

STAT nnn  
George McCabe  
Name:

EXAM II.1  
mm dd, yyyy

Show all work on these pages.

- (25) 1. (a) Assume a one-way anova with the same number of observations for each treatment. Suppose the degrees of freedom for treatments is 4 and the degrees of freedom for error is 45. Find (i) the number of treatments, and (ii) the number of observations for each treatment.

$$\boxed{I = 5} \quad \boxed{J = 10}$$

- (b) Give the degrees of freedom for the three-way interaction in a  $2 \times 3 \times 4$  three-way anova.

$$1 \times 2 \times 3 = \boxed{6}$$

- (c) For a one-way anova with three groups, the sample sizes are 10, 10, and 20; and the corresponding variances are 50, 70, and 40. Find the MSE.

$$MSE = \frac{9(50) + 9(70) + 19(40)}{9 + 9 + 19} = \boxed{49.72}$$

- (d) In a one-way anova with three groups, you are interested in the difference between the mean of the first group and the average of the means of the other groups. If the MSE is 60 and the sample sizes are 10, 6, and 12, find the standard error for this difference.

$$s = \sqrt{\frac{60}{10} + \frac{60}{4(6)} + \frac{60}{4(12)}} = \boxed{3.12}$$

- (e) For a one-way anova with 3 treatments and 5 subjects per treatment, the sum of squares total is 450 and the sum of squares for treatments is 150. Find (i) the F statistic for testing no treatment differences, (ii) the degrees of freedom for the null distribution of this statistic.

Source	SS	df	MS	F
A	150	2	75	3
E	300	12	25	
T	450	14		

$$\boxed{F = 3}$$

$$\boxed{df = 2, 12}$$

- (25) 2. (a) You are using SAS to analyze a  $4 \times 2$  anova using the statements CLASS A B; MODEL Y=A B A\*B;. Write a contrast statement for the difference between  $\mu_{11}$  and  $\mu_{32}$ .

$$\begin{aligned} \mu_{11} &= \mu + \alpha_1 + \beta_1 + (\alpha\beta)_{11} \\ (-) \quad \mu_{32} &= \mu + \alpha_3 + \beta_2 + (\alpha\beta)_{32} \end{aligned}$$

CONTRAST A 1 0 -1 0 B 1 -1 AB 1 0 0 0 0 -1 0 0;

- (b) In a  $2 \times 2$  anova the population means are  $\mu_{11} = 20$ ,  $\mu_{21} = 25$ ,  $\mu_{12} = 31$ , and  $\mu_{22} = 36$ . Is there an interaction in this model? Explain why or why not.

NO

$$\begin{aligned} \mu_{11} - \mu_{21} &= -5 \\ \mu_{12} - \mu_{22} &= -5 \end{aligned}$$

- (c) In a 4-way anova with 5 observations per cell, give the total number of main effects and interactions in the anova table.

4 MAIN

$$\binom{4}{2} = 6 \text{ TWO-WAY}$$

$$\binom{4}{3} = 4 \text{ THREE-WAY}$$

$$\binom{4}{4} = 1 \text{ FOUR-WAY}$$

TOTAL:  $4 + 6 + 4 + 1 = 15$

- (d) An analysis of covariance is used to analyze data from an experiment with five treatments and two covariates. There are ten subjects in each treatment group. Find the degrees of freedom for error.

COV	2
A	4
E	49
TOTAL 49	

$$dfE = 49 - 4 - 2 = 43$$

- (e) You ran a one-way anova with three groups and calculated an  $F$ -statistic, say  $F_{1-2}$ , for comparing the means of groups 1 and 2. An examination of the residuals revealed a very large outlier in group 3. If you rerun the analysis without the outlier, what change in  $F_{1-2}$  and its  $P$ -value do you expect. Explain why you expect this change.

$F \uparrow$ ,  $P \downarrow$  because  $MSE \downarrow$   
and  $MSE$  is in denominator  
of  $F$ .

(25) 3. For this question refer to the output for PROC GLM given as OUTPUT FOR PROBLEM 3.

- (a) Write the anova model for this analysis using the cell means model. (Give explicit numerical values for number of groups being compared and the numbers of observations per group.)

$$y_{ij} = \mu_i + \varepsilon_{ij} \quad i=1, \dots, 4$$

$$\varepsilon_{ij} \text{ iid } N(0, \sigma^2) \quad j=1, \dots, 5$$

- (b) State the usual null hypothesis for this model, give the test statistic with degrees of freedom, the P-value and write one sentence describing the conclusion in words.

$$H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$$

$$H_1: \text{not all } \mu_i \text{ are the same}$$

$$F_{3,16} = 48.20, P = 0.0001$$

TREATMENT MEANS ARE NOT ALL THE SAME.

- (c) Which pairs of means differ significantly according to the Tukey HSD procedure given in the output?

2 & 3 differ from 1 & 4

- (d) What is the standard error for the estimate of the mean of the first group?

$$\frac{1.0596}{\sqrt{5}} = \boxed{0.47}$$

- (e) Give the estimate for the parameter  $\sigma^2$  of your model.

$$MSE = \boxed{1.12}$$

(25) 1. For this question refer to the output for PROC GLM given as OUTPUT FOR PROBLEM 4.

- (a) Write the anova model for the analysis marked **PART A** using the cell means model. (Give explicit numerical values for number of groups being compared and the numbers of observations per group.)

$$Y_{ijkl} = \mu_{ijk} + \epsilon_{ijkl}$$

$$\epsilon_{ijkl} \text{ iid } N(0, \sigma^2)$$

$$i=1,2; j=1,2; k=1,2; l=1, \dots, 5$$

- (b) Summarize the results of the hypothesis tests for the **PART A** analysis.

All main effects plus AB and AC are significant  
BC and ABC are not

- (c) The **PART B** analysis consists of two runs, one for each level of the factor A. Summarize the results for A at level 1.

B sig, C & BC are not

$$\left( \begin{array}{l} F_{1,16} = 757.60 \\ p = .0001 \end{array} \right) \quad \left( \begin{array}{ll} F_{1,16} = 0.16 & P = .69 \\ F_{1,16} = 2.01 & P = .18 \end{array} \right)$$

- (d) Summarize the results for A at level 2.

C sig, B + BC are not

$$\left( \begin{array}{l} F_{1,16} = 687.76 \\ p = .0001 \end{array} \right) \quad \left( \begin{array}{ll} F_{1,16} = 3.76 & P = .07 \\ F_{1,16} = 0.14 & P = .71 \end{array} \right)$$

- (e) Do you have reason to doubt the assumption of equal variances for the analyses done at the two different levels of A. Explain why or why not.

NO.

$$MSE_{A=1} = .70$$

$$MSE_{A=2} = .73$$