HW #2 Solution

1. (a) Midband gain: \( V_o = G_m \cdot U_i \cdot R_L = G_m \cdot \frac{R_i}{R_s + R_i} \cdot U_i \cdot R_L \)
   
   \( A_m = \frac{V_o}{U_i} = \frac{R_i}{R_s + R_i} \cdot G_m \cdot R_L \)
   
   (b) \[
   \begin{align*}
   \omega_{ph} &= \frac{1}{C_L R_L} \\
   \omega_{pl} &= \frac{1}{C_c (R_s + R_x)}
   \end{align*}
   \]
   
   \[ H(s) = \frac{A_m}{(1 + \frac{s}{\omega_{pl}})(1 + \frac{s}{\omega_{ph}})} \]

   (c) \[
   \frac{100k}{20k + 10k} \cdot G_m \cdot 10k = 10
   \]
   \[ G_m = \frac{100}{100k} \cdot \frac{10}{10^6} = 1.2 \text{ [mA/V]} \]

2. (a) \( V_o = -G_m \cdot U_i \cdot R_L = -G_m \cdot \frac{1}{\beta_m + R_s} \cdot U_i \cdot R_L \)
   
   \[ A_v = -\frac{R_L}{\beta_m + R_s} \]

   (b) [Diagram of a circuit with labels and components]
\[ i_t = \frac{V_t - V_S}{(V_S G_S)} \]
\[ i_t + j_m (V_t - V_S) = \frac{V_S}{R_S} \]
\[ V_S = V_t - \left( \frac{1}{G_S} \right) i_t \]
\[ i_t + j_m V_t = (B_m + \frac{j}{R_S}) (V_t - \left( \frac{1}{G_S} \right) i_t) \]
\[ 1 + \left( B_m + \frac{j}{R_S} \right) \left( \frac{1}{G_S} \right) i_t = \left( -B_m + \left( \frac{j}{R_S} + B_m \right) \right) V_t \]

\[ Z_m = \frac{V_t}{i_t} = R_S \left[ 1 + \left( \frac{B_m}{R_S} \right) \left( \frac{1}{G_S} \right) \right] = R_S + (1 + B_m R_S) \left( \frac{1}{G_S} \right) \]

\[ Y_m = \frac{1}{R_S + \frac{1 + B_m R_S}{G_S}} = \frac{S G_S R_S + 1 + B_m R_S}{S G_S} \]

\[ = \frac{S G_S (1 + B_m R_S - j \omega G_S R_S^2)}{(1 + B_m R_S)^2 + \omega^2 G_S^2 R_S^2} \]

\[ = \frac{\omega^2 G_S^2 R_S + j \omega G_S (1 + B_m R_S)}{(1 + B_m R_S)^2 + \omega^2 G_S^2 R_S^2} \]

\[ Z_{m*} = \frac{1}{\frac{1}{Y_{m*}}} = \frac{1}{\frac{R_m}{G_m}} = \frac{1}{\frac{1}{G_m}} = \frac{1}{G_m} \]

\[ \text{Impedance of } m \text{th } \]

\[ R_m = \frac{(1 + B_m R_S)^2 + \omega^2 G_S^2 R_S^2}{\omega^2 G_S^2 R_S} \]

\[ = R_S + \frac{(1 + B_m R_S)^2}{\omega^2 G_S^2 R_S} \]

\[ \text{capacitance of } m \text{th } \]

\[ C_m = \frac{G_m (1 + B_m R_S)}{(1 + B_m R_S)^2 + \omega^2 G_S^2 R_S^2} \]

\[ \text{capacitance of } m \text{th } \]
\# Input pole
\[ R_s \frac{1}{\frac{1}{L} \frac{R_{\text{in}}}{(1-A_n) C_{\text{gd}} + \frac{1}{C_{\text{in}}}}} \]
\[ \omega_{p,\text{in}} = \frac{1}{(R \parallel R_{\text{in}}) \left[ (1-A_n) C_{\text{gd}} + \frac{1}{C_{\text{in}}} \right]} \]
\[ \Leftrightarrow \text{Main pole} \]

\# Output pole
\[ \omega_{p,\text{out}} = \frac{1}{R_{\text{in}} C_{\text{gd}}} \]

\[ H(s) = \frac{A_n}{(1 + \frac{s}{\omega_{p,\text{in}}})(1 + \frac{s}{\omega_{p,\text{out}}})} \]

(c)

<table>
<thead>
<tr>
<th>( R_s )</th>
<th>( A_n )</th>
<th>( \omega_{p,\text{in}} )</th>
<th>G.B.W = ( A_n \times \frac{\omega_{p,\text{in}}}{2\pi} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>100</td>
<td>( \times 50 )</td>
<td></td>
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</tbody>
</table>
3. Midband gain \( A_v = -g_m R_D \)

\[
\begin{align*}
W_p &= \frac{1}{C_s (R_s \parallel g_m)} , \quad W_{p2} \approx \frac{1}{(C_{gd} + C_{db}) R_D} \\
W_Z &\approx \frac{1}{C_s R_s}
\end{align*}
\]

\[
H(s) \approx A_v \cdot \frac{(1 + \frac{s}{W_Z})}{(1 + \frac{s}{W_P}) (1 + \frac{s}{W_{P2}})}
\]

4. Midband gain \( A_v = (-g_m R_D) (-g_m (R_0 \parallel R_L)) = g_m^2 R_0 \cdot R_L \)

# Poles

\[
\begin{align*}
W_{p1} \approx & \quad \frac{1}{R_s [1 + g_m R_D) C_{gd1} + C_{gd1}]}
\quad \text{dominant!} \\
W_{p2} \approx & \quad \frac{1}{R_D [1 + g_m R_D) (g_{d2} + C_{gd2}]}
\quad \text{can be ignored}
\quad \text{optional}
\end{align*}
\]

\[
H(s) = \frac{A_v}{(1 + \frac{s}{W_{p1}})(1 + \frac{s}{W_{p2}})(1 + \frac{s}{W_{p3}})}
\]
$$\lambda_i \rightarrow C_{g_{d1}} \rightarrow U_i \rightarrow C_{g_{d2}} \leftarrow i_o$$

\[i_o = g_m U_g + \frac{U_i - U_g}{1/sC_{gd_1}}\]

\[\frac{U_i - U_g}{1/sC_{gd_1}} = \frac{U_g}{1/s(C_{gs} + g_{ma} + C_{gd_2})}\]

\[\frac{U_g}{1/sC_{gd_2}} + i_o = g_m U_g \Rightarrow \left(2C_{gd} - g_m\right)U_g = -i_o\]

\[U_g = \frac{i_o}{g_m - 2C_{gd}}\]

\[\sigma \quad 2C_{gd} U_i = (2C_{gd} + 5C_{gs} + 5(C_{gd} + sC_{gd})) U_g\]

\[= 2 \left(2C_{gd} + C_{gs}\right) \frac{i_o}{g_m - 2C_{gd}} \Rightarrow U_i = \frac{2\left(C_{gd} + C_{gs}\right) i_o}{g_m - 2C_{gd}}\]

\[\sigma \quad \lambda_i = \left(\frac{g_m - 2C_{gd}}{g_m - 2C_{gd}}\right) i_o + sC_{gd} \cdot \frac{2\left(C_{gd} + C_{gs}\right) i_o}{g_m - 2C_{gd}}\]

\[= \left(1 + \frac{2\left(C_{gd} + C_{gs}\right)}{g_m - 2C_{gd}}\right) i_o = \frac{g_m + 2s(C_{gs} + C_{gd}) - 2C_{gd}}{g_m - 2C_{gd}} i_o\]

\[\sigma \quad H(s) = \frac{\lambda_i(s)}{\lambda_o(s)} = \frac{g_m - 2C_{gd}}{g_m + 2s(C_{gs} + C_{gd}) - 2C_{gd}}\]

\[= \frac{g_m - 2C_{gd}}{g_m + 2s(C_{gd} + 2C_{gs})} = \frac{g_m - 2C_{gd}}{g_m + s(C_{gd} + 2C_{gs})}\]
6. 

\[ A_U = -g_{m1} \cdot R_{o3} \]

# Poles

\[
\begin{align*}
\omega_{PA} &= \frac{1}{\left(\frac{1}{g_{m1}}\right) \left(C_1 + C_{gs2}\right)} \\
\omega_{PB} &= \frac{1}{R_{o3} \cdot C_2}
\end{align*}
\]

\[
H(s) = \frac{A_U}{\left(1 + \frac{s}{\omega_{f1}}\right) \left(1 + \frac{s}{\omega_{f2}}\right)}
\]

\[ Z_{out} = \frac{1}{2} \left( R_{o2} \cdot g_{m2} \cdot R_{o1} \right) \]

\[
\Rightarrow A_U = -g_{m1} \cdot \frac{1}{2} \cdot R_{o1} \cdot R_{o2} \cdot g_{m2} = -\frac{1}{2} g_{m1} \cdot g_{m2} \cdot R_{o1} \cdot R_{o2}
\]

(b) 

\[
\omega_P = \frac{1}{\frac{1}{2} \left( R_{o1} \cdot R_{o2} \cdot g_{m2} \right) \left(C_L + C_{gd2} + C_{dd2}\right)}
\]

\[
\approx \frac{1}{\frac{1}{2} \left( R_{o1} \cdot R_{o2} \cdot g_{m2} \right) C_L}
\]
8. (a) $R_{in}$ is extremely large

\[
\omega_{p1} = \frac{1}{\left( \frac{1}{j \omega} \right) \left[ (1 + g_{m2} \cdot R_{ox}) \cdot g_{d2} + g_{ss2} \right]}
\]

\[
\omega_{p2} = \frac{1}{R_{ox} \cdot C_L}
\]

dominant

and most significant