NAME ______ your name ______ YOUR TA’S NAME ______ my name ______

STUDENT ID # ______ 002##-### ______ RECITATION TIME _______ 7:30 or 8:30 _______

1. You must use a #2 pencil on the mark-sense sheet (answer sheet).

2. If the cover of your question booklet is GREEN, write 01 in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below. If the cover is ORANGE, write 02 in the TEST/QUIZ NUMBER boxes and darken the spaces below.

3. On the mark-sense sheet, fill in your TA’s name and the course number.

4. Fill in your NAME and STUDENT IDENTIFICATION NUMBER and blacken in the appropriate spaces.

5. Fill in your four-digit SECTION NUMBER. If you do not know your section number, please ask your TA.


7. Fill in your name and your instructor’s name on the question sheets above.

8. There are 12 questions, each worth 8 points (you will automatically earn 4 points for taking the exam). Blacken in your choice of the correct answer in the spaces provided for questions 1–12. Do all your work on the question sheets.

9. Turn in both the mark-sense sheets and the question sheets when you are finished.

10. If you finish the exam before 7:20, you may leave the room after turning in the scantron sheet and the exam booklet. If you don’t finish before 7:20, you MUST REMAIN SEATED until your TA comes and collects your scantron sheet and your exam booklet.

11. NO CALCULATORS, PHONES, BOOKS, OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.
EXAM POLICIES

1. Students may not open the exam until instructed to do so.
2. Students must obey the orders and requests by all proctors, TAs, and lecturers.
3. No student may leave in the first 20 min or in the last 10 min of the exam.
4. Books, notes, calculators, or any electronic devices are not allowed on the exam, and they should not even be in sight in the exam room. Students may not look at anybody else’s test, and may not communicate with anybody else except, if they have a question, with their TA or lecturer.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.
6. Any violation of these rules and any act of academic dishonesty may result in severe penalties. Additionally, all violators will be reported to the Office of the Dean of Students.

I have read and understand the exam rules stated above:

STUDENT NAME: ____________
your name

STUDENT SIGNATURE: ________
sign here
1. Given the two series \( A = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \ldots \) and \( B = \sum_{n=1}^{\infty} n^5 e^{-n^6} \), use the Integral Test to determine whether each series is convergent and choose the correct statement from below:

A. Both series are divergent
B. Series A is convergent, series B is divergent
C. Series A is divergent, series B is convergent
D. Both series are convergent
E. The Integral Test is inconclusive

\[
A = \sum_{n=1}^{\infty} \frac{1}{n^2}
\]

\[
\int_1^{\infty} \frac{1}{x^2} \, dx = \lim_{b \to \infty} -\frac{1}{x} \bigg|_1^b
\]

\[
= \lim_{b \to \infty} -\frac{1}{b} - \left(-\frac{1}{1}\right) = 1 \sqrt{\text{A converges}}
\]

\[
B = \sum_{n=1}^{\infty} n^5 e^{-n^6}
\]

\[
\int_1^{\infty} x^5 e^{-x^6} \, dx = \lim_{b \to \infty} -\frac{1}{6} e^{-x^6} \bigg|_1^b
\]

\[
= \lim_{b \to \infty} -\frac{1}{6} e^{-b^6} - \left(-\frac{1}{6} e^{-1}ight) = \frac{1}{6} e \sqrt{\text{B converges}}
\]

2. Given the following table of values for \( e^{-n} \), which partial sum of the series \( s = \sum_{n=1}^{\infty} (-1)^n e^{-n} \) is the first to be within 0.01 of \( s \)?

<table>
<thead>
<tr>
<th>( n )</th>
<th>( e^{-n} )</th>
<th>( \text{Need } e^{-(n+1)} &lt; 0.01 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.3679</td>
<td>( &gt; 0.01 )</td>
</tr>
<tr>
<td>2</td>
<td>0.1353</td>
<td>( &gt; 0.01 )</td>
</tr>
<tr>
<td>3</td>
<td>0.0498</td>
<td>( \approx 0.01 )</td>
</tr>
<tr>
<td>4</td>
<td>0.0183</td>
<td>( &gt; 0.01 )</td>
</tr>
<tr>
<td>5</td>
<td>0.0067</td>
<td>( &lt; 0.01 )</td>
</tr>
</tbody>
</table>

A. \( s_1 \)
B. \( s_2 \)
C. \( s_3 \)
D. \( s_4 \)
E. \( s_5 \)

Need \( e^{-(n+1)} < 0.01 \)

so \( n = 4 \)
3. Consider the following two alternating series:

\[ \sum_{n=1}^{\infty} (-1)^n \frac{10}{n^3} \]

\[ \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2 + 1} \]

I. \[ \sum_{n=1}^{\infty} (-1)^n \frac{10}{n^3} \]

\[ \frac{10}{n^3} > 0 \checkmark \]

\[ \frac{10}{(n+1)^3} < \frac{10}{n^3} \checkmark \]

II. \[ \sum_{n=1}^{\infty} \frac{(-1)^{n-1} n}{n^2 + 1} \]

\[ \lim_{n \to \infty} \frac{10}{n^3} = 0 \checkmark \]

\[ \text{Converges (either abs or cond.)} \]

\[ \text{Look at } \sum_{n=1}^{\infty} \frac{10}{n^3} \]

A. I and II are absolutely convergent
B. I is conditionally convergent and II is absolutely convergent
C. I and II are conditionally convergent
D. I is absolutely convergent and II is divergent
E. I is absolutely convergent and II is conditionally convergent

\[ \sum_{n=1}^{\infty} \frac{(-1)^n}{n^3} \]

\[ \sum_{n=1}^{\infty} \frac{1}{n^3} \]

\[ \text{Converges p-test.} \]

\[ \text{Since } \sum_{n=1}^{\infty} \frac{1}{n^3} \text{ converges, } \sum_{n=1}^{\infty} (-1)^n \frac{10}{n^3} \text{ also converges absolutely.} \]

4. Which of the following series converge absolutely?

\[ \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{2n+1}} \]

\[ \sum_{n=1}^{\infty} \frac{(-1)^n (2n+1)}{3n} \]

\[ \sum_{n=1}^{\infty} \frac{\sin n}{n^2 + 2} \]

A. Only III.
B. I and III.
C. II and III.
D. Only I.
E. All of I, II and III.

\[ \sum_{n=1}^{\infty} \frac{\sin n}{n^2 + 2} \]

\[ \sum_{n=1}^{\infty} \frac{1}{n^n} < \sum_{n=1}^{\infty} \frac{1}{n^2} \]

\[ \text{conv. by p-test} \]

III. abs conv. \checkmark
5. Given the series 

\[ \sqrt{I. \sum_{n=2}^{\infty} \frac{2n+1}{3^n}} \]

\[ \times \sum_{n=1}^{\infty} \frac{2^n}{n^2} \]

Then

A. Both I and II converge

B. Only I converges

C. Only II converges

D. Both I and II diverge

E. Both I and II converge absolutely

6. The interval of convergence of the power series 

\[ \sum_{n=1}^{\infty} \frac{(-1)^n(2n+1)(x-1)^{n+1}}{3^n} \]

is

A. \([-2, 4]\)

B. \((-2, 4)\)

C. \((-3, 3)\)

D. \([-2, 4]\)

E. \((-1, 5)\)

(check endpoints)

\[ x = -2 \]

\[ \frac{(-1)^n(2n+1)(-2)^{n+1}}{3^n} \]

\[ x = 4 \]

\[ \frac{(-1)^n(2n+1)(4)^{n+1}}{3^n} \]

Div. by TFD or AST
7. Find a power series representation for \( \frac{x}{2x^2 + 1} \). What is the term that contains \( x^7 \)?

A. \(-8x^7\)  
B. \(-4x^7\)  
C. \(0x^7\)  
D. \(4x^7\)  
E. \(8x^7\)

\[
\frac{X}{1 + 2x^2} = \frac{X}{1 - (-2x^2)} = \sum_{n=0}^{\infty} (-1)^n (2x)^n \quad \text{when} \quad |2x^2| < 1
\]

\[
\sum_{n=0}^{\infty} (-1)^n 2^n x^{2n+1}
\]

\[\text{want } x^7 \text{ so } 2n+1 = 7 \implies n = 3
\]
\[
(-1)^3 2^3 x^7
\]
\[
= \frac{-8x^7}{1}
\]

8. In the Taylor series for \( e^{2x} \) centered at \( a = 3 \), what is the term that contains \( (x - 3)^3 \)?

A. \( \frac{e^6}{3} (x - 3)^3 \)  
B. \( \frac{2e^6}{3} (x - 3)^3 \)  
C. \( \frac{4e^6}{3} (x - 3)^3 \)  
D. \( \frac{5e^6}{3} (x - 3)^3 \)  
E. \( \frac{7e^6}{3} (x - 3)^3 \)

\[
f(x) = e^{2x} = \sum_{n=0}^{\infty} \frac{f^{(n)}(3)}{n!} (x-3)^n
\]

\[
\begin{array}{c|c|c}
 n & f^{(n)}(x) & f^{(n)}(3) \\
\hline
 0 & e^{2x} & 1 \\
 1 & 2e^{2x} & 2e^6 \\
 2 & 4e^{2x} & 4e^6 \\
 3 & 8e^{2x} & 8e^6
\end{array}
\]

\[
\frac{f^{(3)}(3)}{3!} (x-3)^3 = \frac{8e^6}{3} (x-3)^3 = \frac{4e^6}{3} (x-3)^3
\]
9. In the power series representation of $\int \frac{e^x - 1}{x} \, dx$, what is the term that contains $x^3$?

Note that $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$.

\[ \frac{e^x - 1}{x} = \frac{(x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots)}{x} \]

A. $\frac{x^3}{6}$

B. $\frac{x^3}{9}$

C. $\frac{x^3}{12}$

D. $\frac{x^3}{15}$

E. $\frac{x^3}{18}$

\[ \int \frac{e^x - 1}{x} \, dx = \int \frac{\sum_{n=1}^{\infty} \frac{x^{n-1}}{n!}}{x} \, dx = \sum_{n=1}^{\infty} \frac{x^n}{n \cdot n!} + C \]

Want $n = 3$

\[ \frac{x^3}{3 \cdot 3!} = \frac{x^3}{3 \cdot 6} = \frac{x^3}{18} \]

10. The radius of convergence of the binomial series $(1 + x)^k = 1 + kx + \frac{k(k-1)x^2}{2!} + \frac{k(k-1)(k-2)x^3}{3!} + \cdots$ is 1. If we used it to expand $\frac{x}{\sqrt{16 + x^2}}$, what is the radius of convergence of the resulting power series?

\[ \text{for } |x| < 1 \quad (1 + x)^k = 1 + kx + \frac{k(k-1)x^2}{2!} + \cdots \]

A. 8

B. 1

C. 4

D. 5

E. 0.9

\[ \frac{x}{\sqrt{16 + x^2}} = \frac{x}{(16 + x^2)^{\frac{1}{2}}} = \frac{x}{4} \left(1 + \frac{x^2}{16}\right)^{-\frac{1}{2}} = 1 - \frac{1}{2} \left(\frac{x}{16}\right) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2!} \left(\frac{x}{16}\right)^2 + \cdots \]

If $\left|\frac{x^2}{16}\right| < 1$, so $|x^2| < 16$

\[ |x| < 4 \]

\[ R = 4 \]
11. Eliminate the parameter to find a Cartesian equation of the curve given below:

\[ x = e^t - 1 \quad y = e^{2t} \]

A. \( y = \ln(x + 1), x > -1 \)
B. \( y = (x + 1)^2, x > -1 \)
C. \( y = e^{2x+1}, x > -1 \)
D. \( y = (x - 1)^2, x > -1 \)
E. \( y = \frac{\ln x}{2}, x > 0 \)

Not on this Exam!

12. Find the length of the curve

\[ x = 3t^2 \quad y = 2t^3 \quad 0 \leq t \leq 3 \]

A. \( 20\sqrt{10} - 2 \)
B. \( 2\sqrt{10} - 1 \)
C. \( 2\sqrt{10} - 2 \)
D. \( 10\sqrt{2} - 2 \)
E. \( 20\sqrt{2} - 2 \)

Not on this Exam!