Instructions:

1. Academic Integrity
   a. Students may not open the exam until instructed to do so.
   b. Students must obey the orders and requests by all proctors, TAs, and lecturers.
   c. No student may leave in the first 20 minutes or in the last 10 minutes of the exam.
   d. Books, notes, calculators, or any electronic devices are not allowed on the exam, and they should not even be in sight in the exam room. Students may not look at anybody else’s test, and may not communicate with anybody else except, if they have a question, with their TA or lecturer.
   e. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.
   f. Any violation of these rules and any act of academic dishonesty may result in severe penalties. Additionally, all violators will be reported to the Office of the Dean of Students.

   I have read and understand the exam rules stated above:
   STUDENT NAME (print): ________________
   STUDENT SIGNATURE: __________________

2. Fill in all the information requested above. On the scantron sheet fill in your name, student ID number, and the section number of your recitation with an extra 0 at the left. Blacken the correct circles.

3. This booklet contains 12 problems. Problems 1 through 4 are worth 9 points each. Problems 5 through 12 are worth 8 points each. The maximum score is 100 points.

4. For each problem mark your answer on the scantron sheet and also circle it in this booklet.

5. Work only on the pages of this booklet.

Mark TEST 01 on your scantron!
1. If $R$ is the region in the first quadrant bounded above by $y = 3 - 2x$ and below by $y = x^2$, and if the center of mass is $(\bar{x}, \bar{y})$, compute $\bar{x}$.

A. $\frac{1}{3}$
B. $\frac{7}{18}$
C. $\frac{7}{20}$
D. $\frac{5}{12}$
E. $\frac{5}{14}$

2. Compute $\sum_{n=1}^{\infty} \frac{(-1)^n + 2^{n-1}}{3^n}$.

A. $\frac{11}{4}$
B. $\frac{3}{4}$
C. $\frac{13}{4}$
D. $\frac{7}{6}$
E. $\frac{11}{6}$

\[
\sum_{n=1}^{\infty} \frac{(-1)^n + 2^{n-1}}{3^n} = \frac{\sum_{n=1}^{\infty} (-1)^n}{3^n} + \frac{\sum_{n=1}^{\infty} 2^{n-1}}{3^n} = \text{both geometric}
\]

\[
\begin{align*}
\sum_{n=1}^{\infty} (-1)^n &= \frac{-1}{1 + \frac{1}{3}} = \frac{-1}{\frac{4}{3}} = \frac{-3}{4} \\
\sum_{n=1}^{\infty} 2^{n-1} &= \frac{1}{1 - \frac{2}{3}} = \frac{1}{\frac{1}{3}} = 3
\end{align*}
\]

\[
\Rightarrow \sum_{n=1}^{\infty} \frac{(-1)^n + 2^{n-1}}{3^n} = \frac{-\frac{3}{4}}{1 - \frac{2}{3}} + \frac{3}{1 - \frac{2}{3}} = \frac{-\frac{3}{4}}{\frac{1}{3}} + \frac{3}{\frac{1}{3}} = \frac{-3}{4} \cdot 3 + 3 \cdot 3 = \frac{-9}{4} + 9 = \frac{3}{4}
\]
3. Which of the following limits exist?

I. \( \lim_{n \to \infty} \frac{\sqrt{2n^2 + 3}}{2n - 1} \)

II. \( \lim_{n \to \infty} \frac{\ln n}{\sqrt{n + 2}} \)

III. \( \lim_{n \to \infty} \frac{(-1)^n 2^n}{n^2 + 1} \)

A. I and III only
B. I only
C. III only
D. I, II and III
E. None of them

\( \lim_{n \to \infty} \frac{\ln n}{\sqrt{n + 2}} = \lim_{n \to \infty} \frac{\frac{1}{n}}{\frac{1}{2} (n+2)^{-\frac{1}{2}}} = \lim_{n \to \infty} 2 \cdot \frac{\sqrt{n + 2}}{n} = 0 \)

4. If \( S = \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{2n^2 + 3}} \), which integer \( N \) is the smallest integer such that the Alternating Series Estimate implies that \( |S - S_N| < 0.1 \)

A. 5
B. 6
C. 7
D. 8
E. 9

Need \( \frac{1}{\sqrt{2(n+1)^2 + 3}} < 0.1 \Rightarrow \frac{1}{10} \)

\( 10 \leq \sqrt{2(n+1)^2 + 3} \)

\( 100 \leq 2(n+1)^2 + 3 \)

\( 97 \leq 2(n+1)^2 \)

\( \frac{97}{2} = 48.5 \leq (n+1)^2 \)

\( n = 5 ? \quad (5+1)^2 = 36 \leq 48.5 \quad \times \)

\( n = 6 ? \quad (6+1)^2 = 49 > 48.5 \quad \checkmark \)

\( n = 6 \)
5. Which statements are true about \( \sum_{n=2}^{\infty} \frac{1}{\sqrt{2n^2 + 1}} \)

\( \times \) Using \( \sum b_n = \sum \frac{1}{n} \), the Limit Comparison Test implies the series converges.

\( \circ \) Using \( \sum b_n = \sum \frac{1}{n} \), the Limit Comparison Test implies the series diverges.

\( \times \) Using \( \sum b_n = \sum \frac{1}{n} \), the Comparison Test implies that the series converges.

\( \times \) Using \( \sum b_n = \sum \frac{1}{n} \), the Comparison Test implies that the series diverges.

\( I \) False. Comparing with \( \sum \frac{1}{n} \) will never imply convergence since it diverges.

A. I, III
B. III only
C. II, IV
D. IV only
E. II only

6. Which statements are true?

\( \checkmark \) If \( a_n > 0 \) and \( \lim_{n \to \infty} \frac{a_{n+1}}{a_n} > 1 \), then \( \sum a_n \) diverges. Ratio Test

\( \times \) If \( a_n > 0 \) and \( \lim_{n \to \infty} \frac{a_{n+1}}{a_n} = 1 \), then \( \sum a_n \) converges. Inconclusive

\( \times \) If \( a_n > 0 \) and \( \lim_{n \to \infty} a_n = 0 \), then \( \sum a_n \) converges. Not True!

A. I only
B. I, II
C. II only
D. III only
E. None of them.
7. Which of the following series converges conditionally?

A. \[ \sum_{n=1}^{\infty} \frac{(-1)^n}{n^{3/2}} \] Abs. convergent by AST and p-test

B. \[ \sum_{n=1}^{\infty} (-1)^n n^2 \] Divergent by TFD

C. \[ \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2 + n} \] Abs. convergent by AST and \( \lim_{n \to \infty} \frac{n^2}{n^2 + n} = 1 \)

D. \[ \sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n} \] convergent by AST, but \( \sum_{n=2}^{\infty} \frac{1}{n \ln n} > \sum_{n=2}^{\infty} \frac{1}{n^2} \) so diverges by DC

E. \[ \sum_{n=1}^{\infty} (-1)^n e^n = \sum_{n=1}^{\infty} (-e)^n \] Diverges by GST

8. Let \( f(x) = x^2 \sin 3x \). Find \( f^{(5)}(0) \).

A. \[ \frac{243}{120} \]

\[ f(x) = x^2 \sin 3x = x^2 \left(3x - \frac{(3x)^3}{3!} + \frac{(3x)^5}{5!} - \ldots \right) \]

B. \[ \frac{27}{6} \]

\[ \left. \frac{f(x)}{x} \right|_{x=0} = 3x^3 - \frac{27x^5}{3!} + \frac{243x^7}{5!} - \ldots \]

C. \[ \frac{-27}{6} \]

\[ f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n = f(0) + \ldots + \frac{f^{(5)}(0)}{5!} x^5 + \ldots \]

\[ \frac{f^{(5)}(0)}{5!} = -\frac{27}{6} = -\frac{9}{2} \]

\[ f^{(5)}(0) = 5! \cdot \left(-\frac{9}{2} \right) = -120 \cdot \frac{9}{2} = -540 \]

\[ \boxed{\frac{f^{(5)}(0)}{5!} = -\frac{9}{2}} \]
9. Compute the Taylor Series of

\[ f(x) = x^4 + 3x^2 - x + 1 \]

about \( a = 1 \).

A. \( 4 + 9(x - 1) + 18(x - 1)^2 + 24(x - 1)^3 + (x - 1)^4 \)
B. \( 4 + 9(x - 1) + 9(x - 1)^2 + 24(x - 1)^3 + 24(x - 1)^4 \)
C. \( 4 + (x - 1)^2 + 3(x - 1)^4 \)
D. \( 4 + 9(x - 1) + 9(x - 1)^2 + 4(x - 1)^3 + (x - 1)^4 \)
E. \( 4 + (x - 1)^2 + 6(x - 1)^4 \)

\[
\begin{array}{c|c|c}
 n & f^{(n)}(x) & f^{(n)}(1) \\
 \hline
 0 & x^4 + 3x^2 - x + 1 & 4 \\
 1 & 4x^3 + 6x - 1 & 9 \\
 2 & 12x^2 + 6 & 18 \\
 3 & 24x & 24 \\
 4 & 24 & 24 \\
 5 & 0 & 0 \\
\end{array}
\]

\[
f(x) = 4 + \frac{9}{1!} (x-1)^1 + \frac{18}{2!} (x-1)^2 + \frac{24}{3!} (x-1)^3 + \frac{24}{4!} (x-1)^4
\]

\[
= 4 + 9(x-1) + 9(x-1)^2 + 4(x-1)^3 + (x-1)^4
\]

10. The set of values \( x \) for which the power series \( \sum_{n=0}^{\infty} \frac{n^2}{2} x^n \) converges is

A. \( x = 0 \) only
B. \( -\frac{1}{2} < x < \frac{1}{2} \)
C. \( -1 < x < 1 \)
D. \( -2 < x < 2 \)
E. \( -1 \leq x < 1 \)

\[
\text{Ratio Test}
\]

\[
lim_{n \to \infty} \left| \frac{(n+1)^2 x^{n+1}}{n^2 x^n} \right| = \frac{1}{|x|} \lim_{n \to \infty} \frac{(n+1)^2}{n^2} = \frac{1}{|x|} \left[ \frac{1}{1} \right] = \frac{1}{|x|} < 1
\]

\(-1 < x < 1 \) Need to check endpoints!

\[
\frac{x = -1}{\sum_{n=0}^{\infty} \frac{n^2 (-1)^n}{2}} \text{ Diverges by TFD or AST}
\]

\[
\frac{x = 1}{\sum_{n=0}^{\infty} \frac{n^2}{2}} \text{ Diverges by TFD or } p\text{-test}
\]

\[
\sum_{n=1}^{\infty} n^2 = \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6} - 2 < 1
\]
11. If \((1 + x)e^x\) is written as a Maclaurin series,

\[(1 + x)e^x = c_0 + c_1x + c_2x^2 + c_3x^3 + \cdots\]

then \(c_3\) equals

A. \(\frac{1}{2}\)

B. \(\frac{1}{6}\)

C. \(\frac{1}{3}\)

D. \(\frac{5}{6}\)

E. \(\frac{2}{3}\)

\[c_3 = \frac{1}{3!} + \frac{1}{2!} = \frac{1}{6} + \frac{1}{2} = \frac{2}{3}\]

12. The first 3 non-zero terms of the Taylor series of an antiderivative of \(\cos \sqrt{x}\) are

A. \(x - \frac{x^2}{4} + \frac{x^3}{72}\)

B. \(1 - \frac{x^2}{2} + \frac{x^4}{24}\)

C. \(x - \frac{x^3}{6} + \frac{x^5}{120}\)

D. \(1 - \frac{x^2}{2} + \frac{x^4}{24}\)

E. \(1 - \frac{x^3}{2} + \frac{x}{24}\)

\[\cos \sqrt{x} = 1 - \frac{x^2}{2} + \frac{x^4}{4!} - \frac{x^6}{6!} \ldots \]

\[\Rightarrow \cos \sqrt{x} = 1 - \frac{(\sqrt{x})^2}{2} + \frac{(\sqrt{x})^4}{4!} - \frac{(\sqrt{x})^6}{6!} \ldots = 1 - \frac{x}{2} + \frac{x^2}{24} - \frac{x^3}{720} \ldots \]

\[\int \cos \sqrt{x} \, dx\]

\[= \int \left(1 - \frac{x^2}{24} + \frac{x^3}{720} \ldots \right) \, dx\]

\[= x - \frac{x^2}{4} + \frac{x^3}{24} - \frac{x^4}{2880} \ldots + C\]

First 3