NAME ___________________ YOUR TA'S NAME Emma Reid

STUDENT ID # _______________ RECITATION TIME _______________

1. You must use a #2 pencil on the mark-sense sheet (answer sheet).

2. If the cover of your question booklet is GREEN, write 01 in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below. If the cover is ORANGE, write 02 in the TEST/QUIZ NUMBER boxes and darken the spaces below.

3. On the mark-sense sheet, fill in your TA’s name and the course number.

4. Fill in your NAME and STUDENT IDENTIFICATION NUMBER and blacken in the appropriate spaces.

5. Fill in your four-digit SECTION NUMBER. If you do not know your section number, please ask your TA.


7. Fill in your name and your instructor’s name on the question sheets above.

8. There are 12 questions, each worth 8 points (you will automatically earn 4 points for taking the exam). Blacken in your choice of the correct answer in the spaces provided for questions 1–12. Do all your work on the question sheets.

9. Turn in both the mark-sense sheets and the question sheets when you are finished.

10. If you finish the exam before 7:20, you may leave the room after turning in the scantron sheet and the exam booklet. If you don’t finish before 7:20, you MUST REMAIN SEATED until your TA comes and collects your scantron sheet and your exam booklet.

11. NO CALCULATORS, PHONES, BOOKS, OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.
EXAM POLICIES

1. Students may not open the exam until instructed to do so.

2. Students must obey the orders and requests by all proctors, TAs, and lecturers.

3. No student may leave in the first 20 min or in the last 10 min of the exam.

4. Books, notes, calculators, or any electronic devices are not allowed on the exam, and they should not even be in sight in the exam room. Students may not look at anybody else’s test, and may not communicate with anybody else except, if they have a question, with their TA or lecturer.

5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.

6. Any violation of these rules and any act of academic dishonesty may result in severe penalties. Additionally, all violators will be reported to the Office of the Dean of Students.

I have read and understand the exam rules stated above:

STUDENT NAME: ____________________________________________

STUDENT SIGNATURE: _________________________________________
1. Let $S_1$ by the sphere $x^2 + y^2 + z^2 + 4x - 6y - 2z = -5$ and let $S_2$ be the sphere $x^2 + y^2 + z^2 + 4x - 2y + 4z = -8$. Find the distance between the centers of the spheres.

A. $\sqrt{17}$  
B. $\sqrt{13}$  
C. $\sqrt{10}$  
D. $\sqrt{14}$  
E. $\sqrt{5}$

\[ S_1 = (x^2 + 4x + 4) - 4 + (y^2 - 6y + 9) - 9 + (z^2 - 2z + 1) - 1 \]
\[ (x+2)^2 + (y-3)^2 + (z-1)^2 - 14 = -5 \]
Center (-2, 3, 1)

\[ S_2 = (x^2 + 4x + 4) - 4 + (y^2 - 2y + 1) - 1 + (z^2 + 4z + 4) - 4 = -8 \]
\[ (x+2)^2 + (y-1)^2 + (z+2)^2 = 9 = -8 \]
Center (-2, 1, -2)

\[ \sqrt{(-2 - 2)^2 + (3 - 1)^2 + (1 - 2)^2} = \sqrt{0 + 4 + 1} = \sqrt{13} \]

2. A man is walking on the deck of a ship in the North-East direction at a speed of 2 km/h. The ship is moving East at a speed of 4 km/h. Find the speed, in km/h, of the man relative to the surface of the water.

A. $\sqrt{16 + 8\sqrt{2}}$  
B. $\sqrt{20 + 4\sqrt{2}}$  
C. $\sqrt{20 + 8\sqrt{2}}$  
D. $\sqrt{18 + 4\sqrt{2}}$  
E. $\sqrt{24 + 8\sqrt{2}}$

\[ \text{Man} \quad <20\times45, 2\sin 45> = <\sqrt{2}, \sqrt{2}> \]
\[ \text{Boat} \quad <4, 0> \]

\[ \text{Speed} = \sqrt{(4\sqrt{2})^2 + (\sqrt{2})^2} \]
\[ = \sqrt{16 + 8\sqrt{2} + 2 + 2} \]
\[ = \sqrt{20 + 8\sqrt{2}} \]
3. Find the angle θ between \( \vec{a} = \langle -4, -1, 1 \rangle \) and \( \vec{b} = \langle 2, 2, 1 \rangle \):

A. \( \frac{\pi}{4} \)

\[
\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|} = \frac{-8 - 2 + 1}{\sqrt{18} \cdot \sqrt{9}} = \frac{-9}{9 \sqrt{2}}
\]

B. \( \frac{5\pi}{6} \)

\[
\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|} = \frac{-1}{\sqrt{2}} = -\frac{\sqrt{2}}{2} \quad \Rightarrow \quad \theta = \frac{3\pi}{4}
\]

C. \( \frac{3\pi}{4} \)

D. \( \frac{\pi}{3} \)

E. \( \frac{2\pi}{3} \)

4. Let \( \vec{a} = \langle 2, -1, 1 \rangle \) and \( \vec{b} = \langle 4, 1, -1 \rangle \). If \( \text{proj}_\vec{b} \vec{a} = \vec{c} \), what is \( c \)?

A. \( \frac{2}{3} \)

\[
\text{proj}_\vec{b} \vec{a} = \frac{\vec{a} \cdot \vec{b}}{||\vec{b}||^2} \cdot \vec{b} = \frac{10 - 2 - 1}{18} \cdot \vec{b}
\]

B. \( \frac{4}{9} \)

C. \( \frac{2}{9} \)

D. \( \frac{1}{3} \)

E. 1
5. If \( \vec{a} = <2, -1, 4> \) and \( \vec{b} = <2, 2, 1> \), find the \( \vec{j} \)-component of \( \vec{a} \times \vec{b} \).

\[
\vec{a} \times \vec{b} = \begin{vmatrix}
2 & -1 & 4 \\
2 & 2 & 1 \\
\end{vmatrix}
\]

A. 6  
B. -6  
C. 9  
D. -9 
E. -2

\[ j \text{ component } -(2 \cdot 8) = +6. \]

6. Find the area of the region enclosed by \( y = 8x^2 \) and \( y = 3 + 5x^2 \)

A. 2  
B. 3  
C. 4  
D. 5  
E. 6

\[
\begin{align*}
3+5x^2 &= 8x^2 \\
3 &= 3x^2 \\
x &= \pm 1
\end{align*}
\]

\[
\int_{-1}^{1} 3 + 5x^2 - 8x^2 \, dx
\]

\[
\int_{-1}^{1} 3 - 3x^2 \, dx
\]

\[
3x - x^3 \bigg|_{-1}^{1}
\]

\[
3 - 1 - (-3 + 1) = 4
\]

2 + 2 = 4
7. The region enclosed by the curves $x = y$ and $x = y^2$ is rotated about the $x$-axis. If the Disk/Washer method is used, which of the following integrals would represent the volume of the resulting solid?

A. $\pi \int_0^1 (x - x^2) \, dx$
B. $\pi \int_0^1 (\sqrt{x} - x)^2 \, dx$
C. $\pi \int_0^1 (y^4 - y^2) \, dy$
D. $2\pi \int_0^1 (y^2 - y^3) \, dy$
E. $2\pi \int_0^1 (y^3 - y^2) \, dy$

8. A certain rectangular pyramid has a height of 4 ft. Cross-sections at $x$ ft below the tip are rectangles whose length is $x$ ft and whose width is $\frac{1}{2}x$ ft (see figure). Find the volume of this pyramid.

A. $\frac{16}{3}$
B. $\frac{32}{3}$
C. $\frac{64}{3}$
D. $32$
E. $64$
9. Use the method of cylindrical shells to find the volume of the solid obtained by rotating about the y-axis the region bounded by \( y = x^3 + x, \ y = 0, \) and \( x = 1. \)

A. \( \frac{16\pi}{15} \)
B. \( \frac{17\pi}{15} \)
C. \( \frac{19\pi}{15} \)
D. \( \frac{22\pi}{15} \)
E. \( \frac{23\pi}{15} \)

\[
\begin{align*}
\text{r} &= x \\
\text{h} &= x^3 + x \\
2\pi \int_0^1 x^4 + x^2 \, dx &\quad 2\pi \left( \frac{x^5}{5} + \frac{x^3}{3} \right) \bigg|_0^1 \\
2\pi \left( \frac{1}{6} + \frac{1}{3} \right) &= 2\pi \left( \frac{1}{15} \right) \\
&= \frac{10\pi}{15} \\
\end{align*}
\]

10. An aquarium 2 feet long, 2 feet wide and 2 feet deep is half full of water. Find the work needed to pump the water out of the aquarium. Use the fact that water weighs 62.5 pounds per cubic foot.

A. 437.5 foot-pounds
B. 375 foot-pounds
C. 312.5 foot-pounds
D. 250 foot-pounds
E. 187.5 foot-pounds

\[
\begin{align*}
\text{V} &= 4 dx \\
8V &= 62.5 \cdot 4 dx \\
\int_0^2 250(x) \, dx &\quad 125x^2 \bigg|_1^2 \\
500 - 125 &= 375
\end{align*}
\]
11. Find the integral: \( \int_{0}^{\pi/2} x^2 \cos x \, dx \)

\[ u = x^2 \quad v = \sin x \]
\[ du = 2x \, dx \quad dv = \cos x \, dx \]

\[ x^2 \sin x - \int 2x \sin x \, dx \]

\[ u = 2x \quad v = -\cos x \]
\[ du = 2 \, dx \quad dv = \sin x \, dx \]

\[ x^2 \sin x - \left( -2x \cos x + \int 2 \cos x \, dx \right) \]

\[ x^2 \sin x + 2x \cos x - 2 \sin x \left| _0^{\pi/2} \right. \]

\[ = \frac{\pi^2}{4} - 2 \]

12. Find the integral: \( \int_{1}^{e} x^{-1/2} \ln x \, dx \)

\[ u = \ln x \quad v = 2x^{1/2} \]
\[ du = \frac{1}{x} \, dx \quad dv = x^{-1/2} \, dx \]

\[ \ln(x) \cdot 2x^{1/2} - \int 2x^{-1/2} \ln(x) \, dx \]

\[ \ln(x) \cdot 2x^{1/2} - 4x^{1/2} \left| _1^e \right. \]

\[ 2 \sqrt{e} - 4 \sqrt{e} + 4 \]

\[ -2 \sqrt{e} + 4 \]
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