Cross Time Frequency Spectra Applied to Automotive Brake Noise

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Abstract
We apply a local cross-correlation method to the study of correlating brake sounds perceived inside a car to the vibration produced by the brakes at the source. The method allows one to correlate the time-varying spectral properties of two vibration or acoustic sources. The aim is to identify the possible objectionable sounds by determining the time-varying frequencies that the two sounds have in common. The method utilizes cross-time-frequency distributions, and in particular we use the cross-spectrogram. Examples are given to show the effectiveness of the method.

1 Introduction
The task of developing automotive brakes often includes testing to identify brake noise and associated vibration sources. A key challenge is identifying the source and mechanism by the objectionable sound or vibrations produced. This requires correlating data perceived by the driver with measurements by sensors at the wheels. The problem is complicated by the fact that maintaining the conditions that produce the signal is usually difficult because brake noise and vibration events are generally transient and nonstationary. One approach to handle the nonstationary aspect of the data to be correlated is to perform time-dependent cross-correlation and cross-spectral analysis.

In this paper, we apply time-frequency distributions (TFDs) and cross-time frequency-distributions (XTFDs) to analyze brake noise and vibrations. A TFD of a brake signal reveals the frequencies in the signal, and moreover shows whether or not these frequencies change over time. Cross-time-frequency analysis provides a means to compare the time-varying spectral content of two different brake signals. In the next section we discuss time-frequency distributions and cross time-frequency distributions. In section 3 we apply these methods to accelerometer and microphone data collected during road tests.

2 Background
The standard power spectrum of a signal shows which frequency components were present in the signal, but not when they were present. Time-frequency distributions (TFDs) display the variation of a signal's frequency content as a function of time [2,3]. There are many time-frequency distributions that have been developed, among them are the spectrogram, Wigner distribution, Choi-Williams, and Zhao-Atlas-Marks [4, 3, 14, 16]. Time-frequency analysis has been applied to a variety of machine-type signals for condition-based assessment [9, 10, 11, 12, 13], including automotive signal analysis [7,8].

All time-frequency distributions may be obtained from [2,3]

\[
C(t, \omega) = \frac{1}{4\pi} \iiint e^{-i\omega t - i\omega u + i\phi(\theta, \tau)} s(u - \frac{1}{2} \tau) s(u + \frac{1}{2} \tau) du d\tau d\theta \tag{1}
\]

where \(s(t)\) is the signal and \(\phi(\theta, \tau)\) is a two dimensional function called the kernel. The kernel determines the distribution and its properties. By choosing different kernels different distributions are obtained. If the kernel is not functionally dependent on the signal, as is usually the case, the distribution obtained is “bilinear” in the signal. The spectrogram, which is perhaps the most commonly used method for time-frequency analysis, is given by
\[ P_{sp}(t, \omega) = \left| \frac{1}{\sqrt{2\pi}} \int e^{-i\omega \tau} h(\tau - t) d\tau \right|^2 \]  

(2)

where \( h(t) \) is a function that tapers to zero, typically called a "window". The spectrogram can also be obtained from Eq. (1) by taking the kernel to be

\[ \phi_{sp}(\theta, \tau) = \int h'(u - \frac{1}{2} \tau)e^{-iu\theta}h(u + \frac{1}{2} \tau) du \]  

(3)

although it is typically implemented by way of Eq. (2).

The concept of local correlation and cross time-frequency distributions have been developed and applied by many investigators, e.g., see [1, 6, 15]. Generally speaking, if we have two signals denoted by \( x(t) \) and \( y(t) \), then the cross-TFD (XTFD) is

\[ C_{xy}(t, \omega) = \frac{1}{4\pi} \iint e^{-i\omega \tau} \phi(\theta, \tau)x(t)\ast(u - \frac{1}{2} \tau)y(u + \frac{1}{2} \tau) du d\tau d\theta \]  

(4)

If we take the kernel to be that for the spectrogram in Equation 3, then we obtain the cross-short-time Fourier spectrum, or the cross-spectrogram, of the signals, given by (see Appendix A),

\[ C_{SP}(t, \omega) = X'_x(\omega)Y'_y(\omega) \]  

(5)

where \( X'_x(\omega) \) and \( Y'_y(\omega) \) are the short-time Fourier transforms of \( x(t) \) and \( y(t) \) respectively, i.e.,

\[ X'_x(\omega) = \frac{1}{\sqrt{2\pi}} \int e^{-i\omega \tau} x(\tau)h(\tau - t) d\tau \]  

(6)

and analogously for \( Y'_y(\omega) \). This result is equivalent to taking the Fourier transform of the cross-correlation of windowed segments of the signals \( x(t) \) and \( y(t) \). Specifically, if we denote the windowed sections of the signals by

\[ x_i(t') = x(t')h(t' - t) \]
\[ y_i(t') = y(t')h(t' - t) \]  

(7)

then the local cross-correlation can be calculated by

\[ R_i(\tau) = \int x_i^*(t') y_i(t' + \tau) dt' \]  

(8)

The Fourier transform of this equation, with respect to \( \tau \) gives the local or short-time cross spectrum, which is equivalent to the cross-spectrogram in Eq. (5). (See Appendix B)

Note that if at a given time \( t \), \( X(\omega) \) and \( Y(\omega) \) have different spectral content, then the magnitude of the cross-spectrogram will be small. Conversely, if at that time the two signals have similar spectral content, the cross-spectrogram will show peaks at the frequencies in common to the two signals.

3 Application to Brake Data

We apply these methods to automobile brake data. The data were collected from an instrumented test vehicle during a road test. The brakes exhibited unacceptable self-excited vibrations during the test that resulted in an objectionable noise at the driver’s ear. Accelerometers were mounted on all four suspension corners and a microphone was placed at the driver’s ear. The XTFD can be used to correlate the noise at the driver’s ear with vibrations measured at the brakes in order to determine which corners are most involved in the vibration mechanism.

Figure 1 shows the spectrogram of a microphone signal recorded during a test on a steeply sloped road. It is clear from the time domain plot at the top that the signal is non-stationary. It is also clear from the spectrogram plot that two separate components exist, one at approximately 20 Hz and one at approximately 38 Hz. The challenge is to determine which components produce the recorded sounds.

In addition to the microphone, the vehicle had accelerometers mounted on each of the four brakes and their output was recorded simultaneously with that of the microphone. Figure 2 shows that energy at both 20 Hz and 38 Hz is present in the signals from the microphone and the right rear accelerometers.

Figure 1 - Microphone Signal From Brake Noise Event
Indeed, the problem was eventually traced to unacceptably high vibration levels in the front brakes. The objectionable noise was due to the 20 Hz component and its harmonics. Since 20 Hz is generally accepted as the lower frequency limit of the human hearing range, the objectionable noise must be assumed to be strongly augmented by the harmonics.

4 Conclusions
We have presented a method, namely cross time-frequency analysis, that allows the investigation of the mutual time-frequency content of two nonstationary sources. One important application is the correlation of the acoustic sound perceived by the driver with the acceleration signal generated at the source. In this way one can discover what source is producing the possible objectionable sound. We have applied the method to brake sound and have shown that the nonstationary sound measured at the driver's ear can be correlated with the accelerations at the brake.

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6 References
Appendix A - Derivation of Eq. 5 from Eq. 4

For completeness, we show that Eq. 5 follows from Eq. 4. Substituting Eq. 3 into Eq. 4, we have

\[ C_{xy}(t, \omega) = \frac{1}{4\pi^2} \int \int \int \int x^+(u - \frac{\tau}{2}) y(u + \frac{\tau}{2}) h(u - \tau) h^+(u - \frac{\tau}{2}) e^{-i\omega\tau} du d\theta d\tau \]  

(A1)

The integral over \( \theta \) yields \( \delta(u-u') \), which, by the sifting property of the delta function yields

\[ C_{xy}(t, \omega) = \frac{1}{2\pi} \int x^+(u - \frac{\tau}{2}) y(u + \frac{\tau}{2}) h(u - \tau) h^+(u - \frac{\tau}{2}) e^{-i\omega\tau} du d\tau \]  

(A2)

Make the substitution

\[ \alpha = u - \frac{\tau}{2} \]

\[ \gamma = u + \frac{\tau}{2} \]

(A3)

and using the fact that \( d\alpha d\gamma = du d\tau \) we have

\[ C_{xy}(t, \omega) = \frac{1}{2\pi} \int \int x^+(\alpha) y(\gamma) h(\gamma - t) h^+(\alpha - t) e^{-i\omega(\alpha - \gamma)} d\alpha d\gamma \]  

(A4)

which is Eq. 5.

Appendix B - Derivation of Eq. 5 from Eq. 8

Taking the Fourier transform in \( \tau \) of the local correlation function, Eq. 8, yields

\[ S_1(\omega) = \frac{1}{2\pi} \int R_1(\tau) e^{-i\omega\tau} d\tau \]  

(B1)

\[ S_1(\omega) = \frac{1}{2\pi} \int \int x^+(t') y(t') h(t + \tau) e^{-i\omega\tau} dt' d\tau \]  

By definition of the short time Fourier transform, Eq. 6, we have

\[ S_1(\omega) = \frac{1}{\sqrt{2\pi}} \int x^+(t') e^{i\omega t'} Y_1(\omega) dt' \]  

(B2)

which is Eq. 5.