Gravity and the figure of the Earth

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Objectives

• What is gravity?
• How and why does it vary (in space and time)?
• What do gravity variations tell us about the Earth?
• How can we measure gravity?
• How is gravity related to geological structures and processes?
Definitions

• Gravimetry = measuring and analyzing the Earth’s gravity field and its space and time variations.

• Closely related to Geodesy = measuring and analyzing the shape and dimensions of the Earth.

• Applications of gravimetry:
  – Internal structure of the Earth (from the surface to the core)
  – Exploration for ore, oil, water
  – Isostasy and mechanical properties of the lithosphere
  – Earth tides
  – Transfer of geophysical fluids between reservoirs: water, magma, ice => temporal variations of gravity
  – Artificial satellites: orbitography

• Planetary gravimetry
Example

Right: map of gravity anomalies (France)

Below: corresponding topographic map

- Positive/negative anomalies
- Correlation with topography
- Interpretation?
Gravitation (1/2)

• Newton’s second law: \( \vec{F} = m\vec{a} \)
  - Force (in Newtons) acting on mass \( m \) (in kg), responsible for its acceleration \( a \) (in m.s\(^{-2}\))

• Newton’s law of gravitation:
  - Two masses \( m \) and \( M \) attract each other
  - This attraction results in a force:
    \[
    \vec{F} = G \frac{mM}{r^2} \hat{r}
    \]
  - Where \( r \) is the distance between the 2 masses and \( G \) the constant of universal gravitation, \( r \) the unit vector in the direction of \( r \)
  - \( G = 6.673 \times 10^{-11} \text{ m}^3\text{kg}^{-1}\text{s}^{-2} \)
Gravitation (2/2)

- $\vec{F} = m\vec{\alpha}$ → force(s) acting on $m$
- $\vec{F} = G \frac{mM}{r^2} \vec{r}$ → force exerted by $M$ on $m$
- In the absence of any other force besides the one generated by $M$, one can write:
  \begin{align*}
  m\vec{\alpha} &= G \frac{mM}{r^2} \vec{r} \\
  \Rightarrow \vec{\alpha} &= G \frac{M}{r^2} \vec{r}
  \end{align*}
- $a = \text{gravitational acceleration}$ of mass $m$ due to the attraction of mass $M$
The Earth’s gravitational acceleration

- $a$ is usually called ‘$g$’
- $g$ should be expressed in m.s$^{-2}$, but variations on the order of $10^{-8}$-$10^{-3}$ m.s$^{-2}$
- $g$ is usually expressed in Gals:
  - 1 Gal (for Galileo) = 1 cm.s$^{-2}$ = 10$^{-2}$ m.s$^{-2}$
  - 1 mGal = 10$^{-5}$ m.s$^{-2}$
  - 1 µGal = 10$^{-8}$ m.s$^{-2}$
- $g$ on the surface of the Earth $\sim$9.8 m.s$^{-2}$ = 982 000 mGal
On other planets…

- Table 2.1 L&V
Besides the Earth’s gravitation

- Previous formulas valid if only force = attraction of the mass of the Earth
- But the Earth’s rotates ⇒ 2 effects:
  - Centrifugal acceleration that opposes gravity
  - Deformation of the Earth: polar flattening
- Effects of other celestial bodies, in particular Moon and Sun:
  - Accelerations of the Earth on its orbit
  - Tides
Effect of the Earth’s rotation (1/2)

- Recall that for a spherical, fixed, homogeneous Earth:
  $$g = G \frac{M}{R^2}$$
  \((R = \text{mean Earth radius} = 6371 \text{ km})\)

- Angular rotation = \(\omega\)
  - Let us consider a plane parallel to the equator:
    - Centrifugal acceleration: \(a_c = \omega^2 r\)
    - Equator: \(a_c \max (r=R)\)
    - Pole: \(a_c = 0 (r=0)\)
  - Let us consider the spherical Earth:
    - Radial component of \(a_c\):
      \[a_r = a_c \cos \Phi = \omega^2 r \cos \Phi\]
    - Then:
      \[r = R \cos \Phi \implies a_r = \omega^2 R \cos^2 \Phi\]
Effect of the Earth’s rotation (2/2)

• Because of its rotation, the Earth is not a sphere but is flattened at the poles.

• The effect of the flattening on the gravity is:

\[
\frac{3GMa^2}{2R^4} J_2 \left( 3\sin^2 \Phi - 1 \right)
\]

– (see Turcotte and Schubert p. 199)
– \( J_2 \) = dimensionless coefficient that quantifies the Earth’s flattening \( (J_2 = 1.0827 \times 10^{-3}) \)
– \( a \) = equatorial radius = 6,378 km (polar radius = 6,357 km)
The Earth’s gravity

- Earth’s **gravitational acceleration** = sum of:
  - Earth’s gravitation (= “Newtonian” attraction)
  - Centrifugal acceleration
- Earth’s **gravity** = sum of:
  - Gravitational acceleration + centrifugal acceleration
  - Flattening correction
  
  \[
  g = \frac{GM}{R^2} - \frac{3GMA^2}{2R^4} J_2 \left(3\sin^2\Phi - 1\right) - \omega^2 R \cos^2\Phi
  \]

  - \(g\) depends on latitude only
- Is that all? No…
  - A point at the surface of the Earth is also submitted to the Newtonian attraction of celestial bodies (in particular Moon and Sun)
  - These formulas assume an homogeneous Earth (or spherical symmetry)
Gravity variations

• Consider a spherical volume near the Earth’s surface:
  – Radius $r = 100$ m
  – Depth to center $d = 200$ m
  – Density $\rho_1 = 2,000$ kg/m$^3$
  – Contribution of $m_1$ to the gravity field on the Earth’s surface?

• Substituting a different material in the same spherical volume:
  – Density $\rho_2 = 3,000$ kg/m$^3$ (basalt replacing sandstones)
  – Contribution of $m_2$ to the gravity field on the Earth’s surface?
Gravity variations

\[ g_{m_1} = G \frac{m_1}{d^2} = G \frac{\rho V}{d^2} \]

\[ V_{\text{sphere}} = \frac{4}{3} \pi r^3 \]

\[ \Rightarrow g_{m_1} = \frac{4G \rho_1 \pi r^3}{3d^2} \]

\[ \Rightarrow g_{m_1} = \frac{4 \times 7 \times 10^{-11} \times 2 \times 10^3 \times 3.14 \times 10^6}{3 \times 4 \times 10^4} \]

\[ \Rightarrow g_{m_1} \approx 14 \times 10^{-6} \text{ m.s}^{-2} = 1.4 \times 10^{-5} \text{ m.s}^{-2} = 1.4 \text{ mGal} \]

Density of \( m_2 = 1/3 \) greater than \( m_1 \):

\[ \Rightarrow \text{Contribution to the gravity field } 1/3 \text{ greater} \]

\[ \Rightarrow g_{m2} = 2.1 \text{ mGal} \]

Contributions of \( m_1 \) and \( m_2 \):
- \( m_1 \) ⇒ gravity below average Earth’s gravity
- \( m_2 \) ⇒ gravity below average Earth’s gravity

Small contributions:
- Average gravity on Earth \( \sim 10^6 \) mGals
- Contributions of \( m_1 \) and \( m_2 \) \( \sim 10^{-6} \)
- Precision needed to detect \( m_1 \) and \( m_2 \)
Gravity variations

- Increasing height above sea level:
  - Increasing distance from Earth center
  - Gravity decreases \( \frac{1}{r^2} \)

- What is the magnitude of this elevation effect on gravity?

(hint: differentiate \( g = GM/r^2 \))

Mount Everest: 8,830 m (~ 29,000 feet)
Gravity variations

\[ g = G \frac{M}{r^2} \]

\[ \Rightarrow \frac{dg}{dr} = -2G \frac{M}{r^3} \]

or:

\[ \frac{dg}{dr} = -2 \frac{g}{r} \]

\[ \Rightarrow \frac{dg}{dr} \approx -2 \frac{10}{6.4 \times 10^6} = -0.3 \times 10^{-5} \text{ m.s}^{-2} \text{ per m} \]

At 8,800 m (top of Mt. Everest), gravity variation (decrease):

\[ [0.3 \times 10^{-5}] \times 8,800 = 2640 \times 10^{-5} \text{ m.s}^{-2} = 2640 \text{ mGals} \]
Density

• Physical property of materials = mass/volume, unit = kg/m$^3$ (S.I., but g/cm$^3$ often used instead)

• Density variations of the Earth materials create gravity variations:
  – e.g. denser materials ⇒ higher gravity
  – Higher gravity ⇒ denser materials ??

• Density of some Earth’s materials: see Table

• Density depends on porosity, water content, temperature, and pressure.

• 3 easy to remember values:
  – Earth = 5,500 kg/m$^3$ = 5.500 g/cm$^3$
  – Continental crust = 2,670 kg/m$^3$ = 2.67 g/cm$^3$
  – Mantle = 3,300 kg/m$^3$ = 3,300 g/cm$^3$

<table>
<thead>
<tr>
<th>Material</th>
<th>Density</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dry sand</td>
<td>1.4 - 1.65</td>
</tr>
<tr>
<td>Wet sand</td>
<td>1.9 - 2.05</td>
</tr>
<tr>
<td>Sandstone</td>
<td>2.0 – 2.5</td>
</tr>
<tr>
<td>Salt</td>
<td>2.1 – 2.4</td>
</tr>
<tr>
<td>Shales</td>
<td>2.1 – 2.6</td>
</tr>
<tr>
<td>Limestone</td>
<td>2.4 – 2.8</td>
</tr>
<tr>
<td>Granit</td>
<td>2.5 – 2.7</td>
</tr>
<tr>
<td>Dolerite</td>
<td>2.5 – 2.7</td>
</tr>
<tr>
<td>Gneiss</td>
<td>2.65 – 2.75</td>
</tr>
<tr>
<td>Basalts</td>
<td>2.7 – 3.1</td>
</tr>
<tr>
<td>Gabbros</td>
<td>2.7 – 3.3</td>
</tr>
<tr>
<td>Peridotite</td>
<td>3.1 – 3.4</td>
</tr>
<tr>
<td>Coal</td>
<td>1.2 – 1.8</td>
</tr>
<tr>
<td>Oil</td>
<td>0.6 – 0.9</td>
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<tr>
<td>Sea water</td>
<td>1.01 – 1.05</td>
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<tr>
<td>Ice</td>
<td>0.88 – 0.92</td>
</tr>
<tr>
<td>Iron</td>
<td>7.3 – 7.8</td>
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<tr>
<td>Copper</td>
<td>8.8 – 8.9</td>
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<tr>
<td>Silver</td>
<td>10.1 – 11.1</td>
</tr>
<tr>
<td>Gold</td>
<td>15.6 – 19.4</td>
</tr>
</tbody>
</table>
What have we learned?

• Gravity is an acceleration
• Earth's gravity is caused by its gravitational attraction and its rotation
• Earth's gravity changes from place to place
• Changes reflect, for instance, rock densities, altitude, latitude
• Gravity is expressed in Gals (mGals, 1 mGal = $10^{-5}$ m.s$^{-2}$)