MA 16200
EXAM 1 Form A
September 17, 2015

NAME ______________________  YOUR TA’S NAME ______________________

STUDENT ID # ________________ RECITATION TIME ______________________

1. You must use a #2 pencil on the mark-sense sheet (answer sheet).

2. If the cover of your question booklet is GREEN, write 01 in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below. If the cover is ORANGE, write 02 in the TEST/QUIZ NUMBER boxes and darken the spaces below.

3. On the mark-sense sheet, fill in your TA’s name and the course number.

4. Fill in your NAME and STUDENT IDENTIFICATION NUMBER and blacken in the appropriate spaces.

5. Fill in your four-digit SECTION NUMBER. If you do not know your section number, please ask your TA.


7. Fill in your name and your instructor’s name on the question sheets above.

8. There are 12 questions, each worth 8 points (you will automatically earn 4 points for taking the exam). Blacken in your choice of the correct answer in the spaces provided for questions 1–12. Do all your work on the question sheets.

9. Turn in both the mark-sense sheets and the question sheets when you are finished.

10. If you finish the exam before 7:20, you may leave the room after turning in the scantron sheet and the exam booklet. If you don’t finish before 7:20, you MUST REMAIN SEATED until your TA comes and collects your scantron sheet and your exam booklet.

11. NO CALCULATORS, PHONES, BOOKS, OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.
1. What is the radius of the sphere \( x^2 + y^2 + z^2 + 8x - 6y + 14z - 7 = 0 \)?

A. \( \sqrt{5} \)  
B. \( \sqrt{7} \)  
C. 3  
D. 7  
E. 9

- Want radius, so don’t need to factor
- \( (x^2 + 8x + 16) + (y^2 - 6y + 9) + (z^2 + 14z + 49) - 49 - 7 = 0 \)
- \( r^2 = 81 \)
- \( r = 9 \)

2. If \( \mathbf{a} = \langle 1, -2, 2 \rangle \) and \( \mathbf{b} = \langle -3, 0, 4 \rangle \), find \( |2\mathbf{b} - 3\mathbf{a}| \).

A. 11  
B. 12  
C. 13  
D. 14  
E. 15

- \( 3\mathbf{a} = \langle 3, -6, 6 \rangle \)  
- \( 2\mathbf{b} = \langle -6, 0, 8 \rangle \)  
- \( 2\mathbf{b} - 3\mathbf{a} = \langle -9, 6, 2 \rangle \)  
- \( |2\mathbf{b} - 3\mathbf{a}| = \sqrt{81 + 36 + 4} = \sqrt{121} = 11 \)
3. What is the angle between the vectors \( \mathbf{u} = 2i + 3j - k \) and \( \mathbf{v} = 5i + 4j + k \)?

Potentially useful information: \( \sqrt{14}\sqrt{42} = 14\sqrt{3} \)

A. 0
B. \( \pi/6 \)
C. \( \pi/3 \)
D. \( 2\pi/3 \)
E. \( 5\pi/6 \)

\[
\cos \theta = \frac{\mathbf{u} \cdot \mathbf{v}}{||\mathbf{u}|| ||\mathbf{v}||} = \frac{21}{\sqrt{14}\sqrt{42}} = \frac{3}{2\sqrt{3}}
\]

\[
||\mathbf{u}|| = \sqrt{4 + 9 + 1} = \sqrt{14}
\]

\[
||\mathbf{v}|| = \sqrt{25 + 16 + 1} = \sqrt{42}
\]

\[
\mathbf{u} \cdot \mathbf{v} = 21
\]

\[
\cos \theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{\pi}{6}
\]

4. Consider the following vectors and statements:

\( \mathbf{a} = i + k, \quad \mathbf{b} = 2i + 5j - 2k, \quad \mathbf{c} = -2i + 2j + 3k \)

I. \( \mathbf{a} \) and \( \mathbf{b} \) are orthogonal.
II. \( \mathbf{a} \) and \( \mathbf{c} \) are orthogonal.
III. \( \mathbf{b} \) and \( \mathbf{c} \) are orthogonal.

Choose ALL of the correct statements.

A. II
B. I, II
C. I, III
D. II, III
E. I, II, III

\[
\mathbf{a} \cdot \mathbf{b} = 2 - 2 = 0 \checkmark
\]

\[
\mathbf{a} \cdot \mathbf{c} = -2 + 3 = 1 \times
\]

\[
\mathbf{b} \cdot \mathbf{c} = -4 + 10 - 6 = 0 \checkmark
\]
5. Find the area of the parallelogram with vertices
\[ A(-7, -5), \ B(-3, 7), \ C(7, 14), \ D(3, 2). \]
\[ \overrightarrow{AB} = \langle -4, 12 \rangle \]
\[ \overrightarrow{AD} = \langle 10, 7 \rangle \]
\[ \overrightarrow{AB} \times \overrightarrow{AD} = \begin{vmatrix} i & j & k \\ -4 & 12 & 0 \\ 10 & 7 & 0 \end{vmatrix} = 0i + 0j + (38 - 120)k = \langle 0, 0, -82 \rangle \]
\[ |\overrightarrow{AB} \times \overrightarrow{AD}| = \sqrt{(-82)^2} = 82 \]

6. Which of the following is orthogonal to the plane through the points
\[ P(2, -1, 2), \ Q(-1, 2, -3), \ R(3, 4, -1)? \]
A. \( \langle -3, 3, 5 \rangle \)
\[ \overrightarrow{PQ} = \langle -3, 3, -5 \rangle \]
B. \( \langle 1, 5, -3 \rangle \)
\[ \overrightarrow{PR} = \langle 1, 5, -3 \rangle \]
C. \( \langle 4, 1, 6 \rangle \)
D. \( \langle 5, 6, -9 \rangle \)
E. \( \langle 8, -7, -9 \rangle \)
\[ \overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} i & j & k \\ -3 & 3 & -5 \\ 1 & 5 & -3 \end{vmatrix} = (-9 + 25)i - (9 + 15)j + (15 + 3)k = \langle 16, -24, 18 \rangle \]
\[ = 2 \langle 8, -7, -9 \rangle \]
7. Find the area of the region bounded by \( y = \cos x \), \( y = \sin 2x \), \( x = 0 \), and \( x = \frac{\pi}{2} \). You may need to use the double-angle identity \( \sin 2\theta = 2\sin \theta \cos \theta \).

A. \( \frac{1}{4} \)  
B. \( \frac{1}{2} \)  
C. \( \frac{1}{3} \)  
D. 2  
E. 4

\[
\int_0^{\pi/2} \cos \theta - \sin 2\theta \, d\theta + \int_{\pi/6}^{\pi/2} \sin 2\theta - \cos \theta \, d\theta
\]

\[
= \sin \theta + \frac{1}{2} \cos 2\theta \bigg|_0^{\pi/6} + \frac{1}{2} \cos 2\theta - \sin \theta \bigg|_{\pi/6}^{\pi/2}
\]

\[
= \left( \frac{1}{2} + \frac{1}{4} - \frac{1}{2} \right) - \frac{1}{4} + \left( \frac{1}{2} - 1 - \left( -\frac{1}{4} - \frac{1}{2} \right) \right) = \frac{1}{4} = \frac{1}{2}
\]

8. If the disk/washer method is used to find the volume generated by rotating the region bounded by the curves \( y = x^4 \), \( x = y^4 \) about the \( x \)-axis, which of the following integrals is correct?

A. \( 2\pi \int_0^1 (y^4 - y^{4/4}) \, dy \)  
B. \( 2\pi \int_0^1 (y^{1/4} - y^4) \, dy \)  
C. \( \pi \int_0^1 (x^8 - x^{1/2}) \, dx \)  
D. \( \pi \int_0^1 (x^{1/2} - x^8) \, dx \)  
E. \( \pi \int_0^1 (y^{1/2} - y^8) \, dy \)

\[
\pi \int_0^1 ((x^{1/4})^2 - (x^4)^2) \, dy
\]

\[
= \pi \int_0^1 x^{1/2} - x^8 \, dy
\]
9. The height of a monument is 3 meters. A horizontal cross-section at a distance $x$ meters from the top is an equilateral triangle with each side being $2x$ meters. Find the volume (in m$^3$) of the monument.

A. $\frac{3\sqrt{3}}{2}$
B. $\sqrt{3}$
C. $9\sqrt{3}$
D. 18
E. 36

\[ A \triangle = \frac{1}{2} bh = \frac{1}{2} (2x)(\sqrt{3}x) = \sqrt{3}x^2 \]

\[ \int_0^3 \sqrt{3}x^2 \, dx = \frac{\sqrt{3}x^3}{3} \bigg|_0^3 = 9\sqrt{3} \]

10. Find the volume of the solid generated by revolving the region bounded by $y = x^2$, $y = 0$, $x = 1$, and $x = 2$ about the line $x = 1$.

A. $\frac{17\pi}{6}$
B. $\frac{15\pi}{4}$
C. $\frac{34\pi}{3}$
D. $\left(\frac{128}{7} - \frac{32}{5}\right)\pi$
E. $\left(\frac{256}{7} - \frac{64}{5}\right)\pi$

\[ 2\pi \int_1^2 x^2(x-1) \, dx = 2\pi \int_1^2 x^3 - x^2 \, dx \]

\[ 2\pi \left( \frac{x^4}{4} - \frac{x^3}{3} \right) \bigg|_1^2 \]

\[ 2\pi \left( \frac{16}{4} - \frac{8}{3} - \left( \frac{1}{4} - \frac{1}{3} \right) \right) \]

\[ 2\pi \left( \frac{15}{4} - \frac{7}{3} \right) = 2\pi \left( \frac{45}{12} - \frac{28}{12} \right) = \frac{17\pi}{6} \]
11. A spring has a natural length of 5 m. If a 25-N force is required to keep it stretched to a length of 10 m, how much work (in joules) is required to stretch it from 5 m to 6 m?

A. \( \frac{55}{2} \)
B. \( \frac{55}{4} \)
C. \( \frac{5}{4} \)
D. \( \frac{5}{2} \)
E. 5

\[
KX = 25 \\
X = 5 \\
\Rightarrow K = 5 \\
\]

\[
\int_{5}^{10} 5X \, dX = \frac{5X^2}{2} \bigg|_{5}^{10} \\
\]

12. A metal rod is 4 m long. The temperature (in °C) is 3x at a distance x m from the left end of the rod (where x = 0). What is the average temperature (in °C) of rod?

A. 6
B. 22
C. 24
D. 34
E. 46

\[
\frac{1}{4} \int_{0}^{4} 3X \, dX = \frac{1}{4} \left( \frac{3X^2}{2} \right) \bigg|_{0}^{4} \\
= \frac{1}{4} \left( 24 \right) = 6 \\
\]