1. You must use a #2 pencil on the mark-sense sheet (answer sheet).

2. If the cover of your question booklet is GREEN, write 01 in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below. If the cover is ORANGE, write 02 in the TEST/QUIZ NUMBER boxes and darken the spaces below.

3. On the mark-sense sheet, fill in your TA's name and the course number.

4. Fill in your NAME and STUDENT IDENTIFICATION NUMBER and blacken in the appropriate spaces.

5. Fill in your four-digit SECTION NUMBER. If you do not know your section number, please ask your TA.


7. Fill in your name and your instructor's name on the question sheets above.

8. There are 12 questions, each worth 8 points (you will automatically earn 4 points for taking the exam). Blacken in your choice of the correct answer in the spaces provided for questions 1–12. Do all your work on the question sheets.

9. Turn in both the mark-sense sheets and the question sheets when you are finished.

10. If you finish the exam before 7:20, you may leave the room after turning in the scantron sheet and the exam booklet. If you don't finish before 7:20, you MUST REMAIN SEATED until your TA comes and collects your scantron sheet and your exam booklet.

11. NO CALCULATORS, PHONES, BOOKS, OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.
EXAM POLICIES

1. Students may not open the exam until instructed to do so.

2. Students must obey the orders and requests by all proctors, TAs, and lecturers.

3. No student may leave in the first 20 min or in the last 10 min of the exam.

4. Books, notes, calculators, or any electronic devices are not allowed on the exam, and they should not even be in sight in the exam room. Students may not look at anybody else's test, and may not communicate with anybody else except, if they have a question, with their TA or lecturer.

5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.

6. Any violation of these rules and any act of academic dishonesty may result in severe penalties. Additionally, all violators will be reported to the Office of the Dean of Students.

I have read and understand the exam rules stated above:

STUDENT NAME: ____________________________

STUDENT SIGNATURE: _________________________
1. Find the centroid of the region bounded by \( y = x^2, \ y = 1, \ -1 \leq x \leq 1 \).

A. \( \left( 0, \frac{3}{5} \right) \)

B. \( \left( 0, \frac{4}{5} \right) \)

C. \( \left( \frac{3}{5}, 0 \right) \)

D. \( \left( \frac{3}{5}, \frac{4}{5} \right) \)

E. \( \left( \frac{4}{5}, 0 \right) \)

2. Consider the sequence \( \left\{ \frac{1 - n}{1 + 2n} \right\}_{n=1}^{\infty} \). Which of the following statements are true?

I. The sequence is increasing. \( \times \)

II. The sequence is bounded. \( \checkmark \)

III. The sequence is convergent. \( \checkmark \)

A. I and II only

B. I and III only

C. II and III only

D. I, II, and III

E. None are true.

\[
\lim_{n \to \infty} \frac{1 - n}{1 + 2n} = -\frac{1}{2} \Rightarrow \text{convergent} \checkmark
\]

\[
\text{convergent so bounded} \checkmark
\]

\[
\frac{\frac{d}{dx} \left[ \frac{1-x}{1+2x} \right]}{1+2x} = \frac{(1+2x)(-1) - (1-x)(2)}{(1+2x)^2} = \frac{-1-2x-2+2x}{(1+2x)^2} = \frac{-3}{(1+2x)^2} < 0
\]

\[
\Rightarrow \text{decreasing}
\]

Was covered last exam, but you should still know how to do this problem.
3. Determine whether the series converges or diverges. If it is convergent, find its sum.

\[-4 + 3 - \frac{9}{4} + \frac{27}{16} - \frac{81}{64} + \cdots\]

A. converges to \(-\frac{3}{4}\)
B. converges to 0
C. converges to \(-\frac{16}{7}\)
D. converges to 4
E. diverges

**Geometric series!**

with \(r = -\frac{3}{4}\)

\[1 - \left(-\frac{3}{4}\right) = \frac{3}{4} < 1 \text{ so it converges}\]

\[-4 + 3 - \frac{9}{4} + \cdots = \frac{a}{1-r} = \frac{-4}{1 - (-\frac{3}{4})} = \frac{-4}{\frac{7}{4}} = \frac{-16}{7}\]

4. For which values of \(p\) does the series \(\sum_{n=2}^{\infty} \frac{1}{n(ln n)^p}\) converge? Hint: consider using the Integral Test.

<table>
<thead>
<tr>
<th>Integral Test</th>
<th>(\int_{\ln 2}^{\infty} \frac{du}{u^p})</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. (p &lt; 1)</td>
<td>(\lim_{b \to \infty} \frac{b^{1-p}}{1-p} \ln b)</td>
</tr>
<tr>
<td>B. (p &lt; 2)</td>
<td>(\lim_{b \to \infty} \frac{b^{1-p}}{1-p} \ln b)</td>
</tr>
<tr>
<td>C. (p \neq 0)</td>
<td>(\lim_{b \to \infty} \frac{b^{1-p}}{1-p} \ln b)</td>
</tr>
<tr>
<td>D. (p &gt; 1)</td>
<td>(\lim_{b \to \infty} \frac{b^{1-p}}{1-p} \ln b)</td>
</tr>
<tr>
<td>E. (p &gt; 2)</td>
<td>(\lim_{b \to \infty} \frac{b^{1-p}}{1-p} \ln b)</td>
</tr>
</tbody>
</table>

\[\begin{array}{c}
\text{\(p > 1\)} \\
\lim_{b \to \infty} \frac{b^{1-p}}{1-p} \ln b
\end{array}\]

\[\begin{array}{c}
= (\frac{(\ln b)^{1-p}}{1-p})
\end{array}\]

\[\begin{array}{c}
= (\ln b)^{1-p}
\end{array}\]

\[\begin{array}{c}
= +\infty
\end{array}\]
5. Which comparison could be used to determine whether the series \( \sum_{n=1}^{\infty} \frac{\sqrt{n^4 + 1}}{n^3 + n^2} \) converges or diverges?

A. converges by comparison with \( \sum_{n=1}^{\infty} \frac{1}{n^2} \)

B. converges by limit comparison with \( \sum_{n=1}^{\infty} \frac{1}{n^2} \)

C. diverges by comparison with \( \sum_{n=1}^{\infty} \frac{1}{n} \)

D. diverges by comparison with \( \sum_{n=1}^{\infty} \frac{1}{n} \)

E. diverges by limit comparison with \( \sum_{n=1}^{\infty} \frac{1}{n} \)

\[
\lim_{n \to \infty} \frac{\frac{\sqrt{n^4 + 1}}{n^3 + n^2}}{\frac{1}{n}} = \lim_{n \to \infty} \frac{n \cdot \sqrt{n^4 + 1}}{n^3 + n^2} = \lim_{n \to \infty} \frac{\sqrt{1 + \frac{1}{n^4}}}{2 + \frac{1}{n}} = 0 < 1 \Rightarrow \text{Behave the same.}
\]

6. What is the minimum number of terms needed to estimate the sum of the series \( \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^5} \) with \( |\text{error}| \leq 4^{-5} \)?

A. 2
B. 3
C. 4
D. 5
E. 6

\[
\begin{array}{c|c}
 n & b_n \\
 1 & 1 \\
 2 & \frac{1}{2^5} \\
 3 & \frac{1}{3^5} \\
 4 & \times \quad \frac{1}{4^5} = 4^{-5} \quad \text{but is not} \leq 4^{-5} \\
 5 & \checkmark \quad \frac{1}{5^5} < 4^{-5} \\
\end{array}
\]

so 4 terms as \( n=4 \) is the previous term.
7. Which of the following statements is/are true?

\[ \sqrt{\text{I. } \sum_{n=1}^{\infty} \frac{n}{5^n} \text{ is absolutely convergent} \quad \lim_{n \to \infty} \left| \frac{n+1}{5^{n+1}} \cdot \frac{5^n}{n} \right| = \lim_{n \to \infty} \frac{1}{5} \left| \frac{n+1}{n} \right| = \frac{1}{5} \cdot 1 \leq 1 \quad \checkmark} \]

\[ \times \text{II. } \sum_{n=3}^{\infty} \frac{(-7)^{n-1}}{\sqrt{n}} \text{ is conditionally convergent} \quad = \frac{1}{5} < 1 \quad \checkmark \]

\[ \sqrt{\text{III. the Ratio Test is inconclusive with } \sum_{n=3}^{\infty} \frac{6\sqrt{n}}{1+n^2}} \]

A. I only
B. II only
C. III only
D. I and III only
E. I, II, and III

\[ \text{III} \quad \lim_{n \to \infty} \left| \frac{6\sqrt{n+1}}{1+(n+1)^2} \cdot \frac{1+n^2}{6\sqrt{n}} \right| = \lim_{n \to \infty} \left| \left( \frac{\sqrt{n+1}}{\sqrt{n}} \right) \left( \frac{1+n^2}{2+2n+n^2} \right) \right| = 1 \cdot 1 = 1 \]

8. Find the interval of convergence of \( \sum_{n=1}^{\infty} \frac{x^n}{n8^n} \)

A. \((-8, 8]\)
B. \((-1, 1)\)
C. \([-1, 1]\)
D. \([-8, 8]\)
E. \([-8, 8)\)

\[ \lim_{n \to \infty} \left| \frac{x^{n+1}}{(n+1)8^{n+1}} \cdot \frac{8^n}{x^n} \right| = \lim_{n \to \infty} \left| \frac{x}{8} \cdot \left( \frac{n}{n+1} \right) \right| = \left| \frac{x}{8} \right| < 1 \quad \Rightarrow \quad |x| < 8 \]

\[ \text{Check endpoints} \]

\[ x = 8 \quad \sum_{n=1}^{\infty} \frac{x^n}{n8^n} = \sum_{n=1}^{\infty} \frac{1}{n} \text{ diverges} \quad \checkmark \]

\[ x = -8 \quad \sum_{n=1}^{\infty} \frac{(-8)^n}{n8^n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \quad \text{Alt. harmonic series} \quad \checkmark \]

\[ \text{converges (conditionally) by AST.} \]
9. In the power series representation of \( \frac{x}{2x^2 + 1} \), what is the term that contains \( x^9 \)?

A. \( 16x^9 \)
B. \( 32x^9 \)
C. \( 0x^9 \)
D. \( -32x^9 \)
E. \( -16x^9 \)

\[
\frac{x}{1 + 2x^2} = x \left( \frac{1}{1 - (-2x^2)} \right) = x \sum_{n=0}^{\infty} (-2x^2)^n = x \sum_{n=0}^{\infty} (-1)^n 2^n x^{2n} = \sum_{n=0}^{\infty} (-1)^n 2^n x^{2n+1}
\]

\[n+1 = 9 \implies n = 4, \text{ term}\]

10. In the Taylor series for \( x^{-1} \) centered at \( a = 2 \), what is the term that contains \( (x-2)^3 \)?

A. \( -\frac{3}{8}(x-2)^3 \)
B. \( -\frac{1}{16}(x-2)^3 \)
C. \( -\frac{3}{16}(x-2)^3 \)
D. \( -\frac{5}{8}(x-2)^3 \)
E. \( -\frac{11}{8}(x-2)^3 \)

\[
\frac{f^{(3)}(2)}{3!} (x-2)^3
\]

\[
-\frac{6}{2^4} = -\frac{6}{16} = \frac{f^{(3)}(2)}{3!}
\]

\[
\left( -\frac{6}{16} \right) \cdot \frac{1}{3!} (x-2)^3 = \left( -\frac{6}{16} \right) \left( \frac{1}{3!} \right) (x-2)^3
\]

\[
= -\frac{1}{16} (x-2)^3
\]
11. In the power series representation of \( \int \frac{e^x - 1}{x} \, dx \), what is the coefficient of the term containing \( x^3 \)? Note that \( e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \cdots \)

\[
\phi^n \frac{x^n}{n!} = \sum_{n=1}^{\infty} \frac{x^n}{n!} \\
\Rightarrow \int \frac{e^x - 1}{x} \, dx = \int \left( \sum_{n=1}^{\infty} \frac{x^n}{n!} \right) \, dx \\
\text{coeff of } x^3 = \int \left( \sum_{n=1}^{\infty} \frac{x^{n-1}}{n!} \right) \, dx \\
= \sum_{n=1}^{\infty} \frac{x^n}{n!} + C
\]

\[
\text{coeff of } x^3 = \frac{1}{18}
\]

A. \( 1/18 \)  
B. \( 1/9 \)  
C. \( 1/6 \)  
D. \( 1/4 \)  
E. \( 1/3 \)

12. An unknown function \( f(x) \) is expressed as a Maclaurin series as \( \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} \). What is the value of \( f^{(3)}(0) \) (that is, the third derivative of \( f(x) \) at \( x = 0 \))?

A. \( -1 \)  
B. \( -2 \)  
C. \( 1 \)  
D. \( 2 \)  
E. \( 6 \)

\[
f(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} \\
f^{(3)}(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1} \\
= f(0) + \frac{f'(0)}{1!} x + \frac{f''(0)}{2!} x^2 + \frac{f^{(3)}(0)}{3!} x^3 + \cdots
\]

\[
\frac{f^{(3)}(0)}{3!} = -\frac{1}{3} \\
\Rightarrow f^{(3)}(0) = -\frac{1}{3} \cdot 3! = \frac{6}{-3} = -2
\]