1. You must use a #2 pencil on the mark-sense sheet (answer sheet).

2. If the cover of your question booklet is GREEN, write 01 in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below. If the cover is ORANGE, write 02 in the TEST/QUIZ NUMBER boxes and darken the spaces below.

3. On the mark-sense sheet, fill in your TA’s name and the course number.

4. Fill in your NAME and STUDENT IDENTIFICATION NUMBER and blacken in the appropriate spaces.

5. Fill in your four-digit SECTION NUMBER. If you do not know your section number, please ask your TA.


7. Fill in your name and your instructor’s name on the question sheets above.

8. There are 12 questions, each worth 8 points (you will automatically earn 4 points for taking the exam). Blacken in your choice of the correct answer in the spaces provided for questions 1–12. Do all your work on the question sheets.

9. Turn in both the mark-sense sheets and the question sheets when you are finished.

10. If you finish the exam before 7:20, you may leave the room after turning in the scantron sheet and the exam booklet. If you don’t finish before 7:20, you MUST REMAIN SEATED until your TA comes and collects your scantron sheet and your exam booklet.

11. NO CALCULATORS, PHONES, BOOKS, OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.
EXAM POLICIES

1. Students may not open the exam until instructed to do so.
2. Students must obey the orders and requests by all proctors, TAs, and lecturers.
3. No student may leave in the first 20 min or in the last 10 min of the exam.
4. Books, notes, calculators, or any electronic devices are not allowed on the exam, and they should not even be in sight in the exam room. Students may not look at anybody else’s test, and may not communicate with anybody else except, if they have a question, with their TA or lecturer.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.
6. Any violation of these rules and any act of academic dishonesty may result in severe penalties. Additionally, all violators will be reported to the Office of the Dean of Students.

I have read and understand the exam rules stated above:

STUDENT NAME:  

STUDENT SIGNATURE:  

007
MA 16200 – Spring 2015

Exam 1

GREEN Test – Version 01

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>B</td>
</tr>
<tr>
<td>2</td>
<td>D</td>
</tr>
<tr>
<td>3</td>
<td>D</td>
</tr>
<tr>
<td>4</td>
<td>B</td>
</tr>
<tr>
<td>5</td>
<td>D</td>
</tr>
<tr>
<td>6</td>
<td>C</td>
</tr>
<tr>
<td>7</td>
<td>C</td>
</tr>
<tr>
<td>8</td>
<td>B</td>
</tr>
<tr>
<td>9</td>
<td>E</td>
</tr>
<tr>
<td>10</td>
<td>E</td>
</tr>
<tr>
<td>11</td>
<td>A</td>
</tr>
<tr>
<td>12</td>
<td>A</td>
</tr>
</tbody>
</table>
1. What is the center of the sphere $2x^2 + 2y^2 + 2z^2 - 12y + 16z + 2 = 0$?

A. $(0, -3, 4)$
B. $(0, 3, -4)$
C. $(0, -6, 8)$
D. $(0, 6, -8)$
E. $(0, 0, 0)$

First divide by 2

\[
\begin{align*}
  x^2 + y^2 + z^2 - 6y + 8z &= -1 \\
  x^2 + (y^2 - 6y + 9) + (z^2 + 8z + 16) &= -1 + 9 + 16 \\
  (x^2 - 9)^2 &= 16 \\
  x^2 + (y-3)^2 + (z+4)^2 &= 24
\end{align*}
\]

Center: $(0, 3, -4)$

2. Find the vector that is 6 units long in the opposite direction to $\langle -2, 1, -2 \rangle$

A. $\langle -12, 6, -12 \rangle$
B. $\langle 12, -6, 12 \rangle$
C. $\langle -4, 2, -4 \rangle$
D. $\langle 4, -2, 4 \rangle$
E. $\langle -4, -2, -4 \rangle$

\[
\begin{align*}
  |\langle -2, 1, -2 \rangle| &= \sqrt{(-2)^2 + 1^2 + (-2)^2} = \sqrt{9} = 3 \\
  \text{Unit vector} &= \frac{1}{3} \langle -2, 1, -2 \rangle \\
  &= \langle -\frac{2}{3}, \frac{1}{3}, -\frac{2}{3} \rangle
\end{align*}
\]

\[
\begin{align*}
  (-1)(6) \langle -\frac{2}{3}, \frac{1}{3}, -\frac{2}{3} \rangle &= \langle 4, -2, 4 \rangle
\end{align*}
\]

opposite direction \hspace{1cm} length 6
3. What is the angle between the vectors $\vec{u} = -3\vec{i}$ and $\vec{v} = \vec{i} + \sqrt{2}\vec{j} - \vec{k}$?

A. 0 
B. $\pi/6$
C. $\pi/3$
D. $2\pi/3$
E. $5\pi/6$

\[ \vec{u} \cdot \vec{v} = |\vec{u}| |\vec{v}| \cos \theta \]
\[ |\vec{u}| = \sqrt{(-3)^2} = 3 \]
\[ |\vec{v}| = \sqrt{1^2 + (\sqrt{2})^2 + (-1)^2} = \sqrt{4} = 2 \]
\[ \vec{u} \cdot \vec{v} = \vec{u} \cdot \vec{v} = -(3)(1) + 0\sqrt{2} + 0(-1) = -3 \]

\[-3 = 3 \cdot 2 \cos \theta \]
\[ \Rightarrow \cos \theta = -\frac{1}{2} \]
\[ \theta = \frac{2\pi}{3} \]

4. Which of the following vectors is perpendicular to $\vec{a} = \langle 3, 5, 1 \rangle$ and to $\vec{b} = \langle -5, 2, -2 \rangle$?

A. $\langle -10, -5, 6 \rangle$
B. $\langle -12, 1, 31 \rangle$
C. $\langle 2, -6, -25 \rangle$
D. $\langle -8, -11, -19 \rangle$
E. $\langle -16, -23, 25 \rangle$

\[ \vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 5 & 1 \\ -5 & 2 & -2 \end{vmatrix} \]
\[ = \begin{vmatrix} 5 & 1 \\ -2 & -2 \end{vmatrix} \vec{i} - \begin{vmatrix} 3 & 1 \\ -5 & -2 \end{vmatrix} \vec{j} + \begin{vmatrix} 3 & 5 \\ -5 & 2 \end{vmatrix} \vec{k} \]
\[ = (-10-2)\vec{i} - (6+5)\vec{j} + (6+25)\vec{k} \]
\[ = -12\vec{i} + 31\vec{k} \]
\[ = \langle -12, 0, 31 \rangle \]
5. A spring has a natural length of 5 m. If a 25-N force is required to keep it stretched to a length of 10 m, how much work (in joules) is required to stretch it from 5 m to 6 m?

A. \( \frac{55}{2} \)
B. \( \frac{55}{4} \)
C. \( \frac{5}{4} \)
D. \( \frac{5}{2} \)
E. 5

\[
\begin{align*}
F &= kx \\
F &= k \cdot 5 = 25 N \\
\Rightarrow \quad k &= 5
\end{align*}
\]

\[
W = \int_5^6 5x \, dx
\]
\[
= \left. \frac{5}{2} x^2 \right|_5^6 = \boxed{\frac{5}{2}}
\]

6. Find the area of the region bounded by \( y = x + 1 \) and \( y = (x - 1)^2 \).

A. \( \frac{11}{6} \)
B. \( \frac{7}{2} \)
C. \( \frac{13}{2} \)
D. \( \frac{13}{2} \)
E. \( \frac{5}{2} \)

\[
\begin{align*}
(x-1)^2 &= x^2 - 2x + 1 = x+1 \\
\Rightarrow \quad x^2 - 3x &= 0 \\
x(x-3) &= 0 \\
x &= 0, \quad x = 3
\end{align*}
\]

\[
\int_0^3 ((x+1) - (x^2 - 2x + 1)) \, dx
\]
\[
= \int_0^3 (-x^2 + 3x) \, dx
\]
\[
= \left. \left(-\frac{1}{3} x^3 + \frac{3}{2} x^2\right)\right|_0^3
\]
\[
= \left(-\frac{27}{3} + \frac{27}{2}\right) - \left(0 + 0\right)
\]
\[
= \frac{3(27)}{6} - \frac{2(27)}{6} = \frac{27}{6} = \boxed{\frac{9}{2}}
\]
7. The region bounded by \( y = e^x \), \( y = 1 \), and \( x = 2 \) is revolved about the \( x \)-axis. If the washer method is used, the volume of the solid generated is given by the integral

A. \( \pi \int_0^2 e^{2x} \, dx \)

B. \( 2\pi \int_1^2 (2 - \ln y)(y - 1) \, dy \)

C. \( \pi \int_0^2 (e^{2x} - 1) \, dx \)

D. \( 2\pi \int_0^2 y(2 - \ln y) \, dy \)

E. \( \pi \int_0^2 (e^x - 1)^2 \, dx \)

8. Find the volume generated by rotating the region bounded by the curves

\( y = x^2, \quad y = 0, \quad x = 1 \)

about the line \( x = -1 \).

A. \( \frac{5\pi}{6} \)

B. \( \frac{7\pi}{6} \)

C. \( \frac{11\pi}{6} \)

D. \( \frac{13\pi}{6} \)

E. \( \frac{17\pi}{6} \)
9. Which of the following is a possible value of the number \(a\) such that the average value of the function \(f(x) = 51 - 26x + 3x^2\) in the interval \([0, a]\) is equal to 9?

A. \(-6\)  
B. \(3\)  
C. \(-3\)  
D. \(-7\)  
E. \(6\)

\[
q = \frac{1}{a-0} \int_0^a (3x^2 - 26x + 51) \, dx \\
= \frac{1}{a} \left[ x^3 - 13x^2 + 51x \right]_0^a \\
= \frac{1}{a} \left( a^3 - 13a^2 + 51a - 0 \right) \\
= a^2 - 13a + 51 \\
\Rightarrow a^2 - 13a + 42 = 0 \\
(a-6)(a-7) = 0 \\
\boxed{a=6, \ a=7}
\]

10. Evaluate

\[
\int_0^1 2x \sin(\pi x) \, dx
\]

A. \(\frac{\pi}{2}\)  
B. \(\frac{\pi}{4}\)  
C. \(\frac{4}{\pi}\)  
D. \(\frac{\pi - 2}{\pi^2}\)  
E. \(\frac{2}{\pi}\)

\[
2 \int_0^1 x \sin(\pi x) \, dx \\
\left. \begin{array}{l}
\quad u = x \\
\quad dv = \sin(\pi x) \, dx \\
\quad du = dx \\
\quad v = -\frac{1}{\pi} \cos(\pi x)
\end{array} \right|_0^1 \\
\Rightarrow 2 \left[ -\frac{x}{\pi} \cos(\pi x) - \int -\frac{1}{\pi} \cos(\pi x) \, dx \right]_0^1 \\
= 2 \left[ -\frac{x}{\pi} \cos(\pi x) + \frac{1}{\pi^2} \sin(\pi x) \right]_0^1 \\
= 2 \left[ \left( \frac{1}{\pi} (1) + 0 \right) - \left( 0 + 0 \right) \right] \\
= \boxed{\frac{2}{\pi}}
\]
11. Find the integral
\[ \int_0^1 x^2 e^x \, dx \]
\[ u = x^2 \quad dv = e^x \, dx \]
\[ du = 2x \quad v = e^x \]
\[ x^2 e^x - \int x e^x \, dx \]
\[ \int_0^1 x^2 e^x - 2\int x e^x \, dx \]
\[ u_1 = x \quad dv_1 = e^x \, dx \]
\[ du_1 = dx \quad v_1 = e^x \]
\[ x^2 e^x - 2(\int e^x \, dx) \]
\[ = (e^x - 2e) \bigg|_0^1 \]
\[ = e - 2 \]

12. Find the integral
\[ \int_0^{\pi/2} \sin^2 x \cos^3 x \, dx \]
\[ A. \frac{2}{15} \]
\[ B. \frac{4}{15} \]
\[ C. \frac{7}{15} \]
\[ D. \frac{8}{15} \]
\[ E. \frac{11}{15} \]