INTRODUCTION

The Problem

The aim of the assignment is to compute the lift on a wing-body combination in subsonic flow by using panel methods.

Theory

The lift of a wing-body combination is not simply obtained by simply adding the lift of the wing alone to the lift of the body alone.Rather,as soon as the wing and body are mated,the flow field over the body modifies the flow field over the wing-this is called the wing-body interaction.

There is no analytical solution which can predict the lift of a wing-body combination,properly taking into account the nature of the wing-body aerodynamic interaction.Either the configuration must be tested in a wind tunnel,or a computational fluid dynamic calculation must be made.We cannot even predict whether the lift will more or less than the sum of the two parts because interaction can be positive or negative.

Methodology of the solution

In our assignment we have a wing-body combination of given characteristics.The wing is a slender delta wing and the body is a slender body of revolution.The combined configuration is exposed to low subsonic flow at a small angle of attack.The lift of the combination in the low subsonic flow field has to be calculated.The method adopted is one of panel method where the system is divided into a specific number of panels.The aim is to model the combination by some vortex distribution and the using that to find the lift on the combination.Therfore a suitable vortex distribution is distributed on the panels and then the right distribution is obtained by applying the zero normal flow condition on the solid boundaries.The lift could be found out by considering the conditions prevailing in the Trefftz plane which will be explained later.

Thus the problem has been set up.

Mathematical Formulation of the Problem

Since the problem is one of ideal fluid flow it should satisfy the Laplace equation.Since we have selected a vortex distribution this condition and the condition at infinity are automatically satisfied.The other condition needed to solve the problem is the boundary condition of zero normal flow on the surface.This boundary condition is applied at the midpoint of each panel.

Useful Mathematical relations

Velocity induced by a vortex:

$$U_\theta = \mathbf{\Gamma/2\pi r}$$

Velocity induced by a constant strength vortex distribution :

$$U = (\gamma/2\pi)[\tan^{-1}((x-x_1)/y)-\tan^{-1}((x-x_2)/y)]$$
$$V = (\gamma/4\pi)\ln[((x-x_1)^2+y^2)/((x-x_2)^2+y^2)]$$

Where
X1 and x2 are the coordinates of the end
Points of the distribution.

Lift on a body :

$$L=-2\rho V\cos\alpha\left[\int_0^b (\phi_u-\phi_l)dy\right]$$

Where the potential distribution is the distribution
In the Trefftz plane.

The potential jumps across a vortex sheet and the jump is equal to the total circulation enclosed by a contour joining these two points(This can be proved from stokes theorem).

Therefore the basic mathematical tools required for solving the problem have been obtained.

The Solution to the problem

The solution to the problem consists of a series of steps given below.

1.Choice of singularity element

The singularity element used to model the flow can be chosen by the programmer viz source,vortex,doublet.In our case a distribution of sources and sinks cannot be used because the body is not closed.However a doublet distribution can be used .But we will resort to a vortex distribution.

2.Generation of panel coordinates and collocation points

The given shape should be divided into a number of panels .The end points of the panels are found out .The collocation points are the points where the zero normal flow condition is applied.In our scheme the panel midpoints are selected to be the collocation points.

3.Computation of influence coefficients

The influence coefficient a_{ij} is defined to be the velocity induced at the i^{th} panel by a unit gamma distribution at the j^{th} panel.These coefficients are computed by using the formulas given before for constant gamma distribution.

4.Obtaining gamma distribution

The equations are setup by enforcing the boundary condition at the collocation points.

$$V_{\infty\, n} + V_{gamma)n} = 0$$

A set of equations are obtained namely

$$\sum \gamma_j a_{ij} = 0$$

These equations are solved using any standard method for solving n x n equations.

The lift on the body is obtained by using the formula given before.
This outlines the solution methodology of the problem assigned.

Discussion of Results

Before proceeding with the given problem a number of cases having exact analytical solution were solved to justify the code of the panel method. 3 known cases have been solved namely

- ✓ Flat plate at an angle of attack
- ✓ Cylinder in cross flow
- ✓ Flat plate in cross flow

It may be noted that all three cases have known analytical solution.
The results obtained have been mentioned below

1.Flat plate at an angle of attack

The solution was worked out for various number of panels.
It was observed that the solution converged for

$$N > 60.$$

For this case ,the boundary condition was not applied at the first panel alone as it gave wrong results.Therefore the BC provides n-1 equations .The equation was obtained by satisfying the kutta condition that the value of γ at the trailing edge is zero.

Hence the solution was obtained .It was plotted with the theoretical value and was found to coincide with the theoretical value almost exactly.

It maybe noted that the theoretical value obtained by thin airfoil theory for a flat plate at an angle of attack α is

$$\gamma = 2V\sin\alpha\,((1-\xi_*)/(1+\xi_*))^{1/2}$$

where $\xi_* = 2\xi/c$ c-chord length

The obtained solutions have been plotted in the form of charts attached.

2.Cylinder in Cross flow

The solution was worked out for various number of panels.

It was observed that the solution converged for

$$N > 80$$

There was no specific difficulty in the solution and only the boundary conditions were needed to obtain the solution.

Hence the solution was obtained .It was plotted with the theoretical value and was found to coincide with the theoretical value .

It maybe noted that the theoretical value for a cylinder is

$$\gamma = 2V \sin \theta$$

where θ is the angle measured from the flow direction.

The obtained solutions have been plotted in the form of charts attached .

3.Flat plate in Cross flow

The solution for this case was not obtained by having constant strength vortex in all panels since it would give problems for applying the boundary condition in the panels at the two ends of the flat plate.Therefore these two panels alone were modeled by linearly varying γ distribution.Then the solution was obtained for various number of panels.It was observed that the solution converged for

$$N > 60$$

The obtained solution was plotted with theoretical value of γ distribution obtained from complex variables.
It maybe noted that the theoretical solution is

$$\gamma = 2Vx_*/(1-x_*)^{1/2} \text{ where } x_*=2x/c$$

4.Wing Body Combination

Finally the assignment problem was solved using panel methods.The solution was obtained with the span being two times the diameter.The solution obtained for various number of panels was found to converge for

$$N > 96$$

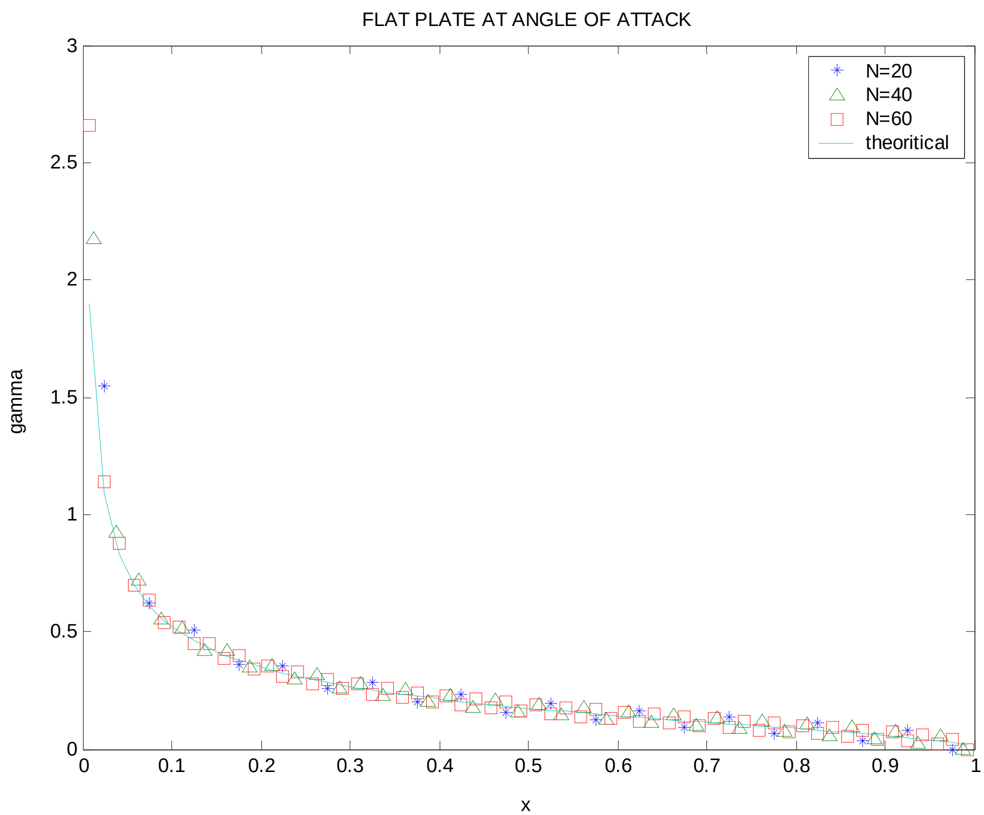
The effect of the size of the body was also observed by repeating the computations for various diameter/span ratio.As expected the solution came closer and closer to the flat plate solution as the diameter was reduced.The γ value was close to zero at the stagnation points which confirms the correctness of the solution.

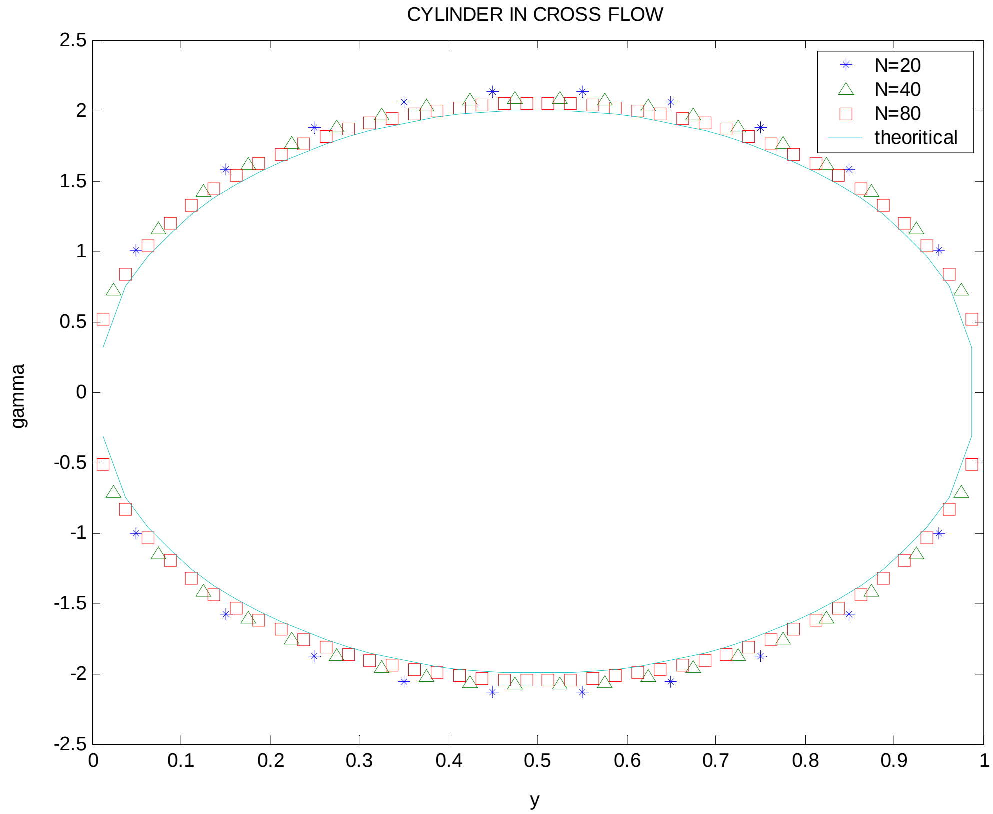
The lift was calculated with the γ distribution obtained and the lift coefficient was calculated for various angle of attacks(small).All these results have been plotted in the charts attached.

Conclusion

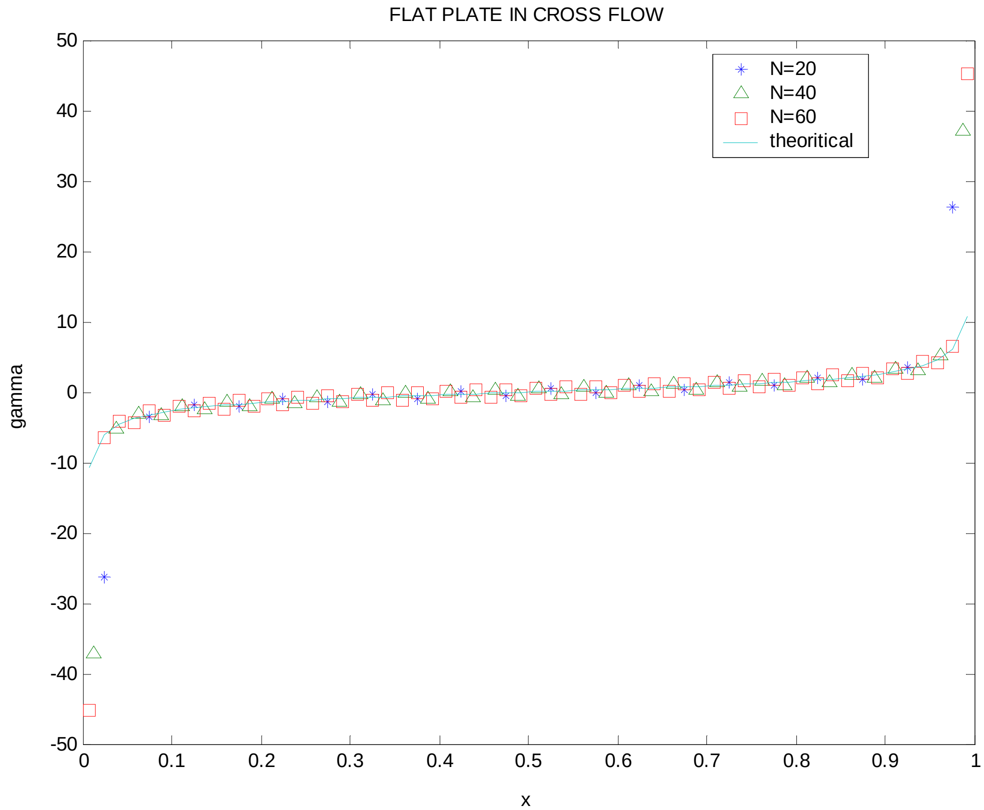
The results indicate that the lift on the wing body combination is almost close to the lift on the body itself.For a wide range of a/b ratios varying from 0(wing only) to 6(fat fuselage with a stubby wing) the variation in C_l is about 5 % only.This is confirmed by the experimental results published in the book “FLUID DYNAMIC LIFT” by Hoerner where he mentions the same fact.

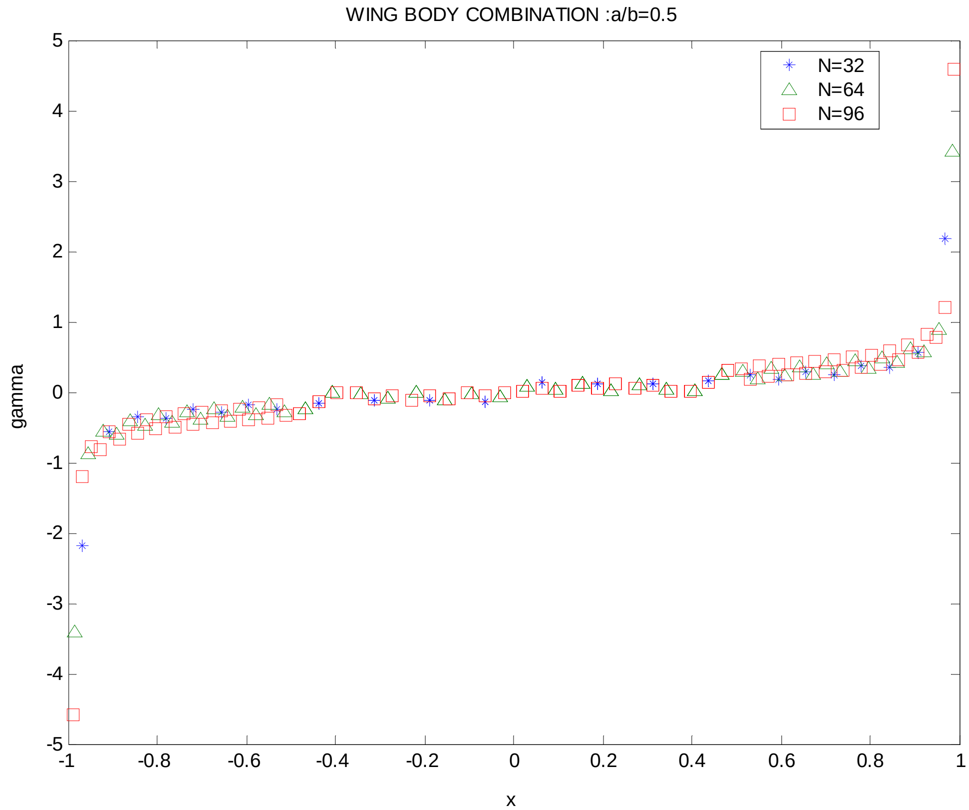
Thus to conclude the lift of the wing body combination is not very different from the lift on wing

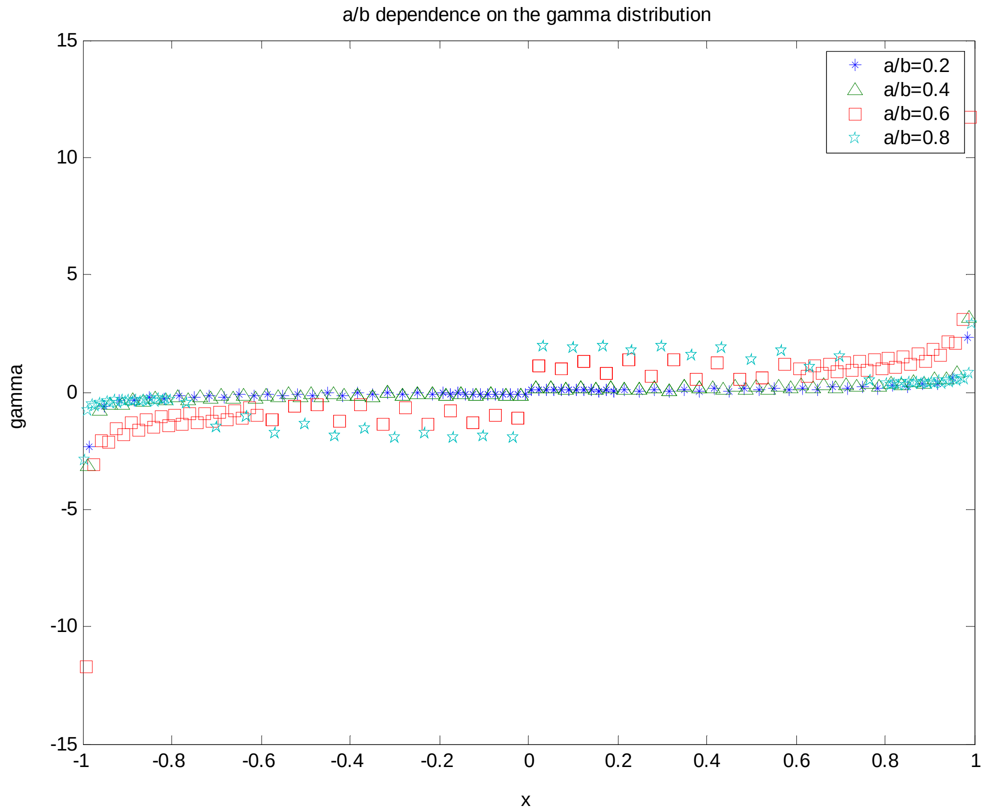

FLAT PLATE AT ANGLE OF ATTACK
N=20
N=40
N=60
theoritical
gamma
x
0
0.5
1
1.5
2
2.5
3
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

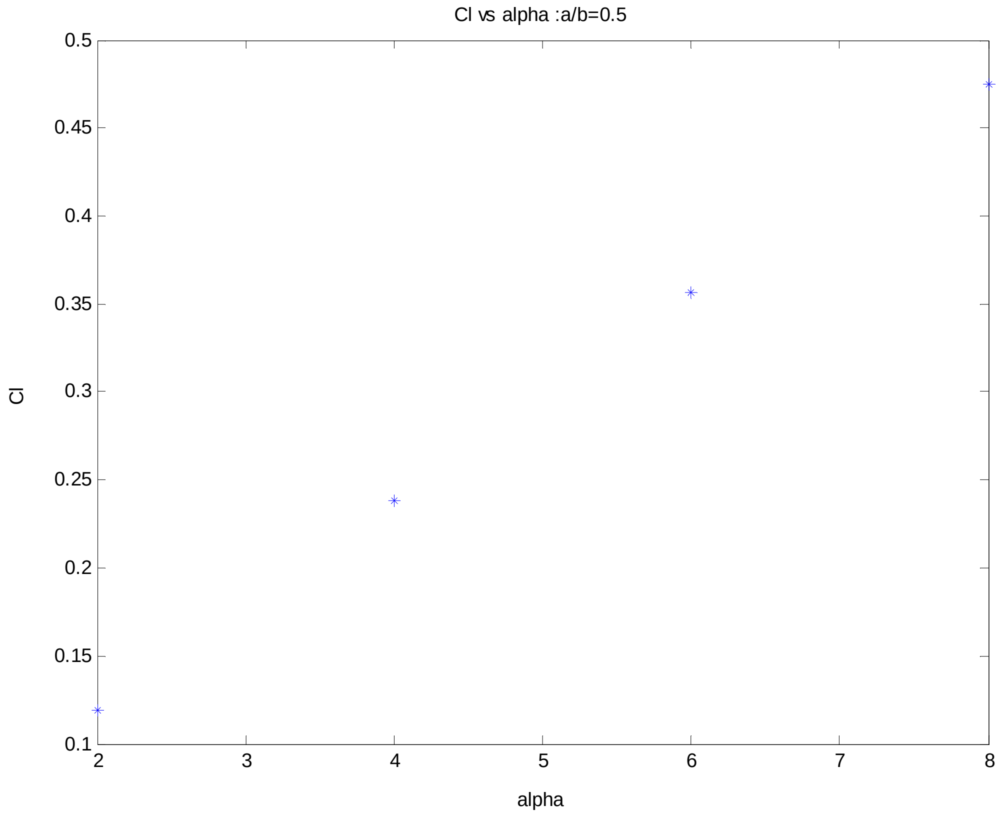


CYLINDER IN CROSS FLOW
N=20
N=40
N=80
theoritical
gamma
y
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


FLAT PLATE IN CROSS FLOW
N=20
N=40
N=60
theoritical
gamma
x
50
40
30
20
10
0
-10
-20
-30
-40
-50
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


WING BODY COMBINATION :a/b=0.5
N=32
N=64
N=96
gamma
x
5
4
3
2
1
0
-1
-2
-3
-4
-5
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1


a/b dependence on the gamma distribution
a/b=0.2
a/b=0.4
a/b=0.6
a/b=0.8
gamma
x
15
10
5
0
-5
-10
-15
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1



Cl vs alpha :a/b=0.5
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
Cl
2
3
4
5
6
7
8
alpha