of sacrifice in Lacan, via Hegel, in particular in the *Phenomenology*, and, then, Alexandre Kojève and George Bataille, both of whom are clearly on Lacan's mind here as well.

²¹ The difference between the erectile organ and the phallus would be inscribed accordingly, as indicated earlier.

within, and of, the Lacanian system itself in the symbolic order of its operation, or, again, more accurately, of a certain formalization of that system. "Thus the erectile organ comes to symbolize [again, also in Lacan's sense of the symbolic] the place of jouissance, not in itself, or even in the form of an image, but as part lacking in desired image: that is why it is equivalent to the square root of -1 of the signification produced above, of the jouissance that it restores by the coefficient of its statement to the function of the lack of signifier -1 " (p. 320).²² Accordingly, "the signification of the phallus" so conceived conforms to the economy of the inaccessible signifier. It can be shown that neither the signified nor the referent are simply suspended here, and are in fact conceived of as ultimately inaccessible as well, via the Lacanian Real. For as Lacan says, "if [the erectile organ's role], therefore, is to bind the prohibition of jouissance, it is nevertheless not [only] for these formal reasons" (*Écrits*, p. 320). Instead it is due primarily to a complex materiality, ultimately related to the Real and its epistemology. Indeed the Real in Lacan's sense may be seen as this materiality (rather than reality). It may be best conceived as a certain radical (but not absolute) alterity inaccessible to a metaphysical configuring, oppositional or other, in particular as anything that can be seen as possessing any attributes (perhaps even the attribute of existence in any way that is or will ever be available to us) independently of our engagement with it.

To summarize, within the Lacanian psychoanalytical situation, the *image* or the *signifier* of the erectile organ is a scandal – in either sense, but most crucially in terms if its psychoanalytic management, or the difficulty or even impossibility of thereof. In this latter sense it is not unlike what the square root of -1 in mathematics was epistemologically at some point. Lacan's approach is to refigure it as a symbolic object – specifically in Lacan's sense of the juxtaposition between the symbolic and the imaginary. In the register of the imaginary "the signification of the phallus," while conceivably involving to *inaccessible* signifiers and referents, may be seen as defined by *accessible signifiers*, but (in part as a consequence) is psychoanalytically useless. In the new system (in the symbolic register) the Lacanian signifiers themselves, in particular the erectile organ, are ultimately inaccessible. By the same token, a symbolic system (in Lacan's sense of the symbolic) is introduced as the dialectic of desire and castration, which enables the subject defined by this systems and/as the Lacanian analytical situation to function. The symbolic object itself in question is given a specific formal structure, just as the square root of -1 is in mathematics. From this perspective, the

erectile organ is not a real unity or oneness, positive or negative, neither " 1 " nor " -1 ," or even anything merely fragmented, analogous to either mathematical real rational or real irrational. Instead it is a "solution" of the psychoanalytic equation which contains oneness – " 1 " – and the negative of oneness – " -1 " – as terms but which makes the "solution" itself, while in a certain sense formalizable, inaccessible even at the level of the signifier, which, to the degree they offer us any image of it, all our imaginaries and visualizations ineluctably "miss," along with the signified and the referent – the Real, keeping in mind the qualifications made earlier (Note 20).

I am not certain to what degree Lacan's epistemological ideas were derived from the epistemology of mathematical complex numbers. It is not inconceivable, especially given his statements cited here. He also knew enough mathematics and mathematicians to draw this parallel and to use it. I suspect that he was at least aware of these epistemological connections, as some of the statements cited above would indicate, even if he did not actually derive his scheme from the epistemology of complex numbers. There are, however, other candidates or the sources of this epistemology of the inaccessible at all levels of signification – the signifier, the signified, the referent – in more immediate semiotic terms, in the work of Saussure and Hjelmslef, on the one hand, and C.S. Peirce, on the other, or in more philosophical terms in the radical philosophy in the wake of Kant and Hegel, certainly in Nietzsche, although one can trace some of these ideas to Plato and the pre-Socratics.

It is, then, only in the sense of the square root of -1 of Lacan's system, as just delineated, and not of the mathematical system of complex numbers, that the erectile organ is the square root of -1 . This argument would clearly invalidate a kind of critique that Sokal and Bricmont level at Lacan, were their critique to survive far lesser levels of scrutiny. Unwittingly, Sokal and Bricmont's own comment in fact says as much: "Even if [Lacan's] 'algebra' made sense, the 'signifier', and 'signified' and the 'statement' contained in it are not numbers" (*Impostures intellectuelles*, p. 32). Of course not, this is the whole point.²³ It is clear even from the most cursory reading that Lacan never says they are. Indeed in a sentence introducing the formula in question, the sentence cited by Sokal and Bricmont, Lacan says: "Thus by calculating that signification according to the algebraic method used here" (*Écrits*, p. 317; emphasis added) – that is, according to Lacan's "algebra," not the actual mathematics of complex numbers, which is my point here.

²² It is significant that Lacan says, "equivalent" and not "identical," which, again, suggest the difference between Lacan's "algebra" and that of the actual mathematical complex numbers, rather than a claim of their identity on Lacan's part.

²³ The reader may be spared the rest of Sokal and Bricmont's sentence, equally ironic in its confirmation of my point here and equally remarkable in its naiveté and blindness.

Lacan's constructions here considered may have been designed primarily for the psychoanalytical purposes, although such purposes in Lacan is a complex matter. Either way, this construction is accomplished by way of an invention and construction of philosophical concepts in Deleuze and Guattari sense, which activity defines philosophy itself, according to their *What is Philosophy?* It is in my view this construction that gives us the best sense of Lacan's usages of mathematics. This point also allows me to close here by giving a reasonable definite, although not definitive, answer to the question what is the place of mathematics in Lacan. Mathematics sometimes function in Lacan's texts in a more direct and less complicated fashion of metaphor, illustration, and the like. For example, on some occasions certain constructions of modern topology, such as the Möbius strip and the Klein bottle, serve Lacan to find that which "we [can] propose to intuition in order to show" certain complex configurations entailed, Lacan argues, by neurosis, or psychosis ("Of Structure as an Inmixing," 192), although the overall situation is ultimately more complex on these occasions as well. I do think, however, the primary and most significant usage of mathematical concepts in Lacan is as components of his own multilayered – irreducible nonsimple – concepts, conforming to Deleuze and Guattari's definition (or concept) of the philosophical concept. The presence and role of such concepts in Lacan is, in my view, unquestionable. Virtually any given sample of Lacan's text manifest such concepts. In "The Subversion of the Subject," imaginary numbers is only a portion of conceptual and metaphorical conglomerate, many components of which are borrowed from various domains – literature, religion, philosophy, or whatever – and many of them would require a kind of analysis just given.

As I said, this view shifts Lacan's usage of mathematics into the philosophical from the psychoanalytic register, in accord with Deleuze and Guattari's ideas in *What is Philosophy?* It is of some interest that the book, while examining the difference between philosophy (defined by deployment of concepts), and other fields, in particular mathematics, science, and art, Deleuze and Guattari omit psychoanalysis from this argument altogether. The relationships between psychoanalysis and philosophy have of course been subject of important recent investigations, such as Derrida's, especially in *The Post Card* (where Lacan is the main subject, along with Freud, and Heidegger), or elsewhere in Deleuze and Guattari, especially in *Anti-Oedipus*, and indeed in Lacan's essay in question, the essay also on Hegel. We know from these investigations that philosophy and psychoanalysis are multiply and perhaps irreducibly entangled, both historically and conceptually. This entanglement, however, is not symmetrical, and part of this asymmetry may entail a different (and more fundamental) role mathematical concepts and mathematics play in the philosophical vs. psychoanalytic thought and

discourse. Indeed, one may see Lacan's usage of mathematics as in part an attempt to change this asymmetry, at least at a certain point, as part of his attempt to make psychoanalysis more scientific or, with Freud, to affirm its scientific character. In the process, Lacan did, I think, manage to enrich our understanding of the nature and complexity of the project of mathematics and science. The success of his deployment of mathematics and science in psychoanalysis qua psychoanalysis is a different question, in part given the very nature of his thought, work, and text. This argument may make mathematics primarily a part of Lacan's work as an inventor of concepts and, hence, as a philosopher, rather than a psychoanalyst (to the degree that these can be distinguished in Lacan's case). At the very least, the role of mathematics in Lacan is fundamentally philosophically mediated, also in Hegel's sense of mediation [*Vermittlung*]. That, however, may well be how mathematics has always functioned outside its own sphere, and indeed often within it.

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Comptes rendus

Alain Génetiot (sous la direction de), *L'Éloge lyrique*. Nancy: Presses Universitaires de Nancy, coll. «CEMLA», 2008. 503 p.

Dans sa brièveté, le titre dit tout. Ce volume offre un aperçu solidement documenté de l'éloge en vers dans son ampleur et sa durée, comme le précise la belle préface d'Alain Génetiot, «De l'ode encomiastique au chant du monde» (p. 5–14). Tout part de Pindare, pour arriver à nous.

Le parcours historique du volume se déroule du XVI^e siècle au nôtre, dans une série de trente-trois articles sur les formes et les visées de l'éloge lyrique. Rien donc au Moyen Âge? En fait, l'article de Dominique Millet-Gérard (p. 391–401), s'il mentionne Claudel, étudie pour l'essentiel le commentaire de Bernard de Clairvaux sur le *Cantique des Cantiques* (l'éloge de l'Époux et de l'Épouse) mis en parallèle avec le modèle.

Mais on lira d'abord la préface, qui pose le sujet: «Comme le discours encomiastique en prose, l'éloge lyrique procède d'un même réservoir de sujets et de lieux parcourus et actualisés à travers l'histoire littéraire. C'est cet invariant, qui ressortit à une forme de sensibilité universelle, que le présent recueil entend mettre en lumière à travers l'exemple de la poésie française, de la Renaissance à l'extrême contemporain» (p. 8). Le poète lyrique, en effet, ne s'arrête pas au présent car sa tâche «est de voir plus loin que lui-même et de s'élever, dans une adhésion empathique et euphorique, à la contemplation des vérités éternelles et transcendentales» (p. 14).

Les articles sont rangés selon la chronologie: on en compte 8 pour le XVI^e siècle, 6 pour le XVII^e, 5 pour le XVIII^e, 7 pour le XIX^e et 7 encore pour les XX–XXI^e. D'un siècle à l'autre, ce sont les mêmes différences d'approche: d'une part les considérations d'ensemble sur le genre, d'autre part des études sur des auteurs (les plus nombreuses), enfin – difficiles parfois à distinguer des précédentes – des examens de cas particuliers, voire de détails.

Pour la première série (les textes s'attachant à la définition ou à l'illustration de l'éloge lyrique), on a l'étude de Nathalie Dauvois sur les commentaires humanistes des odes d'Horace définissant la poétique de l'éloge lyrique (p. 15–27), et celle d'Isabelle Pantin sur «la relation entre célébrer et décrire à la Renaissance» à propos de la «poésie des choses» (p. 95–106). Anne Mantero s'intéresse à «la louange de Dieu dans la poésie du XVII^e siècle»