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Chapter 1

JOINT ENERGY-BANDWIDTH ALLOCATION FOR MULTIPLE BROADCAST CHANNELS WITH ENERGY HARVESTING

***Vaneet Aggarwal*[*1], *Xiaodong Wang*[†2] *and Zhe Wang*[‡3]**
[1]Purdue University
[2]Columbia University
[3]Columbia University

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Abstract

In this chapter, we consider the energy-bandwidth allocation for a network with multiple broadcast channels, where the transmitters access the network orthogonally on the assigned frequency band and each transmitter communicates with multiple receivers orthogonally or non-orthogonally. We assume that the energy harvesting state and channel gain of each transmitter can be predicted for K slots *a priori*. To maximize the weighted throughput, we formulate an optimization problem with $O(MK)$ constraints, where M is the number of the receivers, and decompose it into the energy and bandwidth allocation subproblems. An iterative algorithm is proposed that alternatively solves two sub-problems in each iteration for both the orthogonal and non-orthogonal broadcast channel. We also propose efficient algorithms to solve the two subproblems, so that the optimal energy-bandwidth allocation can be obtained with an overall complexity of $\mathcal{O}(MK^2)$, even though the problem is non-convex when the broadcast channel is non-orthogonal. For the orthogonal broadcast channel, we further formulate a proportionally-fair (PF) throughput maximization problem and derive the equivalence conditions such that the optimal solution can be obtained by solving a weighted throughput maximization problem. Further, the algorithm to obtain the proper weights is proposed. Simulation results show that the proposed algorithm can make efficient use of the harvested energy and the available bandwidth,

[*]E-mail address: vaneet@purdue.edu
[†]E-mail address: wangx@ee.columbia.edu
[‡]E-mail address: zhewang@ee.columbia.edu

and achieve significantly better performance than some heuristic policies for energy and bandwidth allocation. Moreover, it is seen that with energy-harvesting transmitters, non-orthogonal broadcast offers limited gain over orthogonal broadcast.

1. Introduction

With the rapid development of energy harvesting technologies, a new paradigm of wireless communications that employs energy harvesting transmitters has become a reality [1]. The renewable energy source enables the flexible deployment of the transmitters and prolongs their lifetimes [1, 2]. State-of-the-art techniques can provide fairly accurate short-term prediction of the energy harvesting process, which can be used to assist energy scheduling [3, 4]. To make the best use of the harvested energy for wireless communications, many challenging research issues arise [5, 6, 7, 8, 9, 10]. In particular, optimal resource (energy, bandwidth, etc.) scheduling is key to the design of an efficient wireless system powered by renewable energy sources.

For a single transmitter with energy harvesting, a number of works addressed the energy scheduling problem with non-causal channel state information. For static channels, [11] proposed a shortest-path-based algorithm for the energy scheduling. [8, 7] analyzed the optimality properties based on the energy causality and the optimal energy scheduling algorithm was also provided. For fading channels, a staircase water-filling algorithm was proposed in [12] for the case of infinite battery capacity; with finite battery capacity, [13] studied the energy flow behavior with an energy harvesting device and proposed a directional water-filling method. Taking the maximum transmission power into account, [6, 5] proposed a dynamic water-filling algorithm to efficiently obtain the energy schedule to maximize the achievable rate. Energy scheduling for multiuser systems with energy harvesting transmitters has also been considered. In [10], the general capacity region for a static multiple-access channel (MAC) was characterized without considering the constraints on the battery capacity and the maximum transmission power. [14] discussed the optimal power policy for energy harvesting transmitters in a two-user Gaussian interference channel. In [15], the optimal energy scheduling algorithm was proposed for a static broadcast channel with finite battery capacity constraint. Considering both the finite battery capacity and the finite maximum transmission power, the iterative dynamic water-filling algorithm was extended to the fading MAC channel [5]. Moreover, the scheduling problem in the Gaussian relay channel with energy harvesting was discussed in [9].

In this chapter, we consider a multiuser system with multiple transmitters, each powered by a renewable energy source. Each transmitter communicates with its designated receivers and is constrained by the availability of the energy, the capacity of the battery, and the maximum (average) transmission power. Moreover, a frequency band is shared by all transmitters and we assume both orthogonal channel access to avoid interference and non-orthogonal channel access. We aim to obtain the optimal joint energy-bandwidth allocation over a fixed scheduling period based on the available information on the channel states and energy harvesting states at all transmitters, to maximize the weighted sum of the achievable rate. This chapter combines parts of different results in [16, 17, 18, 19, 20, 21].

Consider the special case of equal weights and each transmitter communicates with only one receiver. Then, without energy harvesting, TDMA is optimal for the maximum

unweighted sum-rate, *i.e.,* at any time the link with the highest rate takes all bandwidth. However, for energy harvesting transmitters, TDMA is no longer optimal. This is because the finite battery capacity leads to energy discharge and waste by some transmitters that are not scheduled to transmit in a time slot. Therefore, to make the best use of the harvested energy, multiple transmitters should split the frequency band and transmit in a same slot. In this chapter, we assume that the channel is flat fading and therefore each transmitter only needs to be allocated a portion of the total bandwidth.

We consider a network with multiple transmitters, each powered by the renewable energy source. We assume that the transmitters are assigned orthogonal frequency bands to avoid interfering from each other. In orthogonal broadcast, the frequency band assigned to the transmitter is further split for the transmission to each designated receiver orthogonally (*i.e.,* no interference); on the other hand, in non-orthogonal broadcast, the transmissions to all designated receivers take place on the same frequency band assigned to the transmitter.

We first consider the orthogonal multiple broadcast channel, and formulate a convex optimization problem for weighted throughput maximization with $\mathcal{O}(MK)$ variables and constraints, where M is the number of receivers and K is the number of scheduling time slots. Since the computational complexity of a generic convex solver becomes impractically high when the number of constraints is large [22], we will develop an iterative algorithm that alternates between energy allocation and bandwidth allocation. We will show that this algorithm converges to the optimal solution of the joint energy-bandwidth scheduling problem. We also develop algorithms for solving the two subproblems. Moreover, for a single (non-orthogonal) broadcast channel with energy harvesting transmitter, the optimal energy scheduling over static and two-user fading channels was discussed in [23] and [24], respectively. In this chapter, we treat the energy-bandwidth allocation problem for multiple broadcast channels, including both orthogonal and non-orthogonal broadcast. We also reveal that the gain by non-orthognoal broadcast over orthogonal broadcast is limited with energy harvesting transmitters.

Taking the proportional fairness into account, [25] discussed the convergence of the general proportionally-fair scheduling without energy harvesting. For energy harvesting transmitters with unbounded battery capacity, heuristic algorithms have been proposed in [26] to find the time-power allocations under the proportional fairness. We formulate a proportionally-fair (PF) throughput maximization problem with orthogonal broadcast. In point-to-point channels without energy harvesting, in slot k, the optimal PF scheduler schedules the link with $\max_m R_m^k/A_m^k$, where R_m^k is the rate achievable by link m in slot k and A_m^k is the average rate of link m up to slot k. The average rate is computed over a time window as a moving average: $R_m^{k+1} = (1-\alpha)A_m^k + \alpha R_m^k$ if link m is scheduled in slot k, and $A_m^{k+1} = (1-\alpha)A_m^k$ otherwise [25]. However, in the presence of energy harvesting, using a single link is not optimal and thus scheduling multiple links in a slot and splitting the bandwidth is essential. To efficiently solve the PF throughput maximization problem, we convert it to a weighted throughput maximization problem with proper weights. The algorithm to obtain such weights is also proposed.

2. Weighted Throughput Maximization for the Multiple Orthogonal Broadcast Channel

In this section, we will optimize the weighted throughput for the multiple orthogonal broadcast channels.

2.1. System Model

Consider a network consisting of N transmitters and M receivers sharing a total bandwidth of B Hz, where $N \leq M$ and each transmitter may communicate with multiple receivers. We assume a scheduling period of K time slots and no two transmitters can transmit in the same time slot and the same frequency band. Denote $a_m^k \in [0,1]$ as the normalized bandwidth allocation for link m in time slot k. We consider a flat and slow fading channel, where the channel gain is constant within the entire frequency band of B Hz and over the coherence time of T_c seconds, which is also the duration of a time slot. Assuming that each time slot consists of T time instants, we denote X_{mki} as the symbol sent to the receiver of link m at instant i in slot k. The corresponding received signal at receiver m is given by

$$Y_{mki} = h_{mk} X_{mki} + Z_{mki} \tag{1}$$

where h_{mk} represents the complex channel gain for link m in slot k, and $Z_{mki} \sim \mathsf{CN}(0,1)$ is the i.i.d. complex Gaussian noise. We denote $H_m^k \triangleq |h_{mk}|^2$ and denote $p_m^k \triangleq \frac{1}{T_c}\sum_i |X_{mki}|^2$ as the transmission energy consumption for link m in slot k. Without loss of generality, we normalize both T_c and B to 1; then, p_m^k and a_m^k become the transmission power and the allocated bandwidth of link m in slot k, respectively. For link m, the upper bound of the achievable channel rate in slot k can be written as $a_m^k \log(1+p_m^k H_m^k/a_m^k)$ [27]. Moreover, we denote $\mathcal{K} \triangleq \{1,2,\ldots,K\}$ as the scheduling period, $\mathcal{N} \triangleq \{1,2,\ldots,N\}$ as the set of transmitters, and $\mathcal{M} \triangleq \{1,2,\ldots,M\}$ as the set of receivers. Further, we denote $\mathcal{M}_n \triangleq \{m \mid m \text{ is the receiver of transmitter } n, m \in \mathcal{M}\}$ as the set of receivers of transmitter n, where $\mathcal{M}_n \bigcap \mathcal{M}_{n'} = \phi$ for all $n \neq n' \in \mathcal{N}$.

Assume that each transmitter is equipped with an energy harvester and a buffer battery, as shown in Fig. 1. The energy harvester harvests energy from the surrounding environment. We denote E_n^k as the total energy harvested up to the end of slot k by transmitter n. Since in practice energy harvesting can be accurately predicted for a short period [3][4], we assume that the amount of the harvested energy in each slot is known. Moreover, the short-term prediction of the channel gain in slow fading channels is also possible [28]. Therefore, we assume that $\{H_m^k\}$ and $\{E_n^k\}$ are known non-causally before scheduling. Note that such non-causal assumption also leads to the performance upper bound of the system.

For transmitter n, assuming that the battery has a limited capacity $B_n^{\max}$ and is empty initially, then the battery level at the end of slot k can be written as

$$B_n^k = B_n^{k-1} + \left(E_n^k - E_n^{k-1}\right) - \sum_{\kappa=1}^{k} \sum_{m \in \mathcal{M}_n} p_m^\kappa - \sum_{\kappa=1}^{k} D_n^\kappa, \tag{2}$$

where $D_n^k \geq 0$ represents the energy discharge (waste) for transmitter n in slot k. Moreover, B_n^k must satisfy $0 \leq B_n^k \leq B_n^{\max}$ for all $k \in \mathcal{K}$.

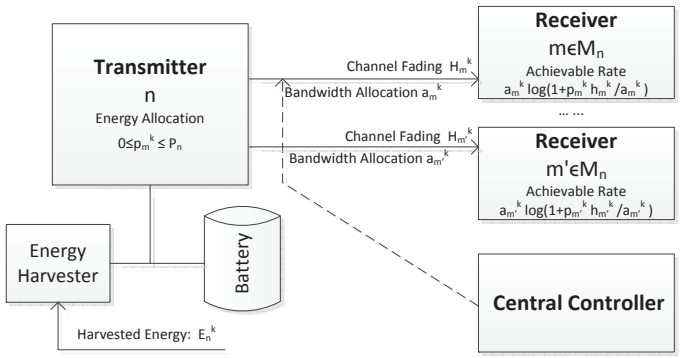

Figure 1. The system block diagram.

We assume that each transmitter n has a maximum per-slot transmission energy consumption, P_n, such that $\sum_{m\in\mathcal{M}_n} p_m^k \leq P_n$ for $k \in \mathcal{K}$. With the maximum transmission energy and the limited battery capacity, some of the harvested energy may not be able to be utilized, and is therefore wasted, *i.e.,* D_n^k may necessarily be strictly positive in some slots. Then, the constraints on the battery level can be written as

$$0 \leq E_n^k - \sum_{\kappa=1}^{k} \sum_{m\in\mathcal{M}_n} p_m^\kappa - \sum_{\kappa=1}^{k} D_n^\kappa \leq B_n^{\max}. \tag{3}$$

Moreover, we denote $\mathcal{D} \triangleq \{\boldsymbol{D}_n \mid \boldsymbol{D}_n \triangleq [D_n^1, D_n^2, \ldots, D_n^K], n \in \mathcal{N}\}$ as the *discharge allocation*. Note that, we assume controllable energy discharge, *i.e.,* the energy can be discharged and wasted anytime, even when the battery is not full.

Remark 1. *In the transmitter model, both the maximum transmission energy and the battery capacity are finite. If the harvested energy is ample, part of the energy has to be discharged even if the transmitter transmits at the maximum (available) transmission energy in each slot. That is,* $D_n^{k*} > 0$ *is due to the incoming energy being large enough that it cannot be used for transmission or storage.*

2.2. Problem Formulation

Define $0 \cdot \log(1 + \frac{x}{0}) \triangleq 0$. We use upper bounds of the achievable channel rate over a weighted sum of the M links and K slots as the performance metric, given by

$$C_{\mathcal{W}}(\mathcal{P}, \mathcal{A}) = \sum_{m\in\mathcal{M}} W_m \sum_{k\in\mathcal{K}} a_m^k \log(1 + \frac{p_m^k H_m^k}{a_m^k}),\ a_m^k \in [0,1], p_m^k \in [0,\infty), \tag{4}$$

where $\mathcal{P} \triangleq \{p_m^k, \forall m \in \mathcal{M}, k \in \mathcal{K}\}$ is the *energy allocation*, $\mathcal{A} \triangleq \{a_m^k, \forall m \in \mathcal{M}, k \in \mathcal{K}\}$ is the *bandwidth allocation*, and $\mathcal{W} \triangleq \{W_m, m \in \mathcal{M}\}$ is the *weight set*. In particular, when $W_m = 1$ for all $m \in \mathcal{M}$, $C_{\mathcal{W}}(\mathcal{P}, \mathcal{A})$ becomes the throughput of the network.

Note that, both a_m^k and p_m^k can be zero in (4). However, if $a_m^k = 0$, the channel rate of link m in slot k is zero, even if the energy allocation $p_m^k > 0$, thus p_m^k is actually wasted. However, we still treat the pair $(a_m^k = 0, p_m^k > 0)$ as feasible as long as $\sum_{m\in\mathcal{M}_n} p_m^k \leq P_n$.

We formulate the following energy-bandwidth allocation problem:

$$\mathsf{P}_{\mathcal{W}}(\epsilon): \quad \max_{\mathcal{P},\mathcal{A}} C_{\mathcal{W}}(\mathcal{P},\mathcal{A}) \tag{5}$$

subject to

$$\begin{cases} \tilde{E}_n^k - B_n^{\max} \leq \sum_{\kappa=1}^k \sum_{m\in\mathcal{M}_n} p_m^\kappa \leq \tilde{E}_n^k \\ \sum_{m=1}^M a_m^k = 1 \\ \sum_{m\in\mathcal{M}_n} p_m^k \leq P_n \\ p_m^k \geq 0 \\ a_m^k \geq \epsilon \end{cases} \tag{6}$$

for all $n \in \mathcal{N}, m \in \mathcal{M}, k \in \mathcal{K}$.

2.3. Optimal Energy Discharge Allocation

To efficiently solve the problem in (5)-(6), we consider a two-stage procedure. In the first stage, we obtain the optimal energy discharge allocation $\mathcal{D}^*$ such that

$$\max_{\mathcal{P},\mathcal{A},\mathcal{D}=\mathcal{D}^*} C_{\mathcal{W}}(\mathcal{P},\mathcal{A}) = \max_{\mathcal{P},\mathcal{A},\mathcal{D}} C_{\mathcal{W}}(\mathcal{P},\mathcal{A}) \tag{7}$$

with the constraints in (6). In the second stage, we use $\mathcal{D}^*$ and define the energy expenditure for transmission as

$$\tilde{E}_n^k \triangleq E_n^k - \sum_{\kappa=1}^k D_n^{\kappa *} . \tag{8}$$

Then we solve the following problem:

$$\max_{\mathcal{P},\mathcal{A}} C_{\mathcal{W}}(\mathcal{P},\mathcal{A}) \tag{9}$$

subject to

$$\begin{cases} \tilde{E}_n^k - B_n^{\max} \leq \sum_{\kappa=1}^k \sum_{m\in\mathcal{M}_n} p_m^\kappa \leq \tilde{E}_n^k \\ \sum_{i=1}^M a_i^k = 1 \\ \sum_{m\in\mathcal{M}_n} p_m^k \leq P_n \\ p_m^k \geq 0 \\ a_m^k \geq 0 \end{cases} \tag{10}$$

for all $n \in \mathcal{N}, m \in \mathcal{M}$ and $k \in \mathcal{K}$.

We consider the following greedy strategy to obtain the energy discharge allocation by assuming that each transmitter transmits at the maximum power in each slot, *i.e.,*

$$\begin{cases} D_n^k = \max\{B_n^{k-1} + E_n^k - E_n^{k-1} - \sum_{\kappa=1}^k \sum_{m\in\mathcal{M}_n} p_m^k - B_n^{\max}, 0\} \\ \sum_{m\in\mathcal{M}_n} p_m^k = \min\{P_n, B_n^{k-1} + E_n^k - E_n^{k-1}\} \end{cases}, \; k = 1, 2, \ldots, K \tag{11}$$

for all $n \in \mathcal{N}$.

Note that, following (11), the total discharged energy is minimized and thus the amount of the energy used for transmission is maximized. Intuitively, this way the feasible domain becomes the largest, providing the best performance for transmission energy scheduling. Specifically, given a feasible bandwidth allocation $\mathcal{A}$, the achievable rate of each link is non-decreasing with respect to the transmission energy, and the battery of each transmitters operates independently. Therefore, following the same lines of the proof in [5], the optimality of (11) can be established. In particular, using any feasible energy discharge $\mathcal{D}$ corresponding to the minimal energy wastage, the optimal value of (9) is same, which is no less than the optimal value under any feasible energy discharge allocation with non-minimal energy wastage.

Lemma 1. *The discharge allocation given by* (11) *is the optimal* $\mathcal{D}^*$ *to the problem in* (5)-(6)*,* i.e., *it satisfies* (7)*, where the LHS of* (7) *is subject to the constraints in* (10) *and the RHS is subject to the constraints in* (6)*.*

Note that, $C_{\mathcal{W}}(\mathcal{P},\mathcal{A})$ is continuous and jointly concave with respect to $a_m^k \in [0,1]$ and $p_m^k \in [0,\infty)$ for $k \in \mathcal{K}, m \in \mathcal{M}$. Then, the problem in (9)-(10) is a convex optimization problem and can be solved by a generic convex solver, whose complexity becomes impractically high when the number of constraints is large [22], which in this case is $\mathcal{O}(MK)$. To reduce the computational complexity, we will develop an efficient algorithm in this section, which exploits the structure of the optimal solution.

2.4. K.K.T. Conditions for Non-Zero Bandwidth Allocation

The problem in (9)-(10) is a convex optimization problem with linear constraints. When the objective function is differentiable in an open domain, the K.K.T. conditions are sufficient and necessary for the optimal solution [22]. Note that, (4) is non-differentiable at $a_m^k = 0$. To use the K.K.T. conditions to characterize the optimality of the problem in (9)-(10), we consider the following approximation:

$$\mathsf{P}_{\mathcal{W}}(\epsilon): \quad \max_{\mathcal{P},\mathcal{A}} C_{\mathcal{W}}(\mathcal{P},\mathcal{A}) \tag{12}$$

subject to

$$\begin{cases} \tilde{E}_n^k - B_n^{\max} \leq \sum_{\kappa=1}^{k} \sum_{m\in\mathcal{M}_n} p_m^\kappa \leq \tilde{E}_n^k \\ \sum_{i=1}^{M} a_i^k = 1 \\ \sum_{m\in\mathcal{M}_n} p_m^k \leq P_n \\ p_m^k \geq 0 \\ a_m^k \geq \epsilon \end{cases} \tag{13}$$

for all $n \in \mathcal{N}, m \in \mathcal{M}, k \in \mathcal{K}$, where ϵ is a small positive number. In particular, $\mathsf{P}_{\mathcal{W}}(0)$ is the original problem in (9)-(6).

Lemma 2. *When* $\epsilon \to 0^+$*, the optimal value of* $\mathsf{P}_{\mathcal{W}}(\epsilon)$ *converges to the optimal value of the problem in* (9)-(6)*,* i.e., $\lim_{\epsilon\to 0^+} \mathsf{P}_{\mathcal{W}}(\epsilon) = \mathsf{P}_{\mathcal{W}}(0)$.

Proof. Since the objective function $C_{\mathcal{W}}(\mathcal{P},\mathcal{A})$ is continuous with respect to $\mathcal{P}\times\mathcal{A} \in \{[0,\infty)\}\times\{[0,1]\}$ and the constraints in (13) are all linear, we have that the optimal solution of $\mathsf{P}_{\mathcal{W}}(\epsilon)$ is continuous with respect to ϵ, *i.e.,* $\lim_{\epsilon\to 0^+} \arg \mathsf{P}_{\mathcal{W}}(\epsilon) = \arg \mathsf{P}_{\mathcal{W}}(0)$. Therefore, we have $\lim_{\epsilon\to 0^+} \mathsf{P}_{\mathcal{W}}(\epsilon) = \mathsf{P}_{\mathcal{W}}(0)$. □

By introducing the auxiliary variables $\{\lambda_n^k \geq 0\}$, $\{\mu_n^k \geq 0\}$, $\{\beta_m^k \geq 0\}$ and $\{\alpha^k\}$ and converting the constraints in (13) into the Lagrangian multiplier, we can define the Lagrangian function for $\mathsf{P}_{\mathcal{W}}(\epsilon)$ as

$$\begin{aligned}\mathcal{L} \triangleq \quad & \sum_{m=1}^{M} W_m \sum_{k=1}^{K} a_m^k \cdot \log(1+\frac{p_m^k H_m^k}{a_m^k}) - \sum_{n=1}^{N}\sum_{k=1}^{K}\Big(\sum_{m\in\mathcal{M}_n} p_m^k \sum_{\kappa=k}^{K}\lambda_n^\kappa - \lambda_n^k \tilde{E}_n^k\Big) \\ & + \sum_{n=1}^{N}\sum_{k=1}^{K}\Big(\sum_{m\in\mathcal{M}_n} p_m^k \sum_{\kappa=k}^{K}\mu_n^\kappa - \mu_n^k\big(\tilde{E}_n^k - B_n^{\max}\big)\Big) \\ & - \sum_{k=1}^{K}\alpha^k(\sum_{m=1}^{M} a_m^k - 1) + \sum_{k=1}^{K}\sum_{m=1}^{M}\beta_m^k(a_m^k - \epsilon). \end{aligned} \tag{14}$$

Then, the following K.K.T. conditions, which are sufficient and necessary for the optimal solution to the convex optimization problem in (12)-(13), are obtained from the Lagrangian function:

$$\frac{H_m^k}{1+p_m^k H_m^k/a_m^k} = (v_n^k - u_n^k)/W_m,\, k\in\mathcal{K},\ n\in\mathcal{N},\ m\in\mathcal{M}_n \tag{15}$$

$$\log(1+\frac{p_m^k H_m^k}{a_m^k}) - \frac{p_m^k H_m^k}{a_m^k + p_m^k H_m^k} = (\alpha^k - \beta_n^k)/W_m,\, k\in\mathcal{K},\ n\in\mathcal{N},\ m\in\mathcal{M}_n \tag{16}$$

$$\lambda_n^k \cdot (\sum_{\kappa=1}^{k}\sum_{m\in\mathcal{M}_n} p_m^\kappa - \tilde{E}_n^k) = 0,\, k\in\mathcal{K},\ n\in\mathcal{N} \tag{17}$$

$$\mu_n^k \cdot (\sum_{\kappa=1}^{k}\sum_{m\in\mathcal{M}_n} p_m^\kappa - \tilde{E}_n^k + B_n^{\max}) = 0,\, k\in\mathcal{K},\ n\in\mathcal{N} \tag{18}$$

$$\alpha^k \cdot (\sum_{m=1}^{M} a_m^k - 1) = 0,\, k\in\mathcal{K} \tag{19}$$

$$\beta_m^k \cdot (a_m^k - \epsilon) = 0,\, k\in\mathcal{K},\ m\in\mathcal{M} \tag{20}$$

together with the constraints in (13), and $\lambda_n^k, \mu_n^k, \beta_m^k \geq 0$ for all $k \in \mathcal{K}$, $n \in \mathcal{N}$, and $m \in \mathcal{M}$, where in (27)

$$u_n^k \triangleq \sum_{\kappa=k}^{K}\mu_n^\kappa,\ v_n^k \triangleq \sum_{\kappa=k}^{K}\lambda_n^\kappa\,. \tag{21}$$

2.5. Iterative Algorithm

In this subsection, we will first decompose the energy-bandwidth allocation problem $\mathsf{P}_{\mathcal{W}}(\epsilon)$ in (12)-(13) into two subproblems, and then propose an iterative algorithm to solve $\mathsf{P}_{\mathcal{W}}(\epsilon)$. We will prove that the iterative algorithm converges to the optimal solution to the problem in (9)-(10).

To efficiently solve problem $\mathsf{P}_{\mathcal{W}}(\epsilon)$ in (12)-(13), we first decompose it into two groups of subproblems, corresponding to energy allocation and bandwidth allocation, respectively.

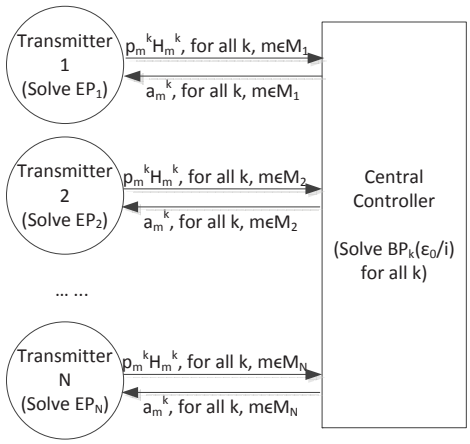

Figure 2. The block diagram of Algorithm 1.

- Given the bandwidth allocation $\mathcal{A} = \{\boldsymbol{a}^k \mid k \in \mathcal{K}\}$, for each $n \in \mathcal{N}$, obtain the energy allocation $\boldsymbol{p}_m$ by solving the following subproblem:

$$\mathsf{EP}_n: \quad \max_{\boldsymbol{p}_m, m\in\mathcal{M}_n} \sum_{m\in\mathcal{M}_n} W_m \sum_{k=1}^{K} a_m^k \cdot \log(1+\frac{p_m^k H_m^k}{a_m^k}) \tag{22}$$

subject to

$$\begin{cases} \tilde{E}_n^k - B_n^{\max} \le \sum_{\kappa=1}^{k} \sum_{m\in\mathcal{M}_n} p_m^{\kappa} \le \tilde{E}_n^k \\ \sum_{m\in\mathcal{M}_n} p_m^k \le P_n \\ p_m^k \ge 0, m \in \mathcal{M}_n \end{cases}, \ k \in \mathcal{K}\,. \tag{23}$$

- Given the energy allocation $\mathcal{P} = \{\boldsymbol{p}_m \mid m \in \mathcal{M}\}$, for each $k \in \mathcal{K}$, obtain the bandwidth allocation $\boldsymbol{a}^k$ by solving the following subproblem:

$$\mathsf{BP}_k(\epsilon): \quad \max_{\boldsymbol{a}^k} \sum_{m=1}^{M} W_m \cdot a_m^k \cdot \log(1+\frac{p_m^k H_m^k}{a_m^k}) \tag{24}$$

subject to

$$\begin{cases} \sum_{i=1}^{M} a_i^k = 1 \\ a_m^k \ge \epsilon, \ m \in \mathcal{M} \end{cases}. \tag{25}$$

To obtain the optimal solution to the original problem in (9)-(10), we propose an iterative algorithm that alternatively solves EP_n for all $n \in \mathcal{N}$ and $\mathsf{BP}_k(\epsilon)$ for all $k \in \mathcal{K}$, with a diminishing ϵ over the iterations. To perform the algorithm, we initially set $a_m^k = 1/M, \forall m, k$, and solve EP_n to obtain the initial $\mathcal{P}$. In each iteration i, we first solve $\mathsf{BP}_k(\epsilon_0/i)$ to update $\boldsymbol{a}^k \in \mathcal{A}$ for all $k \in \mathcal{K}$, where ϵ_0 is a pre-specified positive value; with the updated $\mathcal{A}$, we then solve EP_n to update $\boldsymbol{p}_m \in \mathcal{P}$ for all $m \in \mathcal{M}$.

The proposed iterative algorithm is summarized in Algorithm 1 and its block diagram is shown in Fig. 2.

Algorithm 1 - Iterative Energy-Bandwidth Allocation Algorithm

1: Initialization
$i = 0$, $\mathcal{A} = 1/M$, $V^{(0)} = 0$, Choose any $\epsilon_0 > 0$, Solve EP_n for all $n \in \mathcal{N}$ to generate the initial $\mathcal{P}$
Specify the maximum number of iterations I, the convergence tolerance $\delta > 0$
2: Energy-Bandwidth Allocation
REPEAT
$i \leftarrow i + 1$, $\epsilon \leftarrow \epsilon_0/i$
Solve $\mathsf{BP}_k(\epsilon)$ to update $\boldsymbol{a}^k \in \mathcal{A}$ for all $k \in \mathcal{K}$
Solve EP_n to update $\{\boldsymbol{p}_m \mid m \in \mathcal{M}_n\} \subset \mathcal{P}$ for all $n \in \mathcal{N}$
$V^{(i)} = C_{\mathcal{W}}(\mathcal{P}, \mathcal{A})$
UNTIL $|V^{(i)} - V^{(i-1)}| < \delta$ **OR** $i = I$

In the next subsection, we will show that Algorithm 1 converges and the pairwise optimal $\mathcal{A}$ and $\mathcal{P}$ can be obtained, which is also the optimal solution to the problem in (9)-(10).

We note that, $\mathcal{P}_{\mathcal{W}}(\epsilon)$ is a convex optimization problem with $O(MK)$ variables and constraints. The computational complexity of using the generic convex solver is non-linear with respect to the number of the variables and constraints, which may be impractically high when M and K become large. Using Algorithm 1, the optimal solution to $\mathcal{P}_{\mathcal{W}}(\epsilon)$ can be obtained by solving $\mathcal{O}(N + K)$ convex optimization subproblems which contains $\mathcal{O}(K|\mathcal{M}_n|)$ or $\mathcal{O}(M)$ variables and constraints. Therefore, the overall computational complexity can be significantly reduced with Algorithm 1 for large M and K.

2.6. Proof of Optimality of Iterative Algorithm

We first give the following proposition.

Proposition 1. *Given any bandwidth allocation $\{a_m^k > 0 \mid k \in \mathcal{K}\}, m \in \mathcal{M}$, the optimal energy allocation for the problem EP_n is unique. Also, given the energy allocation $\{p_m^k \mid m \in \mathcal{M}\}, k \in \mathcal{K}$ such that $\sum_{m=1}^{M} p_m^k > 0$, the corresponding optimal bandwidth allocation for the problem $\mathsf{BP}_k(\epsilon)$ is unique.*

Proof. This proposition can be obtained by verifying the strict concavity of $C_{\mathcal{W}}(\mathcal{P}, \mathcal{A})$ with respect to $\mathcal{P}$ given $\mathcal{A}$, and with respect to $\mathcal{A}$ given $\mathcal{P}$. □

Given a pair $(\mathcal{P}, \mathcal{A})$, if $\boldsymbol{p}_m \in \mathcal{P}$ is the optimal solution to EP_n for all $n \in \mathcal{N}$ given $\mathcal{A}$, and $\boldsymbol{a}^k \in \mathcal{A}$ is the optimal solution to $\mathsf{BP}_k(\epsilon)$ for all $k \in \mathcal{K}$ given $\mathcal{P}$, we say that $\mathcal{P}$ and $\mathcal{A}$ are pairwise optimal for $\mathsf{P}_{\mathcal{W}}(\epsilon)$. We also note that, for each subproblem, its K.K.T. conditions form a subset of those of $\mathsf{P}_{\mathcal{W}}(\epsilon)$ given the other primal variables, where any two subsets contain no common dual variable. Then, if the primal variables are pairwise optimal, the K.K.T. conditions in each corresponding subset are satisfied and hence all K.K.T. conditions of $\mathsf{P}_{\mathcal{W}}(\epsilon)$ are satisfied, *i.e.,* the pairwise optimal solution is also the optimal energy-bandwidth allocation for $\mathsf{P}_{\mathcal{W}}(\epsilon)$.

Theorem 1. *The energy-bandwidth allocation $\{\mathcal{P}, \mathcal{A}\}$ is the optimal solution to $\mathsf{P}_{\mathcal{W}}(\epsilon)$ for any $\epsilon > 0$, if and only if, $\{\boldsymbol{p}_m, m \in \mathcal{M}_n\} \in \mathcal{P}$ is optimal to $\mathsf{P}_{\mathcal{W}}(\epsilon)$ given $\{\mathcal{P} \backslash \{\boldsymbol{p}_m, m \in \mathcal{M}_n\}, \mathcal{A}\}$ for all $n \in \mathcal{N}$, and $\boldsymbol{a}^k \in \mathcal{A}$ is optimal to $\mathsf{P}_{\mathcal{W}}(\epsilon)$ given $\{\mathcal{A} \backslash \boldsymbol{a}^k, \mathcal{P}\}$ for all $k \in \mathcal{K}$.*

We note that, for $\mathsf{BP}_k(\epsilon)$, when $\sum_{m=1}^{M} p_m^k = 0$, the objective value is zero for all feasible bandwidth allocations. Therefore, we can fix $a_m^k = 1/M$ as the optimal bandwidth allocation for this case in Algorithm 1. Then, by Proposition 1, we have that the optimal solution to each subproblem in Algorithm 1 is unique. The next theorem establishes the optimality of Algorithm 1.

Theorem 2. *Algorithm 1 converges; and the converged solution* $(\mathcal{P}, \mathcal{A})$ *is the optimal solution to the problem in* (9)-(10).

Proof. We note that, the feasible domain of $\mathsf{BP}_k(\epsilon_0/i)$ expands with iterations while the feasible domain of EP_n remains unchanged. Since we successively solve the maximization problems EP_n and $\mathsf{BP}_k(\epsilon_0/i)$ in iteration i, we have that the objective value is non-decreasing over the iterations. On the other hand, the objective function is upper bounded by $C_{\mathcal{W}}(\mathcal{P}, \mathcal{A}) \leq \sum_{m=1}^{M} \sum_{k=1}^{K} \log(1 + P_m H_m^k)$ therefore the algorithm converges. Since the feasible domain of $\mathsf{P}_{\mathcal{W}}(\epsilon)$ is a closed set for $\epsilon \geq 0$, at the converged point V, we can find the corresponding $\mathcal{P}_0$ and $\mathcal{A}_0$ which are pairwise optimal for $\mathsf{P}_{\mathcal{W}}(\epsilon)$ otherwise $V = C_{\mathcal{W}}(\mathcal{P}_0, \mathcal{A}_0)$ can be increased by performing another iteration. Specifically, if V is reached within finite iterations m', $\mathcal{P}_0$ and $\mathcal{A}_0$ are pairwise optimal for $\mathsf{P}_{\mathcal{W}}(\epsilon_0/i')$; otherwise, $\mathcal{P}_0$ and $\mathcal{A}_0$ are pairwise optimal for $\mathsf{P}_{\mathcal{W}}(0)$.

We first consider the case that V is reached within finite iterations i'. Since V is reached within finite iterations i', we have that $\mathcal{P}_0$ and $\mathcal{A}_0$ are pairwise optimal for both $\mathsf{P}_{\mathcal{W}}(\epsilon_0/i')$ and $\mathsf{P}_{\mathcal{W}}(\epsilon_0/(i'-1))$. Then, by Theorem 1, $(\mathcal{P}_0, \mathcal{A}_0)$ is the optimal solution to $\mathsf{P}_{\mathcal{W}}(\epsilon_0/i')$ and $\mathsf{P}_{\mathcal{W}}(\epsilon_0/(i'-1))$. Note that, since the feasible domain of $\mathsf{P}_{\mathcal{W}}(\epsilon_0/i')$ is expanded from that of $\mathsf{P}_{\mathcal{W}}(\epsilon_0/(i'-1))$ by decreasing ϵ, $(\mathcal{P}_0, \mathcal{A}_0)$ is not on the boundary of $a_n^k \geq \epsilon_0/i'$, *i.e.,* the equality of $a_m^k \geq \epsilon_0/i'$ does not hold. Therefore, continually expanding the feasible domain of $\mathsf{P}_{\mathcal{W}}(\epsilon)$ by decreasing ϵ from ϵ_0/i' to 0, $(\mathcal{P}_0, \mathcal{A}_0)$ remains at a local optimal point and thus also a global optimal point according to the domain's convexity.

We then consider the case that V can only be approached with infinite iterations. For this case, we have that $\mathcal{P}_0$ and $\mathcal{A}_0$ are pairwise optimal for $\mathsf{P}_{\mathcal{W}}(0)$. However, we note that, even so, $(\mathcal{P}_0, \mathcal{A}_0)$ is not necessarily optimal solution to $\mathsf{P}_{\mathcal{W}}(0)$ when $a_m^k = 0$ for some $m \in \mathcal{M}$ and $k \in \mathcal{K}$. To show the optimality of $(\mathcal{P}_0, \mathcal{A}_0)$, we use the proof by contradiction. Suppose that $\mathcal{P}_0$ and $\mathcal{A}_0$ are pairwise optimal for $\mathsf{P}_{\mathcal{W}}(0)$ but $(\mathcal{P}_0, \mathcal{A}_0)$ is not an optimal solution to $\mathsf{P}_{\mathcal{W}}(0)$. Denote $\mathcal{Z} \triangleq \{(m,k) \mid a_m^k = 0, a_m^k \in \mathcal{A}_0, p_m^k \in \mathcal{P}_0\}$ as the set of the links with zero bandwidth allocation. Since $\mathcal{P}_0$ and $\mathcal{A}_0$ are pairwise optimal for $\mathsf{P}_{\mathcal{W}}(0)$, we have that all K.K.T. conditions hold except for the links $(n,k) \in \mathcal{Z}$, *i.e.,* excluding the links in $\mathcal{Z}$, $(\mathcal{P}_0, \mathcal{A}_0)$ is optimal for $\mathsf{P}_{\mathcal{W}}(0)$ by Theorem 1 (excluding the links in $\mathcal{Z}$, the problem $\mathsf{P}_{\mathcal{W}}(0)$ is equivalent to $\mathsf{P}_{\mathcal{W}}(\epsilon')$ where ϵ' is the remaining smallest bandwidth allocation). However, since we also have that $(\mathcal{P}_0, \mathcal{A}_0)$ is not optimal for $\mathsf{P}_{\mathcal{W}}(0)$, then we know that $\{a_m^k = 0, p_m^k = 0 \mid (m,k) \in \mathcal{Z}\}$ is suboptimal, *i.e.,* we can always reassign an arbitrary small bandwidth from some non-zero bandwidth link to a zero bandwidth link and then perform EP_n to achieve a new objective value which is higher than V. Obviously, due to the increase of the objective value, the energy allocation of the link with the newly assigned bandwidth must increase from zero to a positive value after solving EP_n with the new bandwidth allocation. Specifically, for a link $(m,k) \in \mathcal{Z}$, if reassigning an arbitrary small bandwidth can result in the corresponding p_m^k increased form zero to a positive value, we must have $H_m^k > v_n^k - u_n^k$ such that $m \in \mathcal{M}_n$, *i.e.,* according to the water-filling

solution, $v_n^k - u_n^k$ increases after solving EP_n with the new $a_m^k > 0$ while the new p_m^k must be positive.

However, in each specific iteration, we have $a_m^k > 0$ and the optimal solution to EP_n satisfies $H_m^k \leq v_n^k - u_n^k$ such that $m \in \mathcal{M}_n$ when $p_m^k = 0$. Note that, the objective function is continuous and the problem is a convex optimization problem. Then, following the algorithm, when a_m^k converges to zero, we also have $H_m^k \leq v_n^k - u_n^k$ when $p_m^k = 0$, which is contradiction to the above suboptimal assumption. Therefore, the converged objective value must be the optimal value for problem $\mathsf{P}_{\mathcal{W}}(0)$. □

Note that, the convergence is due to the expansion of the feasible domain by reducing ϵ resulting in the increasing objective value over iterations. The optimality can be proved by first verifying the pairwise optimality of the solution upon convergence and then showing it cannot be suboptimal.

2.7. Example Scenario

For the special case that all links have equal weights, *e.g.,* $\mathcal{W} = \{W_m = 1, m \in \mathcal{M}\}$, the following result states that each transmitter should only use its strongest channel.

Theorem 3. *The problem $P_{\{1\}}(0)$ in multiple orthogonal broadcast channels is equivalent to the energy-bandwidth allocation problem in point-to-point channels formulated as*

$$\max_{\mathcal{P},\mathcal{A}} \sum_{n\in\mathcal{N},k\in\mathcal{K}} a_{m_n^k}^k \log\left(1 + \frac{p_{m_n^k}^k H_{m_n^k}^k}{a_{m_n^k}^k}\right) \tag{26}$$

subject to the constraints in (6)*, where $m_n^k \triangleq \arg\max_{m\in\mathcal{M}_n}\{H_m^k\}$ for each $k \in \mathcal{K}$.*

Proof. The first-order condition is necessary for optimality, which can be written as

$$\frac{H_m^k}{1 + p_m^k H_m^k / a_m^k} = \frac{v_n^k - u_n^k + \xi_n^k}{W_m},\ m \in \mathcal{M}_n\ , \tag{27}$$

with
$$u_n^k \triangleq \sum_{\kappa=k}^{K} \mu_n^\kappa\ ,$$
$$v_n^k \triangleq \sum_{\kappa=k}^{K} \lambda_n^\kappa. \tag{28}$$

By setting $W_m = 1$, we then have

$$p_m^k = a_m^k \left[\frac{1}{v_n^k - u_n^k + \xi_n^k} - \frac{1}{H_m^k}\right]^+ . \tag{29}$$

When $\sum_{m\in\mathcal{M}} p_m^k > 0$ and $\epsilon = 0$, the optimal bandwidth allocation is given as [29]

$$a_m^k = \frac{p_m^k H_m^k}{\sum_{j\in\mathcal{M}} p_j^k H_j^k},\ m \in \mathcal{M}\ . \tag{30}$$

Then, for any transmitter n such that $\sum_{m\in\mathcal{M}_n} p_m^k > 0$ and denoting $\Delta \triangleq \sum_{m\in\mathcal{M}_n} a_m^k$, we further have

$$a_m^k = \frac{p_m^k H_m^k \Delta}{\sum_{j\in\mathcal{M}_n} p_j^k H_j^k},\ m \in \mathcal{M}_n \subseteq \mathcal{M}\ . \tag{31}$$

Substituting (31) into (29), we then have

$$p_m^k = \frac{p_m^k H_m^k}{\sum_{j\in\mathcal{M}} p_j^k H_j^k}\left[\frac{1}{v_n^k - u_n^k + \xi_n^k} - \frac{1}{H_m^k}\right]^+ \Delta,\ m \in \mathcal{M}_n\ . \tag{32}$$

Replacing p_j^k in (32) by (29), we have

$$p_m^k = p_m^k \frac{\left[\frac{1}{v_n^k - u_n^k + \xi_n^k} - \frac{1}{H_m^k}\right]^+ H_m^k \Delta}{a_m^k \left[\frac{1}{v_n^k - u_n^k + \xi_n^k} - \frac{1}{H_m^k}\right]^+ H_m^k + \sum_{j\in\mathcal{M}_n, j\neq m} a_j^k \left[\frac{1}{v_n^k - u_n^k + \xi_n^k} - \frac{1}{H_j^k}\right]^+ H_j^k}\ . \tag{33}$$

When $p_m^k > 0$, $\left[\frac{1}{v_n^k - u_n^k + \xi_n^k} - \frac{1}{H_m^k}\right]^+ > 0$ and (33) can be further written as

$$1 = \frac{\Delta}{a_m^k + \left(\sum_{j\in\mathcal{M}_n, j\neq m} a_j^k \left[\frac{1}{v_n^k - u_n^k + \xi_n^k} - \frac{1}{H_j^k}\right]^+ H_j^k\right) / \left(\left[\frac{1}{v_n^k - u_n^k + \xi_n^k} - \frac{1}{H_m^k}\right]^+ H_m^k\right)}\ , \tag{34}$$

$$\Rightarrow \quad \Delta = a_m^k + \sum_{j\in\mathcal{M}_n, j\neq m} a_j^k \left(\frac{\left[1/(v_n^k - u_n^k + \xi_n^k) - 1/H_j^k\right]^+}{[1/(v_n^k - u_n^k + \xi_n^k) - 1/H_m^k]^+} \frac{H_j^k}{H_m^k}\right)\ . \tag{35}$$

Moreover, according to the definition of Δ, we also have

$$a_m^k + \sum_{j\in\mathcal{M}_n, j\neq m} a_j^k \cdot 1 = \Delta\ . \tag{36}$$

Denoting $m_n^k \triangleq \max_{m\in\mathcal{M}_n}\{H_m^k\}$, by (29) and (31), we have $p_{m_n^k}^k > 0$ when $\sum_{m\in\mathcal{M}_n} p_m^k > 0$. Note that, since

$$\frac{\left[1/(v_n^k - u_n^k + \xi_n^k) - 1/H_j^k\right]^+}{\left[1/(v_n^k - u_n^k + \xi_n^k) - 1/H_{m_n^k}^k\right]^+} \cdot \frac{H_j^k}{H_{m_n^k}^k} \leq 1 \tag{37}$$

for all $j \in \{m \in \mathcal{M}_n \mid m \neq m_n^k\}$, we must have $a_j^k = 0$ for all $j \in \{m \in \mathcal{M}_n \mid m \neq m_n^k\}$ so that (35) and (36) are both satisfied.

Therefore, when $\sum_{m\in\mathcal{M}_n} p_m^k > 0$, we must have $p_{m_n^k}^k > 0$ and $p_j^k = 0$ for $\{\forall j \in \mathcal{M}_n \mid j \neq m_n^k\}$. On the other hand, when $\sum_{m\in\mathcal{M}_n} p_m^k = 0$, we have $p_m^k = 0$ for all $m \in \mathcal{M}_n$ given n and k thus the achievable rate is zero no matter which channel is selected. □

2.8. Optimal Algorithms for Solving Subproblems

For the general weighted sum-rate problem, the iterative algorithm developed in Algorithm 1 decomposes $\mathsf{P}_{\mathcal{W}}(\epsilon)$ as follows.

- Given the bandwidth allocation $\mathcal{A}_n \triangleq \{a_m^k, \forall m \in \mathcal{M}_n, k \in \mathcal{K}\}$, for each $n \in \mathcal{N}$, obtain the energy allocation $\boldsymbol{p}_m \triangleq [p_m^1, p_m^2, \ldots, p_m^K]$ by solving the following subproblem:

$$\mathsf{EP}_n(\mathcal{A}_n, \mathcal{W}): \quad \max_{\boldsymbol{p}_m, m\in\mathcal{M}_n} \sum_{m\in\mathcal{M}_n} W_m \sum_{k=1}^{K} a_m^k \log(1+\frac{p_m^k H_m^k}{a_m^k}) \tag{38}$$

subject to

$$\begin{cases} \tilde{E}_n^k - B_n^{\max} \leq \sum_{\kappa=1}^{k} \sum_{m\in\mathcal{M}_n} p_m^{\kappa} \leq \tilde{E}_n^k \\ \sum_{m\in\mathcal{M}_n} p_m^k \leq P_n \\ p_m^k \geq 0,\ m \in \mathcal{M}_n \end{cases}, \ k \in \mathcal{K}\,. \tag{39}$$

- Given the energy allocation $\mathcal{P}_k \triangleq \{p_m^k, \forall m \in \mathcal{M}\}$, for each $k \in \mathcal{K}$, obtain the bandwidth allocation $\boldsymbol{a}^k \triangleq [a_1^k, a_2^k, \ldots, a_M^k]$ by solving the following subproblem:

$$\mathsf{BP}_k(\mathcal{P}_k, \epsilon, \mathcal{W}): \quad \max_{\boldsymbol{a}^k} \sum_{m=1}^{M} W_m a_m^k \log(1+\frac{p_m^k H_m^k}{a_m^k}) \tag{40}$$

subject to

$$\begin{cases} \sum_{i=1}^{M} a_i^k = 1 \\ a_m^k \geq \epsilon,\ m \in \mathcal{M} \end{cases}. \tag{41}$$

2.8.1. Solving the Bandwidth Allocation Subproblem

Based on the Lagrangian function defined in (14), the first-order condition and the complementary slackness of the bandwidth allocation problem can be written as

$$\log(1+\frac{p_m^k H_m^k}{a_m^k}) - \frac{p_m^k H_m^k}{a_m^k + p_m^k H_m^k} = \frac{(\alpha^k - \beta_m^k)}{W_m}\,, \tag{42}$$

$$\alpha^k(\sum_m a_m^k - 1) = 0, \tag{43}$$

$$\beta_m^k(a_m^k - \epsilon) = 0, \tag{44}$$

which along with the constraints in (41) constitute the K.K.T. conditions of $\mathsf{BP}_k(\mathcal{P}_k, \epsilon, \mathcal{W})$. Since $\mathsf{BP}_k(\mathcal{P}_k, \epsilon, \mathcal{W})$ is a convex optimization problem with linear constraints, its K.K.T. conditions are sufficient and necessary for optimality when $\epsilon > 0$ [22].

Denote $x_m^k = X_m(\alpha^k, \beta_m)$ as the solution to

$$x_m^k - \log(x_m^k) = (\alpha^k - \beta_m^k)/W_m + 1\,, 0 < x_m < 1. \tag{45}$$

Note that, for $x \in (0,1)$, $x - \log(x) \in (1, \infty)$. Then, $x_m^k \in (0,1)$ exists when $\alpha^k - \beta_m^k \geq 0$ and the bandwidth allocation given by

$$a_m^k = p_m^k H_m^k \frac{X_m(\alpha^k, \beta_m^k)}{1 - X_m(\alpha^k, \beta_m^k)}, \ (0 < X_m(\alpha^k, \beta_m^k) < 1) \tag{46}$$

for $p_m^k > 0$ satisfies the first-order condition in (42).

When $p_m^k = 0$, we have $\alpha^k = \beta_m^k \geq 0$ by (42). If $\alpha^k = \beta_m^k > 0$, we have $a_m^k = \epsilon$ by (44). Otherwise, we can set $a_m^k = \epsilon$ and the K.K.T. conditions still hold. Thus the minimal bandwidth should be assigned to the receiver with zero transmission energy.

We note that, if there exists an m such that $p_m^k > 0$, the left-hand-side of (42) is greater than 0 and thus $\alpha^k > 0$. Then, by (43), $\sum_m a_m^k = 1$ must hold. Assigning the minimal bandwidth to the receiver with zero transmission energy and substituting (46), we further have

$$\sum_{m \in \mathcal{Z}_0^c} p_m^k H_m^k \frac{X_m(\alpha^k, \beta_m)}{1 - X_m(\alpha, \beta_m)} + |\mathcal{Z}_0|\epsilon = 1 \ , \tag{47}$$

where $\mathcal{Z}_0 \triangleq \{m \mid p_m^k = 0\} = \{m \mid p_m^k = 0, a_m^k = \epsilon\}$ and $\mathcal{Z}_0^c$ is the complementary set of $\mathcal{Z}_0$. Moreover, by (44), we know that $\beta_m^k = 0$ when $a_m^k > \epsilon$. Then, (47) can be further written as

$$\sum_{m \in \mathcal{Z}_1^c \cap \mathcal{Z}_0^c} p_m^k H_m^k \frac{X_m(\alpha^k, 0)}{1 - X_m(\alpha^k, 0)} + |\mathcal{Z}_1|\epsilon = 1 - |\mathcal{Z}_0|\epsilon \ , \tag{48}$$

where $\mathcal{Z}_1 \triangleq \{m \mid p_m^k > 0, \beta_m^k > 0\}$.

Note that, for any $m \in \mathcal{Z}_1$, we have

$$a_m^k = p_m^k H_m^k \frac{X_m(\alpha^k, \beta_m^k)}{1 - X_m(\alpha^k, \beta_m^k)} = \epsilon, \ (\beta_m^k > 0) \ . \tag{49}$$

According to (45), since $X_m(\alpha, \beta)$ is decreasing with respect to α and increasing with respect to $\beta \geq 0$ when $X_m(\alpha, 0) \in (0,1)$, then so does $\frac{X_m(\alpha,\beta)}{1 - X_m(\alpha,\beta)}$. Hence, we further have

$$p_m^k H_m^k \frac{X_m(\alpha^k, 0)}{1 - X_m(\alpha^k, 0)} \leq p_m^k H_m^k \frac{X_m(\alpha^k, \beta_m^k)}{1 - X_m^k(\alpha^k, \beta_m^k)} = \epsilon, \ m \in \mathcal{Z}_1 \ . \tag{50}$$

Therefore, (48) can be written as

$$\sum_{m \in \mathcal{Z}_0^c} \max\left\{\epsilon, p_m^k H_m^k \frac{X_m(\alpha^k, 0)}{1 - X_m(\alpha^k, 0)}\right\} = 1 - |\mathcal{Z}_0|\epsilon \ . \tag{51}$$

Theorem 4. *Suppose that α^k is the solution to* (51)*. Then, the optimal bandwidth allocation for $\mathsf{BP}_k(\mathcal{P}_k, \epsilon, \mathcal{W})$ is given by*

$$a_m^k = \begin{cases} \epsilon, & \text{if } p_m^k = 0 \\ \max\left\{\epsilon, p_m^k H_m^k \frac{X_m(\alpha^k, 0)}{1 - X_m(\alpha^k, 0)}\right\}, & \text{if } p_m^k > 0 \end{cases} \ . \tag{52}$$

Proof. The first term in (52) follows since the minimal bandwidth should be allocated to the receiver with zero transmission energy. Also, by (50) and (46) we have the second term in (52). Moreover, when α^k satisfies (51), all K.K.T. conditions of the bandwidth allocation problem are satisfied therefore the optimal bandwidth allocation is obtained. □

Denote

$$G(\alpha) \triangleq \sum_{m \in \mathcal{Z}_0^c} \max\left\{\epsilon, p_m^k H_m^k \frac{X_m(\alpha^k, 0)}{1 - X_m(\alpha^k, 0)}\right\} . \tag{53}$$

We note that $X_m(\alpha^k, 0) \in (0, 1)$ is continuous and decreasing with respect to α^k, then so does $\frac{X_m(\alpha^k,0)}{1-X_m(\alpha^k,0)}$. Since $p_m^k H_m^k$ is constant, we have that $G(\alpha^k) \in (0, +\infty)$ is also continuous and decreasing with respect to α^k. Then, we may use the bisection method [30] to find out α^k such that $G(\alpha^k) = 1 - |\mathcal{Z}_0|\epsilon$ and the optimal bandwidth allocation can be obtained by (52).

The procedure for solving the bandwidth allocation is summarized as follows.

Algorithm 2 - Solving bandwidth allocation subproblem $\mathsf{BP}_k(\mathcal{P}_k, \epsilon, \mathcal{W})$

1: Initialization
Specify initial $\alpha_u > \alpha_l > 0$ ($G(\alpha_u) < 1 - |\mathcal{Z}_0|\epsilon < G(\alpha_l)$) and error tolerance $\delta > 0$
2: **REPEAT**
  $\alpha \leftarrow (\alpha_u + \alpha_l)/2$
  FOR all $m \in \mathcal{M}$
    Calculate $X_m(\alpha, 0)$ by solving (45) with $\beta = 0$
  ENDFOR
  Evaluate $G(\alpha)$ using $\{X_m(\alpha, 0), m \in \mathcal{M}\}$
  IF $|G(\alpha) - 1 + |\mathcal{Z}_0|| < \delta$ **THEN** Goto step 3 **ENDIF**
  IF $G(\alpha) > 1 - |\mathcal{Z}_0|\epsilon$ **THEN** $\alpha_l \leftarrow \alpha$ **ELSE** $\alpha_h \leftarrow \alpha$ **ENDIF**
3: **FOR** all $m \in \mathcal{M}$
  Calculate a_m^k by (52)
ENDFOR

Since we need to solve for $X_m(\alpha, 0)$ from (45) repeatedly, we can pre-compute the solutions to $y = x - \log(x),\ x \in (0, 1)$ and store them in a look-up table. Then the overall complexity of Algorithm 2 is $\mathcal{O}(M)$ for solving $\mathsf{BP}_k(\mathcal{P}_k, \epsilon, \mathcal{W})$.

Remark 2. *We note that for the special case of equal weights, the optimal bandwidth allocation can be directly obtained by the iterative bandwidth fitting algorithm [16, Algorithm 2] without solving the dual variable α^k and calculating the intermediate variable $X_m(\alpha^k, 0)$. However, for the general weighted case, we need to solve the equation group consisting of* (45) *for all $m \in \mathcal{M}$ and* (51) *to obtain the dual variable α^k and then calculate the optimal bandwidth allocation given by* (52).

2.8.2. Solving the Energy Allocation Subproblem

$\mathsf{EP}_n(\mathcal{A}_n, \mathcal{W})$ is a convex optimization problem with linear constraints thus its K.K.T. conditions are necessary and sufficient for optimality [22]. Using the Lagrangian function defined in (14), in addition to the first-order condition and the feasibility constraints, the

complementary slackness can be written as

$$\lambda^k(\sum_{\kappa=1}^{k}\sum_{m\in\mathcal{M}_n} p_m^\kappa - E^k) = 0, \tag{54}$$

$$\mu_n^k(\sum_{\kappa=1}^{k}\sum_{m\in\mathcal{M}_n} p_m^\kappa - E^k + B^{\max}) = 0, \tag{55}$$

$$\xi_n^k(\sum_{m\in\mathcal{M}_n} p_m^k - P_n) = 0 \tag{56}$$

constituting the K.K.T. conditions.

Taking the derivative of (14) on p_m^k and using the first-order condition, we have

$$p_m^k = a_m^k\left[\frac{W_m}{v_n^k - u_n^k + \xi_n^k} - \frac{1}{H_m^k}\right]^+ . \tag{57}$$

By (56), when $\sum_{m\in\mathcal{M}_n} p_m^k = P_n$, we have $\xi_n^k \geq 0$ and otherwise $\xi_n^k = 0$. Then, we have

$$p_m^k = a_m^k\left[\frac{W_m}{v_n^k - u_n^k} - \frac{1}{H_m^k}\right]^+ \tag{58}$$

when $\sum_{m\in\mathcal{M}_n} a_m^k\left[\frac{W_m}{v_n^k-u_n^k} - \frac{1}{H_m^k}\right]^+ < P_n$. Otherwise, since the constraint requires $\sum_{m\in\mathcal{M}_n} p_m^k \leq P_n$, given v_n^k and u_n^k, we can determine $\bar{\xi}_n^k \geq 0$ such that

$$\sum_{m\in\mathcal{M}_n} a_m^k\left[\frac{W_m}{v_n^k - u_n^k} - \frac{1}{H_m^k}\right]^+ \geq \sum_{m\in\mathcal{M}_n} a_m^k\left[\frac{W_m}{v_n^k - u_n^k + \bar{\xi}_n^k} - \frac{1}{H_m^k}\right]^+ = P_n\ . \tag{59}$$

Then we can treat

$$\bar{P}_m^k \triangleq a_m^k\left[\frac{W_m}{v_n^k - u_n^k + \bar{\xi}_n^k} - \frac{1}{H_m^k}\right]^+ \tag{60}$$

as the maximum transmission energy for each receiver and thus the optimal energy allocation is

$$p_m^k = \min\left\{\bar{P}_m^k, a_m^k\left[W_m w_n^k - \frac{1}{H_m^k}\right]^+\right\}\ , \tag{61}$$

where $w_n^k \triangleq 1/(v_n^k - u_n^k)$.

We note that, p_m^k in (61) is a function of w_n^k. Then, we have the following proposition:

Proposition 2. *Given any bandwidth allocation $\mathcal{A}_n$, p_m^k is the optimal energy allocation for $\mathsf{EP}_n(\mathcal{A}_n, \mathcal{W})$, if and only if, the feasible allocation p_m^k follows the generalized two-dimensional water-filling formula in* (61)*, where the water level w_n^k may only increase at BDP such that $B_n^k = 0$ and only decrease at BFP such that $B_n^k = B_n^{\max}$.*

We note that, in the orthogonal broadcast channel, each transmitter communicates with multiple receivers and the transmitted energy is drawn from the same battery. Then, according to (61), the water (energy) is not only filled along the time axis but also along the

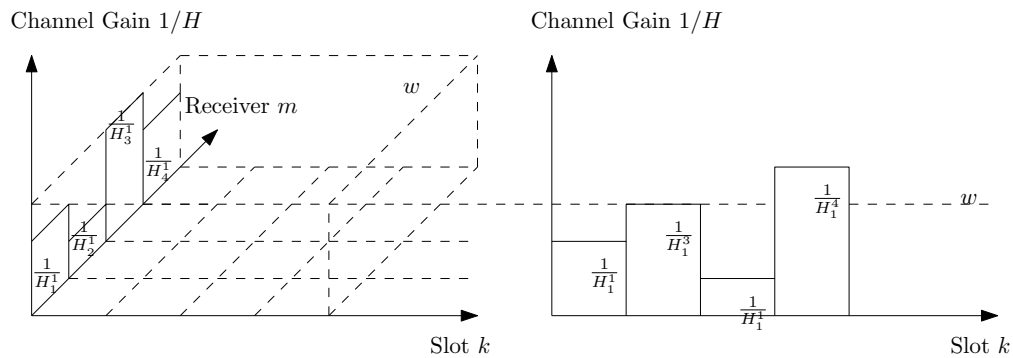

Figure 3. Two-dimensional water-filling. The "water" (energy) is filled over both the receiver-axis (left) and time-axis (right) with the same water level w as interpreted in (61)-(62).

receiver index axis, as shown in Fig. 3. In other words, given two adjacent BDP/BFPs $(a,$ the type of $a)$ and $(b,$ the type of $b)$ where $a \leq b$, the energy allocation p_m^k can be calculated by (61) with the same water level $w_n^k = w^{ab}$ for all receiver $m \in \mathcal{M}_n$ and slot $k \in [a+1, b]$. Then, the water level w^{ab} should be determined by

$$\sum_{k=a+1}^{b} \sum_{m=1}^{M} p_m^k(w^{ab}) = E^b - E^a + (\mathbb{I}(a \text{ is BFP}) - \mathbb{I}(a \text{ is BDP}))\, B_n^{\max} \tag{62}$$

where $\mathbb{I}(\cdot)$ is an indicator function and $p_m^k(w^{ab})$ is calculated by (61) with $w_n^k = w^{ab}$ for $k \in [a+1, b]$.

In [5, 6], a single-user dynamic water-filling algorithm is proposed to find the BDP/BFP set by recursively performing the "forward search" and "backward search" operations with conventional water-filling. Since here the increase/decrease of the water level also occurs at BDP/BFPs, replacing the conventional water-filling used in [5, 6] by the two-dimensional water-filling in (61)-(62), we can obtain the BDP/BFP set for optimal energy allocation in multiple orthogonal broadcast channels. We name this algorithm as the *two-dimensional dynamic water-filling algorithm*. Moreover, after obtaining the optimal BDP/BFP set, the optimal energy allocation can be further calculated by (61)-(62).

3. Weighted Throughput Maximization for the Multiple Non-Orthogonal Broadcast Channel

3.1. Problem Formulation

We consider a system with multiple non-orthogonal broadcast channels, where each transmitter communicates with all its receivers on the same (assigned) frequency band at the same time. Denoting X_{mki} as the symbol sent for receiver m at instant i in slot k, the signal received at receiver m is $Y_{mki} = h_{mk}X_{mki} + \left(h_{mk}\sum_{m_0 \neq m} X_{m_0ki} + Z_{mki}\right)$, where h_{mk} represents the complex channel gain for receiver m in slot k and $Z_{mki} \sim \mathsf{CN}(0,1)$ is

the i.i.d. complex Gaussian noise. We note that, $\sum_{m_0 \neq m} X_{m_0 ki}$ represents the interference and is treated as noise by receiver m. Moreover, we denote the channel gain and the energy consumption in each slot k as $H_m^k \triangleq |h_{mk}|^2$ and $p_m^k \triangleq \frac{1}{T_c} \sum_i |X_{mki}|^2$, respectively.

We denote $\tilde{a}_n^k$ as the amount of bandwidth used by transmitter n. Then, we use the upper bound of the achievable rate over a weighted sum of the M receivers and K slots as the performance metric, given by [27]

$$\tilde{C}_{\mathcal{W}}(\mathcal{P}, \tilde{\mathcal{A}}) \triangleq \sum_{n \in \mathcal{N}} \sum_{k \in \mathcal{K}} \tilde{a}_n^k \sum_{m \in \mathcal{M}_n} W_m \log \left(1 + \frac{p_m^k H_m^k / \tilde{a}_n^k}{\sum_{m_0 \mid H_m^k < H_{m_0}^k} p_{m_0}^k H_m^k / \tilde{a}_n^k + 1} \right) , \tag{63}$$

where $\tilde{\mathcal{A}} \triangleq \{\tilde{a}_n^k, \forall n \in \mathcal{N}, k \in \mathcal{K}\}$. Note that, the rate in each slot is achieved by decoding the messages in the order of the channel quality [31], *i.e.,* we decode the message from a weaker channel prior to that from a stronger channel. Moreover, we assume no two channels have the same gain in the same slot.

We define the energy-bandwidth allocation problem in multiple non-orthogonal broadcast channels as follows:

$$\tilde{\mathsf{P}}_{\mathcal{W}}(\epsilon): \quad \max_{\mathcal{P}, \tilde{\mathcal{A}}} \tilde{C}_{\mathcal{W}}(\mathcal{P}, \tilde{\mathcal{A}}) \tag{64}$$

subject to (6), where $\sum_{m \in \mathcal{M}} a_m^k = 1$ and $a_m^k \geq \epsilon$ is replaced by $\sum_n \tilde{a}_n^k = 1$ and $\tilde{a}_n^k \geq \epsilon$, respectively.

We note that, the above problem is non-convex due to the non-convexity of the objective function. To obtain the energy-bandwidth allocation, we first define $\tilde{p}_n^k \triangleq \sum_{m \in \mathcal{M}_n} p_m^k$ for all $n \in \mathcal{N}$ and rewrite (64) as

$$\max_{\tilde{p}_n^k, \tilde{a}_m^k} \left\{ \sum_n \sum_k \max_{\sum_{m \in \mathcal{M}_n} p_m^k = \tilde{p}_m^k} \left\{ \tilde{a}_n^k \sum_{m \in \mathcal{M}_n} W_m \log \left(1 + \frac{p_m^k H_m^k / \tilde{a}_n^k}{\sum_{m_0 \mid H_m^k < H_{m_0}^k} p_{m_0}^k H_m^k / \tilde{a}_n^k + 1} \right) \right\} \right\} . \tag{65}$$

Denoting

$$F_n^k(p) \triangleq \max_{\pi_m : \sum_{m \in \mathcal{M}_n} \pi_m = 1, \pi_m \geq 0} \sum_{m \in \mathcal{M}_n} W_m \log \left(1 + \frac{\pi_m p H_m^k}{\left(\sum_{m_0 \mid H_m < H_{m_0}} \pi_m \right) p H_m^k + 1} \right) , \tag{66}$$

we further write (65) as

$$\max_{\mathcal{P}, \tilde{\mathcal{A}}} \tilde{C}_{\mathcal{W}}(\mathcal{P}, \tilde{\mathcal{A}}) = \max_{\tilde{p}_n^k, \tilde{a}_m^k} \sum_n \sum_k \tilde{a}_n^k F_n^k(\tilde{p}_n^k / \tilde{a}_n^k) , \tag{67}$$

where $\tilde{\mathcal{P}} \triangleq \{\tilde{p}_n^k, \forall n \in \mathcal{N}, k \in \mathcal{K}\}$ is the total energy allocation.

To solve $\tilde{\mathsf{P}}_{\mathcal{W}}(\epsilon)$, we first solve (67) to obtain the optimal bandwidth allocation $\tilde{\mathcal{A}}$ and the optimal total energy allocation $\tilde{\mathcal{P}}$. Then, given the total energy allocation $\tilde{\mathcal{P}}$, we further optimally split the total energy for each receiver by solving (66).

The optimal solution to (66) is given in [23], which is summarized in the following Lemma:

Lemma 3. *For any (n,k), we have a set of energy cut-off lines $\{L_m^k, \forall m \in \mathcal{M}_n\}$ sorting in ascending order such that $L_a^k \leq L_b^k$ if $H_a^k > H_b^k$ for all $a,b \in \mathcal{M}_n$. For any $a \in \mathcal{M}_n$, the optimal energy splitting is*

$$p_a^k = \begin{cases} L_b^k - L_a^k, & \text{if } L_b^k < \tilde{p}_n^k \\ \tilde{p}_n^k - L_b^k, & \text{if } L_a^k \leq \tilde{p}_n^k \leq L_b^k \\ 0 & \text{if } \tilde{p}_n^k < L_a^k \end{cases}, \tag{68}$$

where $L_a^k \leq L_b^k$ are two adjacent cut-off lines.

The procedure for computing $\{L_m^k, \forall m \in \mathcal{M}_n\}$ is also given in [23].

3.2. Solving the Problem in (67)

The convexity of $F_n^k(p)$ has been shown in [23], given by the following lemma:

Lemma 4. *$F_n^k(p)$ is strictly concave with respect to p, whose first-order derivative is continuous.*

Then, the problem in (67) is still an energy-bandwidth allocation problem with the rate function defined in (66), which is increasing and jointly concave with respect to the total energy and bandwidth allocations. Note that the problem in (67) and the problem in (9)-(10) have the same feasible domain and the corresponding optimal energy allocations both follow the water-filling formula (will be shown later in this section). Then, it is easy to verify that the optimal energy discharge given by (11) and the iterative algorithm given in Algorithm 1 can also give the optimal solution to the problem in (67).

Hence we focus on the energy and bandwidth allocation subproblems as follows:

- Energy allocation subproblem: Denote $\tilde{\mathcal{A}}_n \triangleq \{\tilde{a}_n^k, k \in \mathcal{K}\}$,

$$\tilde{\mathsf{EP}}_n(\tilde{\mathcal{A}}_n, \mathcal{W}): \quad \max_{\tilde{p}_n^k} \sum_n \sum_k \tilde{a}_n^k F_n^k(\tilde{p}_n^k/\tilde{a}_n^k), \tag{69}$$

$$\text{subject to} \begin{cases} \tilde{E}_n^k - B_n^{\max} \leq \sum_{\kappa=1}^k \tilde{p}_n^\kappa \leq \tilde{E}_n^k \\ 0 \leq \tilde{p}_n^k \leq P_n \end{cases}, \ k \in \mathcal{K}. \tag{70}$$

- Bandwidth allocation subproblem: Denote $\tilde{\mathcal{P}}_k \triangleq \{\tilde{p}_n^k, n \in \mathcal{N}\}$,

$$\tilde{\mathsf{BP}}_k(\tilde{\mathcal{P}}_k, \epsilon, \mathcal{W}): \quad \max_{\tilde{a}_n^k} \sum_n \sum_k \tilde{a}_n^k F_n^k(\tilde{p}_n^k/\tilde{a}_n^k), \tag{71}$$

subject to

$$\text{subject to} \begin{cases} \sum_{n=1}^N \tilde{a}_n^k \leq 1 \\ \tilde{a}_n^k \geq \epsilon \end{cases}, \ n \in \mathcal{N}. \tag{72}$$

We can write the Lagrangian function for the problem in (67) as

$$\mathcal{L}_N \triangleq \sum_n \sum_k \tilde{a}_n^k F_n^k(\tilde{p}_n^k/\tilde{a}_n^k) + \mathcal{M}(\tilde{\mathcal{P}}, \tilde{\mathcal{A}}), \tag{73}$$

where

$$\begin{aligned}\mathcal{M}(\mathcal{P},\mathcal{A}) \triangleq & -\sum_{n,k}\lambda_n^k\left(\sum_{\kappa=1}^{k}\sum_{m\in\mathcal{M}_n}p_m^\kappa-\tilde{E}_n^k\right)+\sum_{n,k}\mu_n^k\left(\sum_{\kappa=1}^{k}\sum_{m\in\mathcal{M}_n}p_m^\kappa-\tilde{E}_n^k+B_n^{\max}\right)\\ & -\sum_k\alpha^k(\sum_m a_m^k-1)+\sum_{m,k}\beta_m^k(a_m^k-\epsilon)-\sum_{n,k}\xi_n^k(\sum_{m\in\mathcal{M}_n}p_m^k-P_n)\\ = & -\sum_{n,k}\left(\sum_{m\in\mathcal{M}_n}p_m^k\sum_{\kappa=k}^{K}\lambda_n^\kappa-\lambda_n^k\tilde{E}_n^k\right)+\sum_{n,k}\left(\sum_{m\in\mathcal{M}_n}p_m^k\sum_{\kappa=k}^{K}\mu_n^\kappa-\mu_n^k\left(\tilde{E}_n^k-B_n^{\max}\right)\right)\\ & -\sum_k\alpha^k(\sum_m a_m^k-1)+\sum_{m,k}\beta_m^k(a_m^k-\epsilon)-\sum_{n,k}\xi_n^k(\sum_{m\in\mathcal{M}_n}p_m^k-P_n)\,,\end{aligned} \tag{74}$$

3.2.1. Solving the Energy Allocation Subproblem

Since $\tilde{\mathsf{EP}}_n(\tilde{\mathcal{A}}_n,\mathcal{W})$ is a convex optimization problem with linear constraints, its K.K.T. conditions are sufficient and necessary for optimality when $\epsilon>0$ [22]. With $\mathcal{L}_N$ defined in (73), we can write the first-order condition for the non-orthogonal broadcast channel as

$$\partial\left(\tilde{a}_n^k F_n^k(\tilde{p}_n^k/\tilde{a}_n^k)\right)/\partial\tilde{p}_n^k \triangleq (F_n^k)'(\tilde{p}_n^k/\tilde{a}_n^k)=\upsilon_n^k-u_n^k \tag{75}$$

where υ_n^k and u_n^k are defined in (28), and $(F_n^k)'(p)$ denotes the first-order derivative of $F_n^k(p)$. For all $p\geq 0$, we further derive the derivative of $F_n^k(p)$ in closed-form:

Proposition 3. *For any $p\geq 0$, the derivative of $F_n^k(p)$ is*

$$(F_n^k)'(p)=\max_{m\in\mathcal{M}_n}\left\{\frac{W_m}{p+1/H_m^k}\right\}\,. \tag{76}$$

Proof. By Lemma 3, we have

$$(F_n^k)'(p)=\partial\left(\sum_{m\in\mathcal{M}_n}W_m\log(1+\frac{p_m^k(p)H_m^k}{H_m^k\sum_{m_0\,|\,H_m^k<H_{m_0}^k}p_{m_0}^k(p)+1})\right)/\partial p \tag{77}$$

$$=\partial\left(W_a\log(1+\frac{(p-L_a^k)H_a^k}{L_a^kH_a^k+1})\right)/\partial p \tag{78}$$

$$=\frac{W_a}{p+1/H_a^k}\,,\ p\in[L_a^k,L_b^k] \tag{79}$$

where (78) follows because (68) indicates that, for any $a\in\mathcal{M}_n$, $p_a^k(p)$ is constant when $p<L_a^k$ or $p>L_b^k$.

Hence $(F_n^k)'(p)$ is a piecewise function composed by the segments in the form of $f_m^k(p)\triangleq W_m/(p_m^k+1/H_m^k)$. By Lemma 4, $(F_n^k)'(p)$ is continuous. Thus, for any two adjacent different cutoff lines $L_a^k<L_b^k$, L_a^k is the intersection of the two curves $f_a^k(p)=W_a/(p+1/H_a^k)$ and $f_b^k(p)=W_b/(p+1/H_b^k)$.

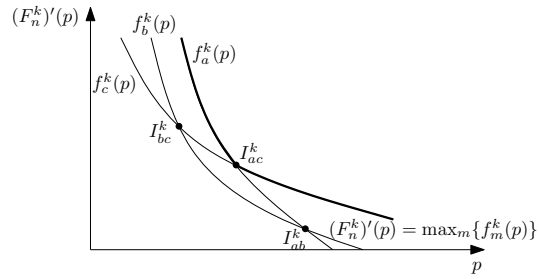

Figure 4. The derivative of $F_n^k(p)$.

Denoting the intersection of $f_a^k(p)$ and $f_b^k(p)$ as I_{ab}^k (*i.e.,* $p = I_{ab}$ such that $f_a^k(I_{ab}^k) = f_b^k(I_{ab}^k)$), we then have

$$L_a^k = I_{ab}^k \triangleq \frac{H_b^k W_b - H_a^k W_a}{H_b^k H_a^k \left(W_a - W_b\right)} \ . \tag{80}$$

Specifically, for any $a, b \in \mathcal{M}_n$, I_{ab}^k is unique if it exists. Then, we can write

$$F_n^k(\tilde{p}) = \int_0^{\tilde{p}} (F_n^k)'(p) dp \tag{81}$$

$$= \max_{\{I_{ab}^k < I_{bc}^k < \ldots < \tilde{p} \mid a,b,c,\ldots \in \mathcal{M}_n\}} \left\{ \sum_{ab} \int_{I_{ab}^k}^{\min\{I_{bc}^k, \tilde{p}\}} f_a^k(p) dp \right\} \ , \tag{82}$$

where (82) follows since $(F_n^k)'(p)$ is a piecewise function with the segments of $f_m^k(p)$ and I_{ab}^k is the intersection of $f_a^k(p)$ and $f_b^k(p)$.

Then, as shown in Fig. 4, we can obtain a set of I_{ab}^k and it is easy to verify that the optimal solution to the problem in (82) forms the derivative of $F_n^k(p)$ as

$$(F_n^k)'(p) = \max_{m \in \mathcal{M}_n} \left\{ \frac{W_m}{p + 1/H_m^k} \right\} \ . \tag{83}$$

□

Moreover, we note that $(F_n^k)'(\tilde{p}_n^k / \tilde{a}_n^k)$ is strictly decreasing with respect to $\tilde{p}_n^k$ due to the strict concavity of $F_n^k(p)$. Then using (75) and Proposition 3, $\tilde{p}_n^k$ can be uniquely determined as follows

$$\tilde{p}_n^k = \tilde{a}_n^k \Big((F_n^k)' \Big)^{-1} (1/w_n^k) \tag{84}$$

$$= \min \left\{ P_n, \tilde{a}_n^k \max_{m \in \mathcal{M}_n} \left\{ \left[W_m w_n^k - \frac{1}{H_m^k} \right]^+ \right\} \right\} \tag{85}$$

where $w_n^k = 1/(v_n^k - u_n^k)$ and $(\cdot)^{-1}$ denotes the inverse function.

We note that, since $\mathsf{P}_{\mathcal{W}}(\epsilon)$ and $\tilde{\mathsf{P}}_{\mathcal{W}}(\epsilon)$ have the same Lagrangian multipliers, by analyzing the K.K.T. conditions and using Proposition 3, it is easy to verify that the changes

of w_n^k still follows Proposition 2, *i.e.,* it may only increase/decrease at the BDP/BFP. Then, we treat (85) as a water-filling formula and the water level is determined by

$$\sum_{k=a+1}^{b} \tilde{p}_n^k(w^{ab}) = E^b - E^a + (\mathbb{I}(a \text{ is BFP}) - \mathbb{I}(a \text{ is BDP}))\, B_n^{\max} \tag{86}$$

where $\tilde{p}_n^k(w^{ab})$ is calculated by (85) with $w_n^k = w^{ab}$ for $k \in [a+1, b]$.

As for the energy allocation problem in multiple orthogonal broadcast channels, since here the water level change also occurs at BDP/BFPs, we can use the water-filling in (85)-(86) to replace the conventional water-filling operation in [5, Algorithm 2], and then the BDP/BFP set can be obtained. After obtaining the BDP/BFP set, using (85)-(86), we obtain the optimal total energy allocation.

3.2.2. Solving the Bandwidth Allocation Subproblem

When $\sum_{n\in\mathcal{N}} \tilde{p}_n^k = 0$, the sum-rate in slot k is zero. Thus, in this subsection we focus on the case $\sum_{n\in\mathcal{N}} \tilde{p}_n^k > 0$.

Since $\widetilde{\mathsf{BP}}_k(\tilde{\mathcal{P}}_k, \epsilon, \mathcal{W})$ is a convex optimization problem with linear constraints, its K.K.T. conditions are sufficient and necessary for optimality when $\epsilon > 0$ [22]. The first-order condition can be written as

$$\frac{\partial\left(\tilde{a}_n^k F_n^k(\tilde{p}_n^k/\tilde{a}_n^k)\right)}{\partial \tilde{a}_n^k} = F_n^k(\tilde{p}_n^k/\tilde{a}_n^k) - (F_n^k)'(\tilde{p}_n^k/\tilde{a}_n^k)\tilde{p}_n^k/\tilde{a}_n^k = \alpha^k\ ,\ n \in \mathcal{N}, k \in \mathcal{K}\ , \tag{87}$$

where the value of $F_n^k(\tilde{p}_n^k/\tilde{a}_n^k)$ can be calculated using the algorithm in [23]. Taking the constraints in (72) into account, $\tilde{a}_n^k$ must satisfy

$$\sum_{n=1}^{N} \max\{\tilde{a}_n^k, \epsilon\} = 1,\ k \in \mathcal{K}\ . \tag{88}$$

We note that, for each $k \in \mathcal{K}$, we have $N+1$ equations [(87) for all $n \in \mathcal{N}$ and (88)] and $N+1$ variables [$\tilde{a}_n^k$ for all $n \in \mathcal{N}$ and α^k]. Therefore, all the variables $\tilde{a}_n^k$ can be uniquely determined by solving the equation group given $k \in \mathcal{K}$.

Since $F_n^k(p)$ is concave by Lemma 4, $aF_n^k(p/a)$ is jointly concave with respect to p and a. Then, $\partial\left(aF_n^k(p/a)\right)/\partial a$ is non-increasing with respect to a given p. Also, the left-hand-side of (88) is non-decreasing with respect to a. Therefore, given α^k, we can use the bisection method to find the corresponding $\tilde{a}_n^k(\alpha^k)$ in (87). Finally we can use the bisection method again to determine the proper α^k such that (88) is satisfied. The procedure for computing the bandwidth allocation is summarized as follows.

Algorithm 3 - Solving bandwidth allocation subproblem $\tilde{\mathsf{BP}}_k(\mathcal{P}_k, \epsilon, \mathcal{W})$

1: Initialization
Specify initial $\alpha_u^k > \alpha_l^k > 0$ such that $\sum_{n=1}^N \max\{\tilde{a}_n^k(\alpha_u^k), \epsilon\} < 1 < \sum_{n=1}^N \max\{\tilde{a}_n^k(\alpha_l^k), \epsilon\}$
Specify error tolerance $\delta > 0$
2: **REPEAT**
$\alpha \leftarrow (\alpha_u^k + \alpha_l^k)/2$
FOR all $n \in \mathcal{N}$
(*) Solve (87) to obtain $\tilde{a}_n^k(\alpha)$ using the bisection method
ENDFOR
IF $|\sum_{n=1}^N \max\{\tilde{a}_n^k(\alpha^k), \epsilon\} - 1| < \delta$ **THEN** Goto step 4 **ENDIF**
IF $\sum_{n=1}^N \max\{\tilde{a}_n^k(\alpha^k), \epsilon\} > 1$ **THEN** $\alpha_l^k \leftarrow \alpha$ **ELSE** $\alpha_h^k \leftarrow \alpha$ **ENDIF**
3: **FOR** all $n \in \mathcal{N}$
Calculate $\tilde{a}_n^k$ by (52)
ENDFOR

The complexity of Algorithm 3 is $\mathcal{O}(N)$.

Remark 3. *Comparing Algorithm 3 with Algorithm 2, the main difference lies in the step marked by "*", where the corresponding bandwidth allocations a_m^k and $\tilde{a}_n^k$ are calculated by solving the same equation [*i.e., (45)*] in Algorithm 2 and multiple different equations [*i.e., (87) *with different $F_n^k(\tilde{p}_n^k/\tilde{a}_n^k)$ for all $n \in \mathcal{N}$] in Algorithm 3.*

3.3. Special Case: Equal Weights

When $W_m = 1$ for all $m \in \mathcal{M}$, by Proposition 3, we have

$$(F_n^k)'(p) = \max_{m \in \mathcal{M}_n} \left\{ \frac{1}{p + 1/H_m^k} \right\} , \tag{89}$$

for all $p \geq 0$. Since, given any $a, b \in \mathcal{M}_n$ such that $H_a > H_b > 0$, we have $1/(p+1/H_a) > 1/(p + 1/H_b)$ for all $p \geq 0$, then we have

$$(F_n^k)'(p) = \max_{m \in \mathcal{M}_n} \left\{ \frac{1}{p + 1/H_m^k} \right\} = \frac{1}{p + 1/\max_{m \in \mathcal{M}_n}\{H_m^k\}} . \tag{90}$$

Therefore, by (66), we must have

$$F_n^k(p) = \log(1 + pH_{m_n^k}^k) \tag{91}$$

where $m_n^k \triangleq \arg\max_{m \in \mathcal{M}_n}\{H_m^k\}$, *i.e.,* each transmitter uses only the strongest channel to transmit in each slot. Then, we have the following corollary.

Corollary 1. *Theorem 3 also holds for the network with multiple non-orthogonal broadcast channels. Moreover, with equal weights, networks with multiple orthogonal and non-orthogonal broadcast channels achieve the same maximum throughput.*

Remark 4. *When the weights are equal, by Corollary 1, the energy-bandwidth allocation for multiple orthogonal broadcast channels is equivalent to that for multiple point-to-point channels treated in [16]. Thus, we can use Corollary 1 along with the algorithms in [16] to solve the energy allocation problem when the weights are equal.*

3.4. Achievable Rate Regions

Denoting $C_{O,m}(\mathcal{P},\mathcal{A})$ and $C_{N,m}(\mathcal{P},\mathcal{A})$ as the sum-rate of receiver m achieved by the energy-bandwidth allocation $(\mathcal{P},\mathcal{A})$ in K slots for multiple orthogonal and non-orthogonal broadcast channels, respectively. Then, the rate region can be defined as $\mathcal{R}_{(\cdot)} \triangleq \{(r_1,r_2,\ldots,r_M) \mid 0 \le r_m \le C_{(\cdot),m}(\mathcal{P},\mathcal{A}),\ \mathcal{P},\mathcal{A} \text{ are feasible}\}$, where $(r_1,r_2,\ldots,r_M)$ is the sum-rate vector for all receivers.

Lemma 5. *The rate region $\mathcal{R}_O$ is strictly convex for the network with multiple orthogonal broadcast channels.*

Proof. Consider two sum-rate vectors $R^1, R^2 \in \mathcal{R}_O$ and the corresponding energy-bandwidth allocation as $(\mathcal{P}^1,\mathcal{A}^1)$ and $(\mathcal{P}^2,\mathcal{A}^2)$. Then, given any $\theta \in (0,1)$ and $\bar{\theta} = 1-\theta$, consider $R^3 = \theta R^1 + \bar{\theta} R^2$, where $R^i \triangleq (r_1^i, r_2^i, \ldots, r_M^i)$. We note that, $C_{O,m}(\mathcal{P},\mathcal{A})$ is sum of a series of log functions which are strictly concave with respect to p_m^k and a_m^k. Then, for $m \in \mathcal{M}$, we have

$$r_m^3 = \theta r_m^1 + \bar{\theta} r_m^2 \tag{92}$$
$$\le \theta C_{O,m}(\mathcal{P}^1,\mathcal{A}^1) + \bar{\theta} C_{O,m}(\mathcal{P}^2,\mathcal{A}^2) \tag{93}$$
$$< C_{O,m}(\theta\mathcal{P}^1 + \bar{\theta}\mathcal{P}^2, \theta\mathcal{A}^1 + \bar{\theta}\mathcal{A}^2) \tag{94}$$

where $\mathcal{P}^3 \triangleq \theta\mathcal{P}^1 + \bar{\theta}\mathcal{P}^2$ and $\mathcal{A}^3 \triangleq \theta\mathcal{A}^1 + \bar{\theta}\mathcal{A}^2$. Note that, since $\mathsf{P}_{\mathcal{W}}(\epsilon)$ is a convex optimization problem and its feasible domain is also convex, $(\mathcal{P}^3,\mathcal{A}^3)$ is a feasible energy-bandwidth allocation. Then, by definition we have $R^3 \in \mathcal{R}_O$ and thus $\mathcal{R}_O$ is a strictly convex set. □

Moreover, for the network with multiple non-orthogonal broadcast channels, we define a convex region

$$\bar{\mathcal{R}}_N \triangleq \left\{ (r_1,r_2,\ldots,r_M) : r_m \le \tilde{\mathsf{P}}_{\{W_m=1,W_i=0,\forall i\neq m\}}(0), \sum_m r_m \le \tilde{\mathsf{P}}_{\{W_m=1,\forall m\}}(0) \right\}. \tag{95}$$

Note that for $\mathcal{W} = \{W_m = 1, W_i = 0, \forall i \neq m\}$, $\tilde{\mathsf{P}}_{\mathcal{W}}(0)$ and $\mathsf{P}_{\mathcal{W}}(0)$ maximize the sum-rate for the single receiver m and the two problems are the same. Then we have

$$\tilde{\mathsf{P}}_{\{W_m=1,W_i=0,\forall i\neq m\}}(0) = \mathsf{P}_{\{W_m=1,W_i=0,\forall i\neq m\}}(0) = \max_{\mathcal{P},\mathcal{A} \text{ are feasible}} C_{(\cdot),m}(\mathcal{P},\mathcal{A}),\ m \in \mathcal{M}. \tag{96}$$

For $W_m = 1, m \in \mathcal{M}$, by Theorem 3 and Corollary 1, $\tilde{\mathsf{P}}_{\{W_m=1,\forall m\}}(0)$ and $\mathsf{P}_{\{W_m=1,\forall m\}}(0)$ have the same solution, which can be denoted as $(\mathcal{P}^*,\mathcal{A}^*)$. For any $(r_1,r_2,\ldots,r_M) \in \mathcal{R}_N$, by definition, we have $r_m \le \max_{\mathcal{P},\mathcal{A} \text{ are feasible}} C_{N,m}(\mathcal{P},\mathcal{A})$ and $\sum_m r_m \le \sum_m C_{N,m}(\mathcal{P}^*,\mathcal{A}^*)$. Then, we have $\mathcal{R}_N \subseteq \bar{\mathcal{R}}_N$ and the sum-rate vectors $(C_{(\cdot),1}(\mathcal{P}^*,\mathcal{A}^*), C_{(\cdot),2}(\mathcal{P}^*,\mathcal{A}^*), \ldots, C_{(\cdot),M}(\mathcal{P}^*,\mathcal{A}^*))$ and $(\ldots,0,\mathsf{P}_{\{W_m=1,W_i=0,\forall i\neq m\}}(0),0,\ldots)$ for all $m \in \mathcal{M}$ can be achieved with both orthogonal and non-orthogonal broadcast.

We give an example for the network with one transmitter and two receivers. According to the above analysis, $\mathcal{R}_O$ and $\bar{\mathcal{R}}_N$ have three common points on the boundary as shown

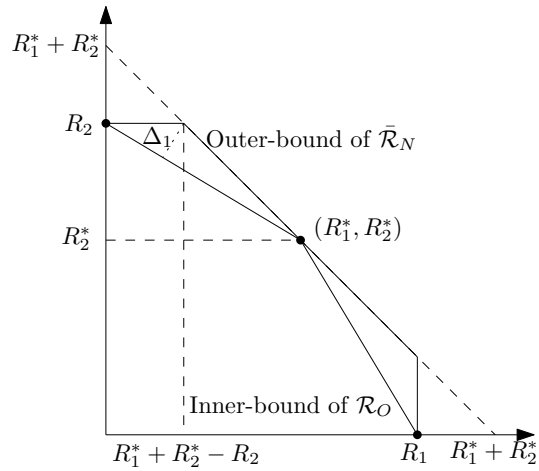

Figure 5. Rate regions of orthogonal and non-orthogonal broadcast channels.

in Fig. 5: $(R_1, 0)$ for $\{W_{11} = 1, W_{12} = 0\}$, $(0, R_2)$ for $\{W_{11} = 0, W_{12} = 1\}$, and (R_1^*, R_2^*) for $\{W_{11} = W_{12} = 0.5\}$. Due to the concavity of $\mathcal{R}_O$ and $\bar{\mathcal{R}}_N$, the maximum improvement (Euclidean distance between boundary of $\mathcal{R}_O$ and $\bar{\mathcal{R}}_N$) of using the non-orthogonal broadcast channel is bounded by

$$\Delta = \max\left\{\frac{(R_2 - R_2^*)(R_1^* + R_2^* - R_2)}{\sqrt{(R_2 - R_2^*)^2 + R_1^{*2}}}, \frac{(R_1 - R_1^*)(R_2^* + R_1^* - R_1)}{\sqrt{(R_1 - R_1^*)^2 + R_2^{*2}}}\right\}. \tag{97}$$

4. Achieving Proportional Fairness in Orthogonal Broadcast Channels

In this section, we formulate a proportionally-fair (PF) throughput maximization problem for the network with multiple orthogonal broadcast channels, and show that it can be converted to a weighted throughput maximization problem with some proper weights.

4.1. PF Throughput Maximization

We consider the following utility function

$$U(\mathcal{P}, \mathcal{A}) \triangleq \sum_{m \in \mathcal{M}} \log\left(\sum_{k \in \mathcal{K}} a_m^k \log(1 + \frac{p_m^k H_m^k}{a_m^k})\right) \tag{98}$$

Then, the PF throughput maximization problem is formulated as

$$\mathsf{F}_\epsilon : \quad \max_{\mathcal{P}, \mathcal{A}} U(\mathcal{P}, \mathcal{A}) \tag{99}$$

subject to the constraints in (6), whose solution is known to result in proportional fairness [26][25]. Without loss of generality, we assume $\tilde{E}_n^K > 0$ for all $n \in \mathcal{N}$ and thus each transmitter achieves a non-zero sum-rate to make the PF throughput lower bounded.

We next convert F_ϵ into a weighted throughput problem $\mathsf{P}_{\mathcal{W}}(\epsilon)$. Specifically, given $\mathcal{W}$, we denote $R_m(\mathcal{W})$ as the sum-rate achieved for receiver m by the optimal solution to $\mathsf{P}_{\mathcal{W}}(\epsilon)$; we also denote $\bar{R}_m$ as the sum-rate achieved for receiver m by the optimal solution to F_ϵ. We note that, since the rate region $\mathcal{R}_O$ is strictly convex, $R_m(\mathcal{W})$, which is the tangent point of a hyperplane (defined by $\mathcal{W}$) to $\mathcal{R}_O$, is continuous in $\mathcal{W}$.

Theorem 5. *Given $\mathcal{W}$, the optimal solution to $\mathsf{P}_{\mathcal{W}}(\epsilon)$ is also optimal to F_ϵ, if and only if, there exists $\theta > 0$ such that $W_m R_m(\mathcal{W}) = \theta$ for all $m \in \mathcal{M}$, where $R_m(\mathcal{W})$ is the sum-rate achieved for receiver m by the optimal solution to $\mathsf{P}_{\mathcal{W}}(\epsilon)$.*

Proof. We note that $\mathsf{P}_{\mathcal{W}}(\epsilon)$ and F_ϵ have the same decision variables and the same constraints and they can use the same Lagrangian multiplier as defined in (74). Then, the Lagrangian functions for $\mathsf{P}_{\mathcal{W}}(\epsilon)$ and F_ϵ can be defined as (14) and

$$\mathcal{L}_F \triangleq \sum_{m\in\mathcal{M}} \log\left(\sum_{k\in\mathcal{K}} a_m^k \log(1+\frac{p_m^k H_m^k}{a_m^k})\right) + \mathcal{M}(\mathcal{P},\mathcal{A}), \tag{100}$$

respectively. Taking the first-order derivatives with respect to p_m^k, we have

$$\frac{\partial \mathcal{L}_P}{\partial p_m^k} = W_m \frac{\partial\left(a_m^k \log(1+\frac{p_m^k H_m^k}{a_m^k})\right)}{\partial p_m^k} + \frac{\partial \mathcal{M}}{\partial p_m^k}\ , \tag{101}$$

$$\frac{\partial \mathcal{L}_F}{\partial p_m^k} = \frac{1}{\bar{R}_m} \frac{\partial\left(a_m^k \log(1+\frac{p_m^k H_m^k}{a_m^k})\right)}{\partial p_m^k} + \frac{\partial \mathcal{M}}{\partial p_m^k}\ ; \tag{102}$$

also, we can obtain the derivative with respect to a_m^k in the same form as above. Note that, for $\mathsf{P}_{\mathcal{W}}(\epsilon)$ and F_ϵ, their K.K.T. conditions are sufficient and necessary for optimality when $\epsilon > 0$. Also, since $\bar{R}_m$ is the sum-rate achieved for receiver m by the optimal solution to F_ϵ and $R_m(\mathcal{W})$ is the sum-rate achieved by the optimal solution to $\mathsf{P}_{\mathcal{W}}(\epsilon)$, when $W_m = 1/R_m(\mathcal{W})$ for all $m \in \mathcal{M}$, the solution satisfies the K.K.T. conditions of F_ϵ also satisfies those of $\mathsf{P}_{\mathcal{W}}(\epsilon)$, and vice versa. Therefore, $\mathsf{P}_{\mathcal{W}}(\epsilon)$ and F_ϵ have the same optimal solution. Moreover, we note that scaling W_m by a positive factor θ does not affect the optimality of $\mathsf{P}_{\mathcal{W}}(\epsilon)$ and thus the above equivalence condition can be further relaxed to $W_m = \theta/R_m(\mathcal{W})$ where $\theta > 0$. Furthermore, since the objective functions of the two problems are both continuous, we can further extend the result to the case of $\epsilon = 0$. □

We call $\mathcal{W}$ the *PF weights* if $\mathsf{P}_{\mathcal{W}}(\epsilon)$ and F_ϵ have the same optimal solution.

4.2. Obtaining the PF Weights

To obtain the PF weights, we first define an optimization problem:

$$\min_{(\frac{1}{W_1},\frac{1}{W_2},\ldots,\frac{1}{W_M})\in\mathcal{R}_O} \max_{\mathcal{P},\mathcal{A}} \left\{\sum_m W_m \left(\sum_k a_m^k \log(1+\frac{p_m^k H_m^k}{a_m^k}) - 1/W_m\right)\right\} \tag{103}$$

subject to

$$\begin{cases} \sum_k a_m^k \log(1+\frac{p_m^k H_m^k}{a_m^k}) \geq 1/W_m, \quad n\in\mathcal{N}, m\in\mathcal{M}_n \\ \text{Constraints in (6)} \end{cases}. \tag{104}$$

We note that, since $1/W_m$ is drawn from the rate region $\mathcal{R}_O$, the optimal value of (103) is zero, and $\sum_k a_m^k \log(1+\frac{p_m^k H_m^k}{a_m^k}) - 1/W_m = 0$. By Theorem 5, the PF weights is also the optimal solution to (103). Then, denoting $\bar{W}_m \triangleq W_m + \lambda_m$ where $\lambda_m \geq 0$ is the dual variable, we convert the inner maximization problem in (103) to its dual problem and (103) can be further written as

$$\min_{(\frac{1}{W_1},\frac{1}{W_2},\ldots,\frac{1}{W_M})\in\mathcal{R}_O,\bar{W}_m\geq W_m} \max_{\mathcal{P},\mathcal{A} \text{ subject to (6)}} \left\{\sum_m \bar{W}_m \left(\sum_k a_m^k \log(1+\frac{p_m^k H_m^k}{a_m^k}) - 1/W_m\right)\right\} . \tag{105}$$

Note that the inner problem of (105) is equivalent to the weighted throughput optimization problem $\mathsf{P}_{\bar{\mathcal{W}}}(\epsilon)$ with an additional constant term $\sum_m \bar{W}_m/W_m$, where $\bar{\mathcal{W}} = \{\bar{W}_m, m \in \mathcal{M}\}$. Thus, when $W_m = \bar{W}_m = 1/R_m(\mathcal{W})$, the problem in (105) is optimally solved (the optimal value is zero, which is same as the problem in (103)) and by Theorem 5 the optimal PF weights are obtained. Then, we can write the subgradient for the outer minimization problem in (105) as [22]

$$g_{\bar{W}_m} = R_m(\bar{\mathcal{W}}) - 1/W_m \ , \tag{106}$$

$$g_{W_m} = \bar{W}_m/W_m^2 > 0 \ . \tag{107}$$

Since the subgradient of W_m is positive, the optimal $1/W_m$ is on the positive boundary of $\mathcal{R}_O$. Note that $(R_1(\bar{\mathcal{W}}), R_2(\bar{\mathcal{W}}), \ldots, R_M(\bar{\mathcal{W}}))$ is on the positive boundary of $\mathcal{R}_O$ and changes continuously as $\bar{W}$ changes. Then, the following update rule

$$\begin{cases} W_m \leftarrow \min\left\{\bar{W}_m, \left[W_m - \delta\cdot(R_m(\bar{\mathcal{W}}) - 1/W_m)\right]^+\right\} \\ \bar{W}_m \leftarrow \max\left\{W_m, \left[\bar{W}_m - \delta\cdot g_{\bar{W}_m}\right]^+\right\} \end{cases} , \tag{108}$$

enforces that W_m always moves closer to the point on the positive boundary of $\mathcal{R}_O$ and $\bar{W}_m$ is updated by the subgradient. Specifically, if we fix W_m (or $\bar{W}_m$) and update $\bar{W}_m$ (or W_m) only using the second (first) term in (108), W_m (or $\bar{W}_m$) can converge and the optimal $\bar{W}_m$ (or W_m) can be obtained for the fixed W_m (or $\bar{W}_m$).

To find the PF weights, we need to obtain the optimal solution to (105) such that $W_m = \bar{W}_m$. Specifically, we choose the same initial condition and step size for W_m and $\bar{W}_m$, and simultaneously update W_m and $\bar{W}_m$ in each iteration. Then, W_m and $\bar{W}_m$ remain the same in each iteration and the update rule becomes

$$W_m^{(i+1)} = \bar{W}_m^{(i+1)} \leftarrow \left[\bar{W}_m^{(i)} - \delta(i)\cdot g_{\bar{W}_m}^{(i)}\right]^+ , \tag{109}$$

where the step size $\delta(i)$ satisfies $\lim_{i\to\infty}\delta(i) = 0$ and $\sum_{i=1}^{+\infty}\delta(i) = +\infty$, *e.g.,* $\delta(i) = 1/i$. In particular, if $W_m^{(i+1)}$ can converge, the problem in (105) is optimally solved and finally we have $\bar{W}_m = W_m$ for all $m \in \mathcal{M}$, *i.e.,* $R_m(\mathcal{W}) = 1/W_m$. By Theorem 5, $\mathcal{W}$ are the PF weights.

The procedure for computing the PF energy-bandwidth allocation is summarized as follows.

Algorithm 4 - PF energy-bandwidth allocating algorithm

1: Initialization
$i = 0$
Specify the initial fairness weights $\mathcal{W}^{(0)}$, convergence threshold δ_0, maximum iteration number I
2: Obtaining the PF weight
REPEAT
$i \leftarrow i + 1$
Solve $\mathsf{P}_{\mathcal{W}^{(i-1)}}(\epsilon)$ to obtain $(\mathcal{P}^{(i)}, \mathcal{A}^{(i)})$
Update $\mathcal{W}^{(i)}$ by (109)
UNTIL $\sum_m |R_m(\mathcal{W}^{(i)}) - 1/W_m^{(i)}| \leq \delta_0$ **OR** $i = I$
3: Choose the energy-bandwidth Allocation
$(\mathcal{P}^{(i)}, \mathcal{A}^{(i)})$ is the obtained energy-bandwidth allocation

Note that, the convergence of the proposed algorithm is highly dependent on the selection of the initial value, *i.e.,* $\mathcal{W}^{(0)}$. Specifically, we can set

$$\frac{1}{W_m^{(0)}} \approx \mathbb{E}_{\{\tilde{E}_n^k, H_n^k\}} \left[\bar{R}_m(K) \right] \ , \tag{110}$$

as the initial PF weights, where $\bar{R}_m(K)$ denotes the sum-rate achieved by the solution to F_ϵ given the realizations $\{\tilde{E}_n^k, H_n^k, n \in \mathcal{N}, k \in \mathcal{K}\}$ in the scheduling period K, and the simulation results in Section 5 demonstrate that the optimal performance is approached closely in a few iterations.

5. Numerical Results

We first focus on a single transmitter and compare the achievable rate regions for orthogonal and non-orthogonal two-user broadcast channels, i.e, $N = 1$ and $M = 2$. For the transmitter, we set the initial battery level $B_n^k = 0$, the battery capacity $B_n^{\max} = 20$ units, and we do not apply the maximum power constraint. We generate the realizations of the harvested energy E_n^k and channel gains H_m^k following the truncated Gaussian distribution $\mathcal{N}(10, 2)$ and the Rayleigh distribution with the parameter 2, respectively. Moreover, we consider two scheduling period, $K = 1$ slot and $K = 10$ slots, and show the sum-rate improvement by the non-orthogonal broadcast over the orthogonal broadcast in Fig. 6 and Fig. 7, respectively. Specifically, we note that when $K = 10$ the improvement is quite marginal. Moreover, in Fig. 7, two curves share three common points corresponding to the sum-rate achieved by the solution to $\mathcal{P}_0(W_1, W_2)$ for $(W_1, W_2) = (1, 0), (0.5, 0.5)$ and $(0, 1)$, respectively. Also, when $W_1 = W_2 = 0.5$, the sum-rates are maximized for both the orthogonal and non-orthogonal broadcast, which are same.

5.1. Weighted Sum-Rate Maximization

We then consider a network with multiple broadcast channels where there are $N = 3$ transmitters and each communicates with 2 receivers, *i.e.,* $\mathcal{M}_1 = \{1, 2\}, \mathcal{M}_2 = \{3, 4\}, \mathcal{M}_3 = \{5, 6\}$. We set the scheduling period as $K = 20$ slots. For each transmitter n, we set the initial battery level $B_n^0 = 0$ and the battery capacity $B_n^{\max} = 20$ units. We assume that the

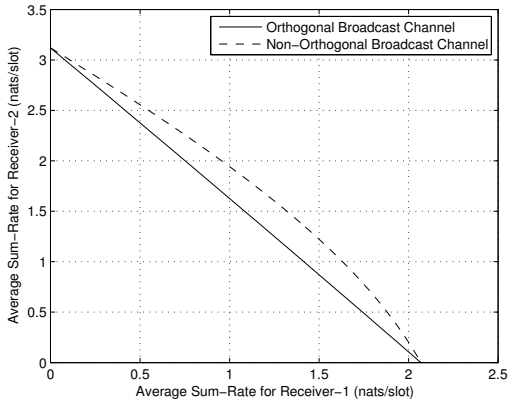

Figure 6. Achievable sum-rate regions of two-user orthogonal/non-orthogonal broadcast channels ($K = 1$).

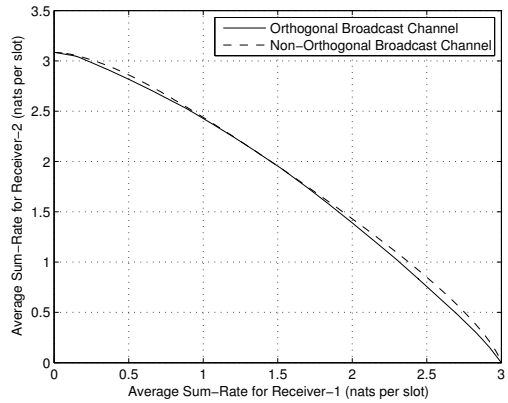

Figure 7. Achievable sum-rate regions of two-user orthogonal/non-orthogonal broadcast channels ($K = 10$).

harvested energy E_n^k follows a truncated Gaussian distribution with mean μ_n and variance of 2. We also assume a Rayleigh fading channel with the parameter σ_m.

For comparison, we consider two simple scheduling strategies, namely, the *greedy energy policy* and the *equal bandwidth policy*. For the greedy energy policy, each transmitter first tries to use up the available energy in each slot. Then, given the available energy for each transmitter, we solve the energy-bandwidth allocation problem slot by slot, *i.e.,* $\mathsf{P}_{\mathcal{W}}(0)$ for $K = 1$, to calculate the energy and bandwidth allocated for each receiver. For the equal bandwidth policy, we first assign the bandwidth for each transmitter equally. Then, given the assigned bandwidth for each transmitter, we solve an energy-bandwidth allocation problem transmitter by transmitter, *i.e.,* $\mathsf{P}_{\mathcal{W}}(0)$ for $N = 1$, to calculate the energy and bandwidth (for orthogonal broadcast channel only) allocated for each receiver.

To compare the performance of the different algorithms and policies, we evaluate the (weighted) sum-rate for the multiple orthogonal broadcast channels (O-BCs) and non-orthogonal broadcast channels (NO-BCs), respectively. We use $\mathcal{W}_1 = \{W_m = 1/6\}$ and $\mathcal{W}_2 = \{W_m = (2(n-1)+m)/21\}$ for the unweighted and weighted sum-rate cases, respectively, and set the channel fading parameter $\sigma_m = 2$. Moreover, we assume the power unconstrained case where the energy harvesting rate is $\mu_n = 6, 7, 8, 9, 10, 11$ units per slot and a power constrained case where the maximum power constraint is $P_n = 10$ and the energy harvesting rate is $\mu_n = 1, 2, 3, 4, 5, 6$ units per slot. We run the simulation 500 times to obtain the performance for the different algorithm and policies, as shown in Figs. 8, 9, and 10 for the power unconstrained case with $\mathcal{W}_1$, the power unconstrained case with $\mathcal{W}_2$, and the power constrained case with $\mathcal{W}_2$, respectively.

As shown in Fig. 8, the maximum throughput in NO-BC is the same as that in O-BC under the optimal energy-bandwidth allocation and the greedy energy policy. This is because in both O-BC and NO-BC, the optimized bandwidth allocation requires that each transmitter only transmit to the receiver with the strongest channel in each slot when the weights are equal (*e.g.,* $\mathcal{W}_1$), as stated in Theorem 3 and Corollary 1. For the equal bandwidth policy, O-BC performs worse than NO-BC since the NO-BC makes better use

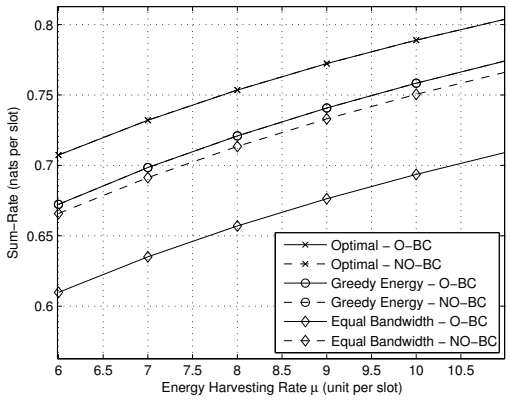

Figure 8. Sum-rate comparisons for different policies without the maximum power ($\mathcal{W}_1$).

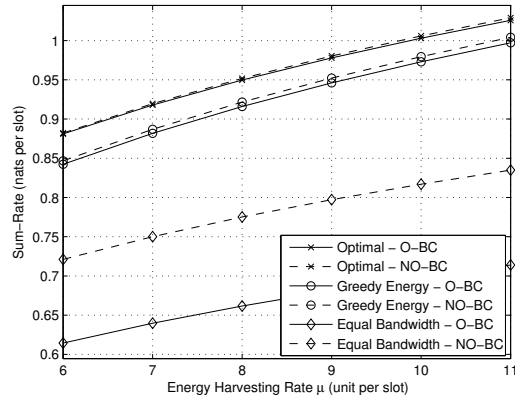

Figure 9. Weighted sum-rate comparisons for different policies without the maximum power ($\mathcal{W}_2$).

of the allocated bandwidth by optimally treating the interference. When we use the unequal weights $\mathcal{W}_2$, it is seen in Figs. 9 and 10 that we may get better performance by using NO-BC instead of O-BC under all policies. However, for the optimal energy-bandwidth allocation, such improvement is quite marginal. Moreover, when the maximum power is constrained, it is seen in Fig. 10 that the gap between the performances of the optimal energy-bandwidth allocation and the greedy energy policy decreases as the energy harvesting rates increases.

5.2. PF Throughput Maximization

We next evaluate the PF throughput performance in the network with multiple orthogonal broadcast channels. For comparison, we consider three scheduling strategies, namely, the *greedy policy*, the *traditional PF policy*, and the *approximate PF policy*. For the greedy policy, the transmitter evenly splits the maximum available energy for the transmission to each receiver in each slot, *i.e.,* $p_m^k = B_n^k/|\mathcal{M}_n|$, and the equal bandwidth is also allocated, *i.e.,* $a_m^k = 1/M$. For the traditional PF policy, the transmitter tries to use the maximum available energy in each slot and one transmission link is chosen to use the entire bandwidth as follows:

$$\arg\max_m \left\{ \log(1 + p_m^k H_m^k)/\tilde{R}_m^k \right\}, \tag{111}$$

where we denote $\tilde{R}_m^k$ as the average sum-rate before slot k [25]. For the approximate PF policy, we use the approximate PF weights given in (110) and then solve a weighted sum-rate maximization problem.

To evaluate the performance of the different algorithm and policies, we consider two scenarios, namely, the *varying EH scenario*, where the different transmitters have different means of the energy harvesting such that $\mu_1 + 2 = \mu_2 + 1 = \mu_3$ and the channel fading parameter is $\sigma = 2$ for all transmitters, and *varying channel scenario*, where the different transmitters have different channel fading parameters such that $\sigma_{1(\cdot)} + 0.5 = \sigma_{2(\cdot)}$ and the mean of the energy harvesting is $\mu = 2$ for all transmitters. In both the scenarios, the

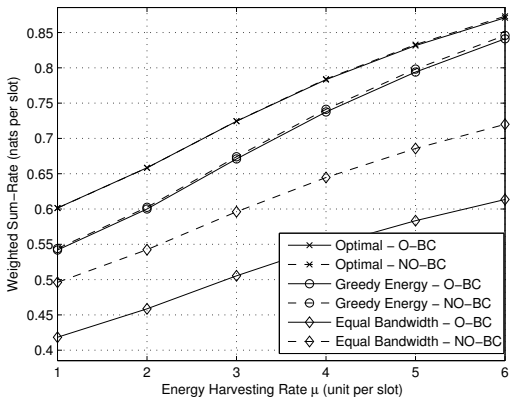

Figure 10. Weighted sum-rate comparisons for different policies with the maximum power ($\mathcal{W}_2$, $P_n = 10$).

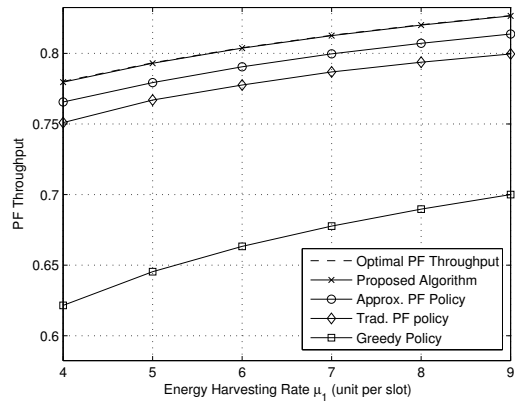

Figure 11. Performance comparisons in the varying EH scenario.

maximum power is unconstrained and we compare the performance of Algorithm 3 and the other three polices with the optimal PF throughput obtained using the generic convex solver. Specifically, in the varying EH scenario and the varying channel scenario, we assume $\mu_1 = 1, 2, 3, 4, 5, 6$ units per slot and $\sigma_{1(\cdot)} = 1, 1.2, 1.4, 1.6, 1.8, 2$, respectively. We run the simulation 500 times to obtain the performance for the different algorithm and policies, as well as the optimal schedule solved by a general convex solver, as shown in Fig. 11 and Fig. 12 for the varying EH scenario and the varying channel scenario, respectively.

From Fig. 11 and Fig. 12, it is seen that for both scenarios Algorithm 3 achieves the same performance as that achieved by the optimal energy-bandwidth allocation solved by the generic convex solver, which is better than the other policies, as excepted. Specifically, the performance of the approximate PF policy is close to the optimal performance and better than that of the traditional PF and greedy policies. It is because the energy harvesting and channel fading processes are stationary and erodic and the sum-rate achieved by the optimal energy-bandwidth allocation is close to the PF weights parameter. Also, the traditional PF policy is optimal for the transmitters without using the renewable energy source. However, due to the energy harvesting process with the finite battery capacity, the potential energy overflow necessitates the bandwidth share to maximize the proportionally-fair throughput. Therefore, the traditional PF policy gives the suboptimal performance for the transmitters powered by the renewable energy source. Moreover, the greedy policy, which does not take the energy and the fairness factors into account, provides the worst performance among the simulated algorithm/polices.

We also evaluate the convergence speed of Algorithm 3 with different initial weights $\mathcal{W}$, *i.e.,* the approximate PF weights and equal weights, as shown in Fig. 13 for $K = 20$. It is seen that, the convergence speed with the initial approximate PF weights is faster than that with the initial equal weights, approaching to the optimal performance after around 10 iterations.

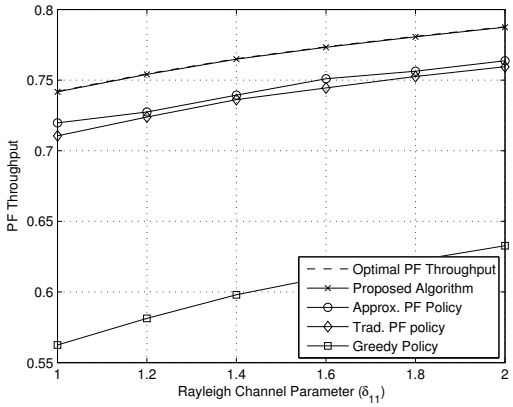

Figure 12. Performance comparisons in the varying channel scenario.

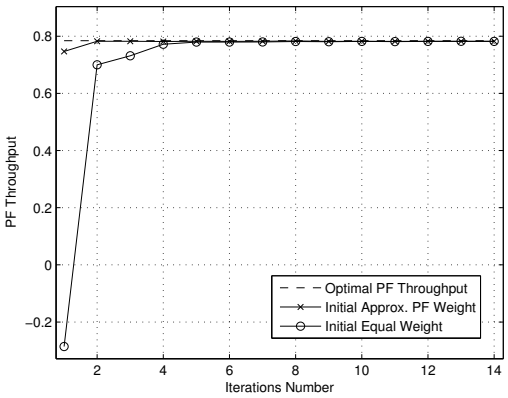

Figure 13. Convergence behavior of Algorithm 3.

6. Conclusions

In this chapter, we have considered the joint energy-bandwidth allocation problem for multiple energy harvesting transmitters over K time slots, to maximize the weighted throughput and the proportionally fair throughput. This problem is formulated as a convex optimization problem with $\mathcal{O}(MK)$ variables and constrains, where M is the number of the receivers and K is number of the slots in a scheduling period, which is hard to solve with a generic convex tool. We have proposed an energy-bandwidth allocation algorithm that iterates between solving the energy allocation subproblem and the bandwidth allocation subproblem, and the convergence and the optimality of the iterative algorithm have been shown. Based on this general iterative algorithm that alternatively solves the energy and bandwidth allocation subproblems, we have developed optimal algorithms for solving the two subproblems for both orthogonal and non-orthogonal broadcast. Moreover, for orthogonal broadcast, we have shown that the PF throughput maximization problem can be converted to the weighted throughput maximization problem with proper weights. Simulation results demonstrate that the proposed algorithms offer significant performance improvement over various suboptimal allocation schemes. Moreover, it is seen that with energy-harvesting transmitters, non-orthogonal broadcast offers limited gain over orthogonal broadcast.

We assumed a longer time slot, where the Shannon capacity formulas hold. The authors of [32, 33, 34] considered approximate capacity with maximum battery constraint. Extension of these results to larger networks, and using these expressions for efficient resource allocation remains open. The approach described in this chapter is not limited to the very particular objective functions and the some of the approaches in this chapter has been found useful for different rate expressions in optical channels [35, 36]. Further, this chapter assumes non-causal information of the channel gains. However, the work can be extended to give algorithms with causal information of the channel gains. The reader is referred to [16, 19, 37] to see extension of some of these algorithms when there is causal information of the channel gains.

This work assumes a flat fading channel, while the work can be extended to frequency-

selective fading channels on the lines of [38, 39], where joint energy and subchannel allocation for multiple links is considered. This chapter solves the different optimization problems based on the structure, while recently [40, 41] studied using Proximal-Jacobian ADMM to solve weighted throughput optimization for the orthogonal model. The interesting result is that the computation complexity using Proximal-Jacobian ADMM is the same as this in this chapter. The advantage of Proximal-Jacobian ADMM is the provable convergence rate. Exploiting the structure to come up with the algorithms as in this chapter can potentially help in lower run-time by having lower pre-constants, and thus finding the convergence rates of the algorithms proposed in this chapter is an important research direction.

References

[1] S. Sudevalayam and P. Kulkarni, "Energy harvesting sensor nodes: survey and implications," *IEEE Commun. Surveys Tuts.*, vol. 13, no. 3, pp. 443–461, Sep. 2011.

[2] J. A. Paradiso and T. Starner, "Energy scavenging for mobile and wireless electronics," *IEEE Trans. Pervasive Computing*, vol. 4, pp. 18–27, Jan. 2005.

[3] G. Maria, W. Aya, and Z. Gil, "Networking low-power energy harvesting devices: measurements and algorithms," *IEEE Trans. Mobile Comput.*, vol. 12, no. 9, pp. 1853–1233, Sep. 2013.

[4] J. Piorno, C. Bergonzini, K. Atienza, and T. Rosing, "Prediction and management in energy harvested wireless sensor nodes," in *VITAE 2009*, May 2009, pp. 6–10.

[5] Z. Wang, V. Aggarwal, and X. Wang, "Iterative dynamic water-filling for fading multiple-access channels with energy harvesting," *IEEE Journal on Selected Areas in Communications*, vol. 33, no. 3, pp. 382–395, March 2015.

[6] ——, "Renewable energy scheduling for fading channels with maximum power constraint," in *Allerton 2013*, Oct. 2013, pp. 1394–1400.

[7] ——, "Power allocation for energy harvesting transmitter with causal information," *IEEE Transactions on Communications*, vol. 62, no. 11, pp. 4080–4093, Nov 2014.

[8] K. Tutuncuoglu and A. Yener, "Optimum transmission policies for battery limited energy harvesting nodes," *IEEE Trans. Commun.*, vol. 11, no. 3, pp. 1180–1189, Mar. 2012.

[9] C. Huang, R. Zhang, and S. Cui, "Throughput maximization for the gaussian relay channel with energy harvesting constraints," *IEEE J. Sel. Areas Commun.*, vol. PP, no. 99, pp. 1–11, Sep. 2012.

[10] J. Yang and S. Ulukus, "Optimal packet scheduling in a multiple access channel with energy harvesting transmitters," *IEEE J. Commun. and Netw.*, vol. 14, no. 2, pp. 140–150, Apr. 2012.

[11] S. Chen, P. Sinha, N. Shroff, and C. Joo, "Finite-horizon energy allocation and routing scheme in rechargeable sensor networks," in *Proc. IEEE 2011 INFOCOM*, Apr. 2011, pp. 2273–2281.

[12] C. Ho and R. Zhang, "Optimal energy allocation for wireless communications with energy harvesting constraints," *IEEE Trans. Signal Process.*, vol. 60, no. 9, pp. 4808–4818, Sep. 2012.

[13] O. Ozel, K. Tutuncuoglu, J. Yang, S. Ulukus, and A. Yener, "Transmission with energy harvesting nodes in fading wireless channels: optimal policies," *IEEE J. Sel. Areas Commun.*, vol. 29, no. 8, pp. 1732–1743, Sep. 2011.

[14] K. Tutuncuoglu and A. Yener, "Sum-rate optimal power policies for energy harvesting transmitters in an interference channel," *IEEE J. Commun., Netw.*, vol. 14, no. 2, pp. 151–161, Apr. 2012.

[15] O. Ozel, Y. Jing, and S. Ulukus, "Optimal broadcast scheduling for an energy harvesting rechargeable transmitter with a finite capacity battery," *IEEE Trans. Wireless Commun.*, vol. 11, no. 8, pp. 2193–2203, Jun. 2012.

[16] Z. Wang, V. Aggarwal, and X. Wang, "Multiuser joint energy-bandwidth allocation with energy harvesting - part I: Optimum algorithm & mulitple point-to-point channels," *available at arXiv 1410.2861*, Oct. 2014.

[17] ——, "Multiuser joint energy-bandwidth allocation with energy harvesting-part II: Multiple broadcast channels & proportional fairness," *arXiv preprint arXiv:1410.2867*, 2014.

[18] ——, "Joint energy-bandwidth allocation in multiple broadcast channels with energy harvesting," *IEEE Transactions on Communications*, vol. 63, no. 10, pp. 3842–3855, Oct 2015.

[19] ——, "Optimal energy-bandwidth allocation for energy-harvesting networks in multiuser fading channels," *IEEE Journal on Selected Areas in Communications*, vol. 34, no. 5, pp. 1565–1577, May 2016.

[20] ——, "Optimal energy-bandwidth allocation for energy harvesting interference networks," in *2014 IEEE International Symposium on Information Theory*, June 2014, pp. 1166–1170.

[21] ——, "Energy-bandwidth allocation in multiple orthogonal broadcast channels with energy harvesting," in *2015 IEEE International Symposium on Information Theory (ISIT)*, June 2015, pp. 2717–2721.

[22] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge: Cambridge University Press, 2009.

[23] J. Yang, O. Ozel, and S. Ulukus, "Broadcasting with an energy harvesting rechargeable transmitter," *IEEE Trans. Wireless Commun.*, vol. 11, no. 2, pp. 571–583, Feb. 2012.

[24] ——, "Optimal scheduling over fading broadcast channels with an energy harvesting transmitter," in *Proc. IEEE CAMSAP 11'*, Dec. 2011, pp. 193–196.

[25] H. Kushner and P. Whiting, "Convergence of proportional-fair sharing algorithms under general conditions," *IEEE Trans. Wireless Commun.*, vol. 3, no. 4, pp. 1250–1259, Jul. 2004.

[26] N. Tekbiyik, T. Girici, E. Uysal-Biyikoglu, and K. Leblebicioglu, "Proportional fair resource allocation on an energy harvesting downlink," *IEEE Trans. Wireless Commun.*, vol. 12, no. 4, pp. 1699–1711, Apr. 2013.

[27] T. Cover and J. Thomas, *Elements of Information Theory*. New York: Wiley, 1991.

[28] A. Duel-Hallen, "Fading channel prediction for mobile radio adaptive transmission systems," *Proc. IEEE*, vol. 95, no. 12, pp. 2299–2313, Dec. 2007.

[29] Z. Wang, V. Aggarwal, and X. Wang, "Optimal energy-bandwidth allocation for energy harvesting interference networks," in *Proc. IEEE ISIT 14'*, Jul. 2014, pp. 1166–1170.

[30] R. Burden and J. Faires, *Numerical Analysis*. Boston: PWS-KENT, 1989.

[31] D. Tse and P. Viswanath, *Fundamentals of Wireless Communication*. Cambridge: Cambridge University Press, 2005.

[32] Y. Dong, F. Farnia, and A. Ozgur, "Near optimal energy control and approximate capacity of energy harvesting communication," *IEEE Journal on Selected Areas in Communications*, vol. 33, no. 3, pp. 540–557, March 2015.

[33] M. Ashraphijuo, V. Aggarwal, and X. Wang, "On the capacity of energy harvesting communication link," *IEEE Journal on Selected Areas in Communications*, vol. 33, no. 12, pp. 2671–2686, Dec 2015.

[34] ——, "Energy harvesting communication using limited battery with efficiency," in *2015 53rd Annual Allerton Conference on Communication, Control, and Computing (Allerton)*, Sept 2015, pp. 301–306.

[35] Z. Wang, V. Aggarwal, X. Wang, and M. Ismail, "Energy scheduling for optical channels with energy harvesting devices," in *2017 IEEE International Conference on Communications (ICC)*, May 2017, pp. 1–6.

[36] V. Aggarwal, Z. Wang, X. Wang, and M. Ismail, "Energy scheduling for optical channels with energy harvesting devices," *IEEE Transactions on Green Communications and Networking*, vol. PP, no. 99, pp. 1–1, 2017.

[37] M. Rezaee, M. Mirmohseni, V. Aggarwal, and M. R. Aref, "Optimal transmission policies for multi-hop energy harvesting systems," *CoRR*, vol. abs/1612.09496, 2016. [Online]. Available: http://arxiv.org/abs/1612.09496

[38] Z. Wang, X. Wang, and V. Aggarwal, "Transmission with energy harvesting nodes in frequency-selective fading channels," *IEEE Transactions on Wireless Communications*, vol. 15, no. 3, pp. 1642–1656, March 2016.

[39] ——, "Energy-subchannel allocation for energy harvesting nodes in frequency-selective channels," in *2015 IEEE International Symposium on Information Theory (ISIT)*, June 2015, pp. 2712–2716.

[40] V. Aggarwal, M. R. Bell, A. Elgabli, X. Wang, and S. Zhong, "Joint energy- bandwidth allocation for multi-user channels with cooperating hybrid energy nodes," in *IEEE International Conference on Communications (Green Communications Systems and Networks Symposium)*, May 2017.

[41] ——, "Joint energy-bandwidth allocation for multiuser channels with cooperating hybrid energy nodes," *IEEE Transactions on Vehicular Technology*, vol. 66, no. 11, pp. 9880–9889, Nov 2017.