Summary so far

• Geodetic measurements $\rightarrow$ velocities $\rightarrow$ velocity gradient tensor (spatial derivatives of velocity)

• Velocity gradient tensor $=$ strain rate (sym.) $+$ rotation rate (antisym.)

• Strain rate tensor can be rotated into coordinate system of its principal axis $\rightarrow$ principal strain rates

• Strain rates typically calculated for polygons (assumes homogeneous strain in each polygon)

• End result: continuous description of the velocity field, to be interpreted in relation with:
  – Geological structures
  – Earthquakes and their mechanisms
  – Continuous versus block-like behavior
  – Link with dynamics?
Other quantitative strain indicators

- Strike, dip, rake (S,D,R)
  - Earthquake faulting
  - Fault slip
- Equivalent to a strain tensor = seismic moment, or “fault” moment:

\[
M_{yy} = -\sin D \cos R \sin(2S) - \sin(2D) \sin R \sin^2 S \\
M_{xx} = \sin D \cos R \sin(2S) - \sin(2D) \sin R \cos^2 S \\
M_{zz} = -M_{xx} - M_{yy} \\
M_{xy} = \sin D \cos R \cos(2S) + 0.5 \sin(2D) \sin R \sin(2S) \\
M_{yz} = -\cos D \cos R \cos(S) + \cos(2D) \sin R \sin S \\
M_{xz} = -\cos D \cos R \sin(S) + \cos(2D) \sin R \cos S
\]

- Recall that moment tensor = scalar moment x unit moment tensor:

\[
M_{ij} = M_o m_{ij}
\]

“magnitude” geometry
“Kostrov summation”

For a given volume or area, one can sum seismic or fault moment = “Kostrov summation” (from Kostrov V.V., Seismic moment and energy of earthquakes, and seismic flow of rocks, Izv. Acad. Sci. USSR Phys. Solid Earth, 1, Eng. Transl., 23-44, 1974):

- Seismic strain rates derived from seismic moment tensors:

\[ e_{ij} = \frac{1}{2\mu VT} \sum_{k=1}^{N} M_{ij}^{k} m_{ij}^{k} \]

where:
- \( N \) = number of events
- \( V \) = cell volume
- \( T \) = time period
- \( M_{ij} \) = seismic moment
- \( m_{ij} \) = unit moment tensor

- Geologic strain rates derived from slip on Quaternary faults:

\[ e_{ij} = \frac{1}{2A} \sum_{k=1}^{N} \frac{L_{ij}^{k} u^{k}}{\sin \delta^{k}} m_{ij}^{k} \]

where:
- \( N \) = number of fault segments
- \( L \) = fault segment length
- \( \delta \) = fault segment dip
- \( A \) = cell area
- \( u \) = slip rate (horizontal)
- \( m_{ij}^{k} \) = unit moment tensor from fault geometry


Result of Kostrov summation of earthquake moment tensors.
Strain beyond Delaunay…

• Delaunay triangulation: easy, but…
  – Assumes homogeneous strain in each triangle
  – Result depends on triangle size
  – Relies on dense coverage with data points, and homogeneous spacing: usually not the case…

• To avoid these problems, one can interpolate the velocity field on a grid, then calculate strain in each grid element: fancier, but…
  – What interpolation function to choose?
    • surface (GMT): “splines in tension” (Smith and Wessel, 1990)
    • Holt et al.’s method
    • England and Molnar’s method
    • Colocation method: not addressed here
  – Propagating errors becomes tricky…
The Holt et al.'s method

- Haines, Holt, Kreemer, Flesch
- Velocity field assumed continuous and parameterized as:

\[ \vec{v} = r \left[ \overrightarrow{W}(x) \times x \right] \]

- Where:
  - \( r \) = Earth’s radius
  - \( W(x) \) = rotation vector function
  - \( x \) = unit position vector
- Key element of the method: \( W(x) \) is expanded using a bi-cubic Bessel interpolation => allows for rapid variations of \( v \).
- Use a curvilinear grid.
- Spatial distribution of \( W(x) \) estimated using least-squares
- Spatial derivatives of \( W(x) \) can be imposed to be zero to force plate rigidity => 1 single plate with one single Euler pole
- Observables = geodetic velocities, plus strain rates from summation of seismic moment of fault slip rates
Application to Asia

![Map of Earthquakes in Asia (1900-1998)]
Application to Asia: Flesch et al., 2001

Principal strain rates derived from a joint inversion of GPS velocities + Quaternary slip rates

Corresponding model velocity field (w.r.t. Eurasia)
Hypothesis: surface deformation field results from sum of 2 contributions: Gravitational Potential Energy (GPE) stresses + plate boundary stresses

Step 1: calculate stresses due to GPE gradients (use geoid or crustal thickness or Airy isostasy)

Step 2: estimate boundary stresses using hypothesis above

Step 3: sum boundary and GPE stresses to get total deviatoric stresses

Step 4: calculate vertically averaged effective viscosity using stress = viscosity x strain rate
GPE stresses: Deviatoric stress field that satisfies force-balance equations, where sources of stress are potential energy differences inferred assuming local isostatic compensation of topography in Asia. Open white principal axes represent tensional stress. Black principal axes are compressional stress.

Best-fit boundary stress contribution from a single rotation of India relative to Eurasia that defines the India boundary (shown as open vectors) with three degrees of freedom.
Total stress field = sum of GPE stresses and boundary stresses.

Vertically averaged effective viscosity obtained dividing the magnitude of vertically averaged total stresses by the magnitude of average strain rates.

Flesch et al., 2001
Flesch et al. 2001: Conclusions

- Modeling procedure assumes force balance between GPS and boundary forces (although not explicitly)
- Assumes continuum, but “zones of high viscosity would behave as rigid blocks (Tarim, Amuria, south China)”
- Comparison with experimental results on rock mechanics => “a significant portion of the strength of the lithosphere resides within the seismogenic portion of the crust”
Your opinion?

• Pros:

• Cons:
An alternate approach: England and Molnar, 1997, 2005

• Divide area into arbitrary triangles: vertices do not have to match data points.

• Assuming:
  – Homogeneous strain within each triangle
  – Velocity varies linearly with latitude and longitude

• Then measured velocities can be modeled as (e.g. at point A):

\[
\mathbf{u}_A = \sum_{m=1}^{3} N_m^A \mathbf{u}_m
\]

\[
N_m^A = a_m + b_m x_A + c_m y_A
\]

\[
x, y = \text{coordinates of vertices } m=1,2,3
\]

\[N = \text{interpolation functions, defined using shape functions:}\]

\[
\begin{aligned}
  a_1 &= 1/3 \\
  b_1 &= (x_2 - y_3) / \Delta \\
  c_1 &= (x_3 - x_2) / \Delta 
\end{aligned}
\]

\((\Delta = \text{triangle area, rotate 1,2,3 to find other } a_2, \text{ etc.})\)
Strain rates

• Differentiating:

\[ u_A = \sum_{m=1}^{3} N_m^A u_m \]

• Gives:

\[ \varepsilon_{xx} = \frac{\partial u_{A,x}}{\partial x} = \sum_{m=1}^{3} b_m u_{m,x} \]
\[ \varepsilon_{yy} = \frac{\partial u_{A,y}}{\partial y} = \sum_{m=1}^{3} c_m u_{m,y} \]
\[ \varepsilon_{xy} = \frac{1}{2} \left( \frac{\partial u_{A,x}}{\partial y} + \frac{\partial u_{A,y}}{\partial x} \right) = \sum_{m=1}^{3} \frac{1}{2} (c_m u_{m,x} + b_m u_{m,y}) \]

• 2 series of linear equations relating observed velocities or strain rates to unknown velocities \( u_m \) => can be solved using least squares

• Observables:
  – GPS velocities
  – Strain rate components computed from:
    • Fault slip rates
    • Seismic moment tensor summation
E&M, 2005: data

Faults: compilation of Quaternary slip rates on major active faults (rate + sense of slip)

Velocities: 4333 **baselines** obtained by Delaunay triangulation of GPS sites from several published studies
E&M 2005: results and conclusions

- Use of Q slip rates only (no GPS): recovers far field motions => strain rates consistent from few years to several 10,000 years.

- Rigid behavior limited to small areas => “kinematics of rigid blocks does not provide a useful description of the motion”

Principal strain rates within triangular regions; contractional principal strain rates = thicker arms, extensional strain rates = thinner arms (calculated = open symbols; estimated = solid symbols). Grey areas = strain rate lower than $3 \times 10^{-9}$ yr$^{-1}$. 
E&M 2005: results and conclusions

Principal compressional strain axis align with topographic gradients => "suggests that Asia deforms as a continuum under the influence of gravity"

Principal compressional strain rates from the velocity field (black bars). Axes of principal horizontal contraction derived from moment tensors of earthquakes in Harvard centroid moment tensor catalogue between 1977 and 2003 (open bars). Grey lines are contours of surface height in km. Inset shows distributions of the moment magnitudes, $M_w$ of the earthquakes.
Your opinion?

• Pros:

• Cons:
Back to GPS data

• Rigorous combination of heterogeneous GPS solution into consistent geodetic frame
Triangulation and strain analysis

- The $3 \times 10^{-9} \text{ yr}^{-1}$ line roughly coincides with the 95% significance level => current precision limit => rigidity “threshold”
- A large part of Asia shows strain rates that are not significant at the 95% confidence level and are lower than $3 \times 10^{-9} \text{ yr}^{-1}$
- It remains true that principal compressional strain axis are perpendicular to topo gradients, but that does not imply continuous deformation…
Your opinion?

• Pros:

• Cons:
A global strain rate map

- Kreemer et al. (2003)
- Use 3,000 geodetic velocities from 50 different studies (mostly GPS)
- Differences in reference frame between studies are handled by solving for a rotation
- Solve for plate motions and strain in predetermined deforming zones
Conclusion

• Strain rate = continuous description of velocity field
• Independent of the reference frame since derived from velocity differences
• Related to:
  – Earthquakes: the higher the strain rate, the more seismic strain release => more and/or larger earthquakes
  – Stresses through constitutive relations => link with dynamics (stress and rheology)