Quiz 16
Ma 16200
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Problem 1:

\[ \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} \]

We see that \( a_n = \frac{1}{n} \) is a sequence of all positive terms and we check the following:

\[ a_{n+1} = \frac{1}{n + 1} < \frac{1}{n} = a_n \]

and \( \lim_{n \to \infty} \frac{1}{n} = 0 \)

Therefore by the Alternating Series Test the series converges.

Problem 2:

\[ \sum_{n=1}^{\infty} (-1)^{n-1} e^{\frac{1}{n}} \]

Again we try to apply the Alternating Series Test however we see for \( b_n = e^{\frac{1}{n}} \):

\[ \lim_{n \to \infty} b_n = \lim_{n \to \infty} e^{\frac{1}{n}} = e^0 = 1 \neq 0 \]

Therefore the series diverges.
Problem 3:

How many terms are needed to approximate the following series within 0.01?

\[
\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^3}
\]

We need to find the first \( n \) value such that:

\[
\frac{1}{n^3} < 0.01 = \frac{1}{100}
\]

Then we will only require \( n-1 \) terms.

\[
\frac{1}{n^3} < \frac{1}{100} \Rightarrow 100 < n^3
\]

Remember \( n \) is always taken to be an integer. We then see:

\[
100 \leq 64 = 4^3 \text{ but } 100 < 125 = 5^3
\]

Therefore because \( n=5 \) is the first term such that the above inequality is satisfied, we only require \( 4 \text{ terms} \) to approximate our series within the given error.