A system dynamics model for a mixed-strategy game between police and driver

Dong-Hwan Kim* and Doa Hoon Kimb

Abstract

Game theorists have recommended many reasonable strategies in policy problems, using, in general, the concept of equilibrium strategy for analyzing the dynamic consequences of available policy options. One of the best known recommendations is that of George Tsebelis, which contends that increasing penalties is not a viable policy tool for decreasing the law-violation tendencies of drivers. This is because the interactions between police and driver can best be represented as a mixed strategy in which the players choose their alternative actions based upon a probability, but the probability of driver's law violation cannot be decreased by increasing the penalty. Our system dynamics model for a mixed-strategy game shows that it takes a very long time for a game-theoretic equilibrium to appear. Therefore, game players cannot, and should not, depend on the equilibrium for choosing their actions. Furthermore, our mixed-game model shows that an increase in penalty can induce compliance from the people, contradictory to the game-theoretic solution, but consistent with real-world behaviors. © 1997 by John Wiley & Sons, Ltd. Syst. Dyn. Rev. 13, 33–52, 1997

Feedback loops in the social world have been regarded as important research areas, not only for system dynamicists, but also for various kinds of social scientists. Social psychologists find feedback loops in self-fulfilling prophesy, group dynamics and social interactions (Weick 1979). Game theorists interpret feedback loops in terms of norms of reciprocity, interdependent choices and tit-for-tat (Komorita et al. 1992; Elster 1989; Axerlord 1984). In particular, game theorists have developed their ideas on reciprocal decisions with a formal logic. They have recommended many reasonable strategies for resolving domestic and international policy problems (Snidal 1985; 1991).

However, the methodology and propositions of game theorists are different to those of system dynamicists in many respects. While the building block of system dynamics modelling is a decision-making activity, game theory consists of a game situation, which is composed of pairs of players, preferences and strategies (Ordeshook 1986). While system dynamicists focus their attentions on dynamic fluctuations of a system, game theorists make efforts to find equilibrium states in game situations. In general, game theorists use a concept of equilibrium strategy for finding consequences of available policy options.

Game theory can be contrasted to decision theory. In decision theory, a decision maker is assumed to make his decision without considering the reactions of others to his decision, while in game theory, players change their decisions in response to...
other players’ actions. Players are supposed to exploit inferior decisions of others. Furthermore, if possible and desirable, players are assumed to violate laws and deceive other players. In these situations, explanations based on decision theory are prone to errors.

One of the most famous policy recommendations suggested by game theorists is that of George Tsebelis (1989). He contends that an increase in penalty against law violation is not a viable policy tool for decreasing the violation tendencies of drivers. This is because the interactions between police and drivers can be best represented as a mixed-strategy game in which each player chooses his or her alternative actions with a probability. In a mixed-strategy game between police and driver, the probability of a driver’s law violation cannot be decreased by increasing penalty against law violation.

In this paper, we propose a system dynamics model for a mixed-strategy game for investigating dynamic processes which are behind the game-theoretic equilibrium. Our SD model for a mixed strategy game shows that it takes a long time for a game-theoretic equilibrium to appear. Therefore, game players cannot and should not depend on the equilibrium state for choosing their actions. Furthermore, our mixed-game model shows that an increase in penalty can induce a compliance from the people. Our model shows behavior that is contradictory to the game-theoretic solution, but consistent with the real-world behavior. We have proposed that these differences between the SD model and the game model come from the lack of dynamic and transient behavior analyses in game theory. In order to generalize our insights from the simulation results, we make a qualitative analysis on the equation structure of our system dynamics model of a mixed-strategy game.

Equilibrium of a mixed-strategy game

For certain two-person games, such as the prisoner’s dilemma, there exist dominant strategies for game players. For other kinds of games, dominant strategies are not present. In these situations, game players often adopt a mixed strategy, in which they choose different actions at random. A runner in a baseball game has no dominant strategy for whether or not he should steal a base. The best runners would start to steal a base when no one can anticipate it. This is a mixed-strategy game. Tsebelis describes it for the case of games between the police and the driver (Tsebelis 1989):

You are driving your car and you are in a hurry ... there are two states of the world: either the police are nearby or they are not. There are two acts to choose from: either to violate the speed limit or to abide by the law. Again, there are four possible outcomes: (a) you can get a ticket for speeding, (b) you can get to work on time without any incident, (c) you can arrive late and avoid a ticket, and finally (d) you can arrive late though there were no policemen on the streets.
Two states mean the acts of other players: decisions of a policeman to patrol or not. Two acts are available to the driver. Four outcomes in this game result from the combination of two states and two acts. They correspond to the cells in the payoff matrix of Table 1. The entry in each cell is the vector of utilities for the policemen and the driver. In keeping with convention, the first entry in each cell \((a_1, b_1, c_1, d_1)\) denotes the driver's utility, the second \((a_2, b_2, c_2, d_2)\) denotes the policeman's utility. Note that \(a_1\) reflects the level of penalty for the driver's law violation.

For the utility of a driver, it is assumed that he would prefer not to speed if a policeman patrols \((c_1 > a_1)\) and he would prefer to speed if a policeman decides not to patrol \((b_1 > d_1)\). For the utility of a policeman, it is assumed that he would prefer to patrol if the driver decides to speed \((a_2 > b_2)\), while he would prefer not to patrol if the driver keeps to the speed limit \((d_2 > c_2)\).

In Table 1, neither the driver nor the policeman has a dominant strategy. If the policeman decides to patrol, the driver prefers not to speed; then the policeman prefers not to patrol and the driver chooses to speed. In order to maximize their expected utility, they should choose their acts with some probabilities. An equilibrium state in a mixed-strategy game means the state in which both players act with a pair of probabilities \((p^*, q^*)\) that can give them the greatest expected utilities (Ordeshook 1986). \(p^*\) is the probability with which the driver chooses to speed and \(q^*\) is the probability with which the policeman decides to patrol. Tsebelis (1989) gives \(p^*\) and \(q^*\) as:

\[
p^* = \frac{(d_2 - c_2)}{(a_2 - b_2 + d_2 - c_2)} \tag{1}
\]

\[
q^* = \frac{(b_1 - d_1)}{(b_1 - d_1 + c_1 - a_1)} \tag{2}
\]

Anyone can derive these formula quickly by maximizing the expected utilities of the driver and the policeman. Note that the probability of the driver's law violation in Eq. 1 is not determined by the payoffs for the driver in Table 1. This means that the penalty for the law-violation \((a_1\) in Table 1) has no effect on the probability of the driver's law violation. From this logic, Tsebelis concludes following theorems:

1. Under assumptions of \(c_1 > a_1, b_1 > d_1, a_2 > b_2\) and \(d_2 > c_2\), the only equilibrium in the police-public game is in mixed strategies as specified by Eqs. 1 and 2.
2. An increase in the penalty leaves the frequency of violation for the law at equilibrium \((p^*)\) unchanged.
3. An increase in the penalty decreases the frequency with which the police enforce the law at equilibrium \((q^*)\).
Table 1. Payoff matrix for policeman and driver

<table>
<thead>
<tr>
<th>Decisions of the driver</th>
<th>Decisions of the policeman</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed</td>
<td>Patrol</td>
<td>a1 a2</td>
</tr>
<tr>
<td></td>
<td>Not Patrol</td>
<td>b1 b2</td>
</tr>
<tr>
<td>Not speed</td>
<td>c1 c2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>d1 d2</td>
<td></td>
</tr>
</tbody>
</table>

Tsebelis concluded that these theorems seem to be contradictory to common sense. For most policy makers, an increase in penalty is conceived as one of the most effective tools for policy implementation. In Korea, the increase in penalty has vastly reduced the number of drunken drivers. In many countries, penalty management is a major policy implementation tool for inducing compliance from the people.

System dynamics model for a mixed-strategy game

Why does game theory produce theorems inconsistent with common sense and the real world? Two kinds of answer can be provided. First, we can reject the hypothesis that the interaction between policeman and driver is a game. Second, if we accept this interaction as a game, we can conjecture that there may be a difference between the equilibrium state of a mixed-strategy game and the real world. In this paper, we focus on the second possibility. We propose a system dynamics model for investigating the potential difference between the game-theoretic recommendation and the dynamics of social systems.

In converting a mixed-strategy game to a system dynamics model, one should preserve the probability of the players' behaviors. The probability of the players' behaviors can be conceptualized in two ways. First, one can interpret the probability of the players' behaviors in terms of the history of previous games. In this method of conceptualization, one can model the mixed-strategy game with two independent players (Kim and Kim 1995). Second, the probability of the players' behaviors can be conceptualized as the proportion of drivers violating the law and the proportion of policemen who patrol. One can call this conceptualization “a population mixed-strategy game”, in which the population as a whole is playing a mixed strategy, even though each individual player uses a pure strategy (Rosenthal 1979; Oechssler 1994). In this paper, we adopt the second method, because it can represent the behavioral changes of players more smoothly and continuously.

Figure 1 shows the STELLA diagram for a mixed-strategy game. We built the model to reflect decision-making processes of both kinds of players. In Figure 1, four stock variables denote the current distributions of policemen and drivers, from which probabilities of their acts can be calculated. Two rate variables, quit_patrol and go_patrol, represent the actions of policemen. Two other rate variables, speed_
up and speed_down repeat the decisions of drivers which change as the actions of policemen change.

Game-players, including policemen and drivers, calculate the expected value of their action, and take an action which will bring them higher utility. For example, if the expected value of patrol (e_up) exceeds the expected value of non-patrol, policemen tend to quit their patrol and go back to their offices. We introduce four table functions to represent how many people change their action according to the expected value of respective acts; patrol_to_office and office_to_patrol for policemen, conform_to_violation and violation_to_conform for drivers. We assume linear relationships between the expected values and the proportion of game-players changing their action, as shown in Figure 2. This assumption will simplify the qualitative analysis of the equation structure of our system dynamics model for a mixed-strategy game.

Eight constant variables correspond to eight payoffs in Table 1. They are summarized in Table 2, in which cells in the third column display values of payoffs assumed in the model. Payoff values are assumed to satisfy the conditions of a mixed-strategy game; $c_1 > a_1$, $b_1 > d_1$, $a_2 > b_2$ and $d_2 > c_2$.

In addition to these basic assumptions, we have added some assumptions for making the model clear and similar to the real world. We have assumed that dpvp (a1) is worst for the driver and dpvnp (b1) is best for the driver. While ppvnp (b2) is assumed worst for the police, ppvp (a2) is assumed best for them. One can
The equilibrium state in the model can be calculated by applying the Eqs. 1 and 2 to the payoffs of Table 2. We get the following probabilities for the behavior of game players:

\[ p^* = \frac{3 - 2}{4 - 1 + 3 - 2} = 0.25 \]

\[ q^* = \frac{4 - 2}{4 - 2 + 3 - 1} = 0.5 \]

We can examine the three theorems presented by Tsebelis by comparing this equilibrium state with model behaviors.
Simulation results

Oscillation rather than equilibrium

The first theorem of Tsebelis says that a mixed-strategy game has an equilibrium state where two players act with the probabilities calculated by Eqs. 1 and 2. However, Eqs. 1 and 2 say nothing about the processes by which two players move to the equilibrium state. Even if the equilibrium state is unique and stable, the path for reaching it should be analyzed. If the path is filled with fluctuations and dissatisfactions, players will give up using the mixed strategy.

Figure 3 shows behavior of the policemen and the drivers during the simulation, with the tendency of game-players to move towards the equilibrium state. However, paths for the equilibrium states are filled with oscillations. To get to the equilibrium state, they should overcome the fluctuating behavior of other players. However, the adjustment time necessary to reach the equilibrium state is more than three years.

In order to examine the existence of the equilibrium states, we have simulated the model to run 10,000 times. We found that the fluctuations shown in Figure 3 disappear in the long run.

The effectiveness of penalty increase

In order to examine the second and third theorems of Tsebelis, we have increased the penalty for violating a speed limit. That is, we have reduced the value of dpvp (driver payoff for violation with police patrol) from 1 to -4. The equilibrium state can be calculated by applying Eqs. 1 and 2 as follows:

\[ p^* (\text{probability of driver's violation}) = \frac{3 - 2}{4 - 1 + 3 - 2} = 0.25 \]

\[ q^* (\text{probability of police patrol}) = \frac{4 - 2}{4 - 2 + 3 + 4} = 0.222 \]

An increase in the penalty reduces the equilibrium probability of patrol from 0.5 to 0.222, but it does not change the equilibrium probability of violation as stated in the second and third theorems of Tsebelis.

Now we introduced the penalty increase at the 300th day to examine its effects on the evolutionary process of a mixed game. Figure 4 shows the time behavior of game players produced in this simulation. The thick line in Figure 4 denotes the increase in the penalty.

In Figure 4, it is clear that the penalty increase changes the system behavior dramatically. The behavior of drivers is vastly changed by the increase in the penalty. The probability of violation is reduced to under 0.25 for a considerable period (about 50 days). Of course, drivers change their violation behavior toward
the equilibrium probability. However, the effects of the penalty increase continue for a considerable period, which may be long enough for the police to believe in the effect of the penalty increase. The fact that game players come back to the equilibrium state after 50 days does not say that the increase in the penalty cannot change the behavior of the driver. The fact is that an increase in the penalty decreases the violation tendency of the driver for more than 50 days, which may be a critical period for policy makers.
The effects of information delay

In the previous SD model, in order to simplify discussions about the game-theoretic equilibrium, we did not include information delays. It is reasonable to assume, however, that there are information delays between the police and the drivers. Now, we have assumed that it takes 10 days for drivers to perceive the changes in the behavior of the police while the police perceive the change of drivers’ behavior within five days. In the model two smooth functions were used to represent information delays for a driver and a policeman in figuring out the probability of the other players’ actions.

Figure 5 displays the simulation results. The response cycles of the players are prolonged and have larger amplitude than the model without information delay. Figure 5 suggests that the equilibrium state may never appear. This is due to the slow adaptiveness of players resulting from the information delay.

With the information delays, it is more difficult for the game players to reach equilibrium actions, and we can conjecture that the penalty increase will change the behavior of the driver for a longer time. Figure 6 shows simulation results with the penalty increase introduced on the 300th day. Compared to Figure 5, it can be seen that the probability of drivers' law violation is slightly reduced.

Effectiveness of automatic penalty management

In our experiments, penalty increase seems to reduce the probability of law violations. However, our experiments deal with a one-time, sudden increase in the penalty for law violations. A one-time increase of the penalty cannot be a promising
tool for reducing law-violation behavior, because drivers and the police adapt to the changed penalty-system environment.

According to our knowledge of the informal rule of police patrolling, the police change the amount of penalty in line with the probability of changes in law violation. If drivers frequently violate the speed-limit law, the police tend to increase the level of the penalty. And if few drivers violate the law, the police impose a low penalty on the violators. The SD model can represent this flexible penalty management system by relating the dpvp variable to the probability of violations.

Figure 7 shows STELLA diagram for automatic penalty management system, which dpvp is calculated as follows:

- \( \text{dpvp} = 1 + \text{penalty_increase} \)
- \( \text{penalty_increase} = \text{perceived_prob_v} \times -5 \)

The amount of penalty for drivers violating the speed-limit law goes to -4 as the probability of violation becomes 1.

Figure 8 shows the effectiveness of the automatic penalty management system applied to the game model without information delays. It can be seen that an equilibrium state is reached after a short period of fluctuations. Contrary to the theorems of game theory, which claim no effectiveness for penalty management, the game-theoretic equilibrium can be attained only through the automatic management of penalties.

Figure 9 shows the performance of the automatic penalty management system in the SD model with information delays. With the introduction of information...
Fig 7. STELLA diagram for automatic penalty management system

Fig 8. The effects of the automatic penalty management system
The automatic penalty management system shows a cyclic behavior in which the game-theoretic equilibrium is never attained. However, if Figure 9 is compared with Figure 5 and 6, it can be seen that the automatic penalty management system performs well in reducing the fluctuating behaviors of both drivers and the police.

Qualitative analysis on an SD model of mixed-strategy game

In this section we will carry out a qualitative analysis on our SD model of the mixed-strategy game. With the help of qualitative analysis, we can generalize the policy implications obtained in the previous simulations and understand the mathematical structure of the mixed-strategy game.

Differential equations of our model of a mixed-strategy game

In our modelling of a mixed-strategy game, we assumed that the number of drivers and policemen who change their action is determined by a linear function of the expected values of violation and patrolling. With this assumption, we can formulate the rate variables in the model as functions of the expected value. If we note that the rate variables in the model can be interpreted as changing rates for the probabilities of violation and patrol, we can construct first-order differential equations for the probability of violation and patrol as follows:

\[
\begin{align*}
\frac{dv}{dt} &= [(a_1 - b_1 - c_1 + d_1)p + b_1 - d_1]F(v) \\
\frac{dp}{dt} &= [(a_2 - b_2 + d_2 - c_2)v + c_2 - d_2]G(p)
\end{align*}
\] (3)

Fig 9. The effects of automatic penalty management system with information delays.
where

\[ F(v) = 1 - v, \quad p \leq \frac{b_1 - d_1}{c_1 - a_1 + b_1 - d_1} \]
\[ F(v) = v, \quad p > \frac{b_1 - d_1}{c_1 - a_1 + b_1 - d_1} \]
\[ G(p) = 1 - p, \quad v > \frac{d_2 - c_2}{a_2 - b_2 + d_2 - c_2} \]
\[ G(p) = p, \quad v \leq \frac{d_2 - c_2}{a_2 - b_2 + d_2 - c_2} \]

where \( p \) and \( v \) correspond to the probabilities of patrol and violation respectively. The left-hand side of Eqs. 3 is simply a formula for calculating the expected value of patrol and violation. When the expected value of violation is greater than zero, drivers who have kept to the speed limit will change their action, which can be represented by migration from the conformance population to the violation population. In this case, the increasing rate for a probability of violation can be represented by multiplication of the expected value and the current probability of conformance \( F(v) = 1 - v \), because, in our model, the proportion of drivers changing their action is determined by the linear function of the expected value. If we apply the same logic to the decreasing rate for the probability of violation, it can be formulated by multiplying the expected value and the current probability of violation \( F(v) = v \), \( G(p) \) in Eqs. 3 thus also alternates between \( p \) and \( 1 - p \).

We can simplify Eqs. 3 by introducing constant values of \( a, b, c \) and \( d \).

\[ a = c_1 - a_1 + b_1 - d_1 > 0 \]
\[ b = b_1 - d_1 > 0 \]
\[ c = a_2 - b_2 + d_2 - c_2 > 0 \]
\[ d = d_2 - c_2 > 0 \]

Now, we can arrange four pairs of differential equations according to different values of \( p \) and \( v \) as shown in Table 3.

The dynamic behaviors of this system can be visualized in the nullcline diagram of Figure 10. It can be seen that four pairs of differential equations are simply Lotka–Volterra equations with different parameters.¹

In Figure 10, it can be seen that the probabilities for the drivers and the police
change at the equilibrium point of \((p,v) = \left(\frac{b}{a}, \frac{d}{c}\right)\). Although a Lotka-Volterra equation shows unstable cyclic behavior, the previous results of our simulation indicate that the mixed-strategy game shows converging behavior. We must analyze our differential equations in order to determine whether or not the convergence toward an equilibrium point is a general property of our SD model of a mixed-strategy game.

### Spiral convergence toward an equilibrium point

To analyze the stability of a critical point \((p,v) = \left(\frac{b}{a}, \frac{d}{c}\right)\), we substitute \(p\) as \(y + \left(\frac{b}{a}\right)\) and \(v\) as \(x + \left(\frac{d}{c}\right)\) in all equations of Table 3. We can then construct Table 4.

All eigenvalues for the linearized system of Table 4 are pure complexes:

\[
\pm \sqrt{\frac{cb}{a}}(1 - \left(\frac{d}{c}\right))^{\frac{1}{2}}, \pm \sqrt{\frac{ac}{a}}(1 - \left(\frac{b}{a}\right))(1 - \left(\frac{d}{c}\right))^{\frac{1}{2}}, \pm \sqrt{\frac{bd}{a}}^{\frac{1}{2}}, \pm \sqrt{\frac{ad}{a}}(1 - \left(\frac{b}{a}\right))^{\frac{1}{2}}.
\]

We can easily show that the trajectories of the linearized system are closed ellipses, with the equilibrium point \((p, v) = \left(\frac{b}{a}, \frac{d}{c}\right)\) as center (Figure 11).

If we consider only the linear terms in Table 4, the probabilities of violation and
Table 4. Differential equations after moving the equilibrium point to the origin

<table>
<thead>
<tr>
<th></th>
<th>$x \leq 0$</th>
<th>$y &gt; 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y \leq 0$</td>
<td>$dx/dt = (-ay)(1 - x - d/c)$</td>
<td>$dx/dt = (-ay)(1 - x - d/c)$</td>
</tr>
<tr>
<td></td>
<td>$dy/dt = (cx)(y + b/a)$</td>
<td>$dy/dt = (cx)(1 - y - b/a)$</td>
</tr>
<tr>
<td>$y &gt; 0$</td>
<td>$dx/dt = (-ay)(x + d/c)$</td>
<td>$dx/dt = (-ay)(x + d/c)$</td>
</tr>
<tr>
<td></td>
<td>$dy/dt = (cx)(y + b/a)$</td>
<td>$dy/dt = (cx)(1 - y - b/a)$</td>
</tr>
</tbody>
</table>

Fig 11. Trajectories of the linearized system and the vector forces of the nonlinear terms

patrol will show cyclic behavior and they will never approach the equilibrium point. However, as eigenvalues of the linearized system are pure complex, a small perturbation in the nonlinear part of the system will change the trajectories of the system. Now we can take into account the nonlinear terms in Table 4, the vector forces of which are not reflected in the ellipses.

In Figure 11, it can be seen that all the vector forces resulting from nonlinear terms move the system toward an equilibrium point. This analysis shows that the mixed-strategy game will ultimately reach its equilibrium point. Note that no learning mechanism is introduced into our system dynamics model of the mixed-strategy game. With the assumption of a linear function between the expected values and the proportion of game players changing their action, the equilibrium point of the mixed-strategy game becomes an asymptotically stable point.

Furthermore, Figure 11 provides insights on how fast the system will approach the equilibrium point. Trajectories of the system toward equilibrium are determined by two kinds of forces. The linear terms of the system induce the system to move around the ellipses, while the nonlinear terms of the system move the system towards the central equilibrium point of the ellipses. Since $p$ and $v$ have values between 0 and 1, the forces towards the equilibrium point produced by the nonlinear terms are relatively weak compared to the forces generated by the linear
terms. That is why our model of the mixed-strategy game was so slow in reaching its equilibrium point.

The slow and spiral convergence to the equilibrium point explains why the penalty increase lowers the probability of violation for a long period without immediate counter-balancing the change in police behavior. The adjustments between the police and drivers are so slow that the effect of the penalty increase continues until the system enters the spiral movement around the new equilibrium point, as shown in Figure 12.

The differential equations of the mixed-strategy game can also be used to explain why the information delay produces a spiral divergence from the equilibrium point, as displayed in Figure 5. The introduction of information delay changes the nullclines of Figure 10 into those of Figure 13. The shaded area of Figure 13 represents the drivers’ misperception of the probability of patrol; even though the probability of patrol is below the critical point of $b/a$, drivers perceive it as greater than $b/a$ and the vectorized forces of movement of drivers and the police move the system away from the equilibrium point within the shaded area.

Our previous simulation results showed that the equilibrium state of a mixed-strategy game with information delay may be never attained. In this situation of continuous cycling around the equilibrium point, the important point for the policy makers is the amplitude of these cycles, rather than the equilibrium point itself. Our simulation results show that penalty management will not change the equilibrium point itself, but can reduce the amplitude of the fluctuating behavior of violation.

Our qualitative analysis on the system dynamics model for a mixed-strategy game does not prove our simulation results in rigorous terms. However, the qualitative analysis implies that our simulation results and their implications for policy
nullclines are not dependent on the specific parameter values assumed in the model.

Policy implications and conclusions

The most important policy implication of our simulation results is that one can, at least temporarily, reduce the violation tendency of drivers by changing the size of the penalty. Another important policy implication is that penalty management can decrease the amplitude of fluctuating behavior of drivers, even under the assumption of information delays.

From the perspective of policy makers, whose interests are not in the long run the state of the system but in performance usually measured in short terms, temporary reductions of speed limit violation will be a sufficient incentive for them to introduce the measure of penalty increase. On the other hand, the policy of penalty management can be justified by the fact that it can reduce the amplitude of fluctuating violations. If we admit that the actual probability of car accidents is affected by the maximum level of violations, rather than by the average probability of speed violation, the policy of penalty management can contribute to reducing the number of car accidents.

Our simulation results and qualitative analysis show that a mixed-strategy equilibrium may be a poor guide in dynamic systems where various kinds of policy interruptions and information delay play crucial roles. This comes from the fact that a game-theoretic equilibrium lacks any analysis of transient behavior. With the help of game theory, one can find the equilibrium state. However, game theory has its limits in showing various paths to the equilibrium state. In the real world, where
game players are myopic and have limited resources, paths to the equilibrium state are more important than the equilibrium point itself. We think that it is too dangerous to judge the effects of penalty increase only on the basis of the game-theoretic equilibrium. The effects should be analyzed within the evolutionary processes toward equilibrium, in which many unanticipated events arise.

We suggest that integration of game theory and system dynamics will provide a safe basis for describing and recommending policies in the real world. System dynamics will simulate evolutionary processes of games toward (non-)equilibrium states, while game theory provides frameworks for modeling the world, which is filled with both competition and cooperation.

Note


References

Appendix: Model equations

\[ \text{drivers\_in\_conforming}(t) = \text{drivers\_in\_conforming}(t - \Delta t) + (\text{speed\_down} - \text{speed\_up}) \times \Delta t \]
\[ \text{INIT drivers\_in\_conforming} = 500 \]

INFLOWS:
\[ \text{speed\_down} = \text{violation\_to\_conform} \times \text{drivers\_in\_violation} / \text{speed\_down\_time} \]

OUTFLOWS:
\[ \text{speed\_up} = \text{conform\_to\_violation} \times \text{drivers\_in\_conforming} / \text{speed\_up\_time} \]

\[ \text{drivers\_in\_violation}(t) = \text{drivers\_in\_violation}(t - \Delta t) + (\text{speed\_up} - \text{speed\_down}) \times \Delta t \]
\[ \text{INIT drivers\_in\_violation} = 500 \]

INFLOWS:
\[ \text{speed\_up} = \text{conform\_to\_violation} \times \text{drivers\_in\_conforming} / \text{speed\_up\_time} \]

OUTFLOWS:
\[ \text{speed\_down} = \text{violation\_to\_conform} \times \text{drivers\_in\_violation} / \text{speed\_down\_time} \]

\[ \text{policemen\_in\_patrolling}(t) = \text{policemen\_in\_patrolling}(t - \Delta t) + (\text{go\_patrol} - \text{quit\_patrol}) \times \Delta t \]
\[ \text{INIT policemen\_in\_patrolling} = 50 \]

INFLOWS:
\[ \text{go\_patrol} = \text{office\_to\_patrol} \times \text{policemen\_in\_the\_office} / \text{patrolling\_time} \]

OUTFLOWS:
\[ \text{quit\_patrol} = \text{patrol\_to\_office} \times \text{policemen\_in\_patrolling} / \text{quitting\_time} \]

\[ \text{policemen\_in\_the\_office}(t) = \text{policemen\_in\_the\_office}(t - \Delta t) + (\text{quit\_patrol} - \text{go\_patrol}) \times \Delta t \]
\[ \text{INIT policemen\_in\_the\_office} = 50 \]

INFLOWS:
\[ \text{quit\_patrol} = \text{patrol\_to\_office} \times \text{policemen\_in\_patrolling} / \text{quitting\_time} \]
OUTFLOWS:

$go\_patrol = office\_to\_patrol\cdot policemen\_in\_the\_office\cdot patrolling\_time$

$\alpha\_difference\_eup\_eunp = eup\_eunp$

$\alpha\_difference\_euv\_eunv = euv\_eunv$

$\alpha\_dpctype = 2$

$\alpha\_dpcp = 3$

$\alpha\_dpvnp = 4$

$\alpha\_dpvvp = 1\cdot\alpha\_\text{penalty\_increase}$

$\alpha\_eup = \text{perceived\_prob\_v}\cdot\alpha\_ppvnp\cdot(1-\text{perceived\_prob\_v})\cdot\alpha\_ppcnp$

$\alpha\_eunv = \text{perceived\_prob\_p}\cdot\alpha\_dpctype\cdot(1-\text{perceived\_prob\_p})\cdot\alpha\_dpcp$

$\alpha\_eup = \text{perceived\_prob\_v}\cdot\alpha\_ppvvp\cdot(1-\text{perceived\_prob\_v})\cdot\alpha\_ppcpc$

$\alpha\_eunv = \text{perceived\_prob\_p}\cdot\alpha\_dpvvp\cdot(1-\text{perceived\_prob\_p})\cdot\alpha\_dpvnp$

$\alpha\_patrolling\_time = 10$

$\alpha\_\text{penalty\_increase} = \text{perceived\_prob\_v}\cdot5$

$\alpha\_\text{perceived\_prob\_p} = \text{SMTH3}(\alpha\_\text{prob\_p}, 10)$

$\alpha\_\text{perceived\_prob\_v} = \text{SMTH3}(\alpha\_\text{prob\_v}, 5)$

$\alpha\_\text{ppcnp} = 3$

$\alpha\_\text{ppcpc} = 2$

$\alpha\_\text{ppvnp} = 1$

$\alpha\_\text{ppvvp} = 4$

$\alpha\_\text{prob\_p} = \text{policemen\_in\_patrolling}/100$

$\alpha\_\text{prob\_v} = \text{drivers\_in\_violation}/1000$

$\alpha\_\text{quitting\_time} = 10$

$\alpha\_\text{speed\_down\_time} = 10$

$\alpha\_\text{speed\_up\_time} = 10$

$\otimes\text{conform\_to\_violation} = \text{GRAPH}(\alpha\_\text{difference\_euv\_eunv})$

$0.00, 0.00, 0.4, 0.1, 0.8, 0.2, 1.20, 0.3, 1.60, 0.4, 2.00, 0.5, 2.40, 0.6, 2.80, 0.7, 3.20, 0.8, 3.60, 0.9, 4.00, 1.00$

$\otimes\text{office\_to\_patrol} = \text{GRAPH}(\alpha\_\text{difference\_eup\_eunp})$

$0.00, 0.00, 0.4, 0.1, 0.8, 0.2, 1.20, 0.3, 1.60, 0.4, 2.00, 0.5, 2.40, 0.6, 2.80, 0.7, 3.20, 0.8, 3.60, 0.9, 4.00, 1.00$

$\otimes\text{patrol\_to\_office} = \text{GRAPH}(\alpha\_\text{difference\_eup\_eunp}^*-1)$

$0.00, 0.00, 0.4, 0.1, 0.8, 0.2, 1.20, 0.3, 1.60, 0.4, 2.00, 0.5, 2.40, 0.6, 2.80, 0.7, 3.20, 0.8, 3.60, 0.9, 4.00, 1.00$

$\otimes\text{violation\_to\_conform} = \text{GRAPH}(\alpha\_\text{difference\_euv\_eunv}^*-1)$

$0.00, 0.00, 0.4, 0.1, 0.8, 0.2, 1.20, 0.3, 1.60, 0.4, 2.00, 0.5, 2.40, 0.6, 2.80, 0.7, 3.20, 0.8, 3.60, 0.9, 4.00, 1.00$